# ANALYTIC HIERARCHY PROCESS AS A RANKING TOOL FOR DECISION MAKING UNITS

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**ABSTRACT**

The paper aims in application of analytic hierarchy process (AHP) for efficiency analysis of the set of decision making units (DMUs). Conventional tool for analysis of efficiency of DMUs is data envelopment analysis (DEA). DEA models allow splitting the set of DMUs into two subsets – efficient and inefficient. The inefficient ones can be ranked according to their efficiency scores given by the DEA model but the efficient units cannot be ranked easily as their have identical maximal efficiency score. Many models have been proposed for ranking of efficient units in DEA models. The aim of the paper is to discuss the possibility of application of AHP models for ranking of efficient or entire set of DMUs. Numerical experiments are realized on the set of 20 commercial banks operating on the Czech financial market. The study is based on a real data set containing financial characteristics of the banks.

Keywords: analytic hierarchy process, data envelopment analysis, efficiency, ranking

## 1. Introduction

Efficiency analysis is an important task that is widely discussed among researchers and practitioners. Efficiency of a unit is measured and analyzed in many various ways from simple financial characteristics and ratios to complex analytical and modelling tools especially developed for this purpose. Most often group of models for efficiency analysis is data envelopment analysis. Its advantage consists in the simplicity of the models and in that the efficiency is evaluated relatively in the group of other units without necessity to deliver any additional information or preferences. The DEA models have many disadvantages that are discussed by many researchers. The decision maker cannot specify any his/her preferences even there are some modifications of conventional DEA models where it is possible in a certain way.

Evaluation of efficiency is always based on multiple criteria and can be considered as multiple criteria decision making problem. The aim of this paper is to verify whether the AHP can be a suitable alternative in evaluation of efficiency to conventional DEA models.

## 2. Literature Review

General and most often used modelling tool for efficiency analysis is data envelopment analysis. Since 1978 when fundamentals of DEA models were introduced by Charnes et al. (1978) this group of models went through a long development. As the conventional DEA models are not able to discriminate among efficient units one direction of this development consists in proposal of models for ranking of efficient or all units under evaluation.

The AHP (and ANP) is one of the most successful tools for analysis of complex decision making problems (and efficiency evaluation belongs among complex problems). The fundamentals of AHP and a detailed description of this method can be found e.g. in (Saaty, 1990).

Several attempts how to join the mentioned two modelling principles – DEA and AHP – in order to evaluate the efficiency in a more convenient way have been done in the past. Adler et al. (2002) present a review of ranking methods including their original procedure for ranking of efficient units based on both AHP and DEA principles. (Jablonsky, 2007) introduces an AHP procedure for ranking of efficient units. Zarei et al. (2012) developed an integrated AHP-DEA model for evaluation of efficiency.

## 3. Hypotheses/Objectives

The main aim of this study is to develop a simple general AHP based model for efficiency analysis of even large data sets in case of presence of several decision makers. The results of the model will be verified on an example with a real economic background – evaluation of efficiency of the set of commercial banks operating on the Czech financial market.

## 4. Research Design/Methodology

Data envelopment analysis (DEA) is a non-parametric technique for evaluation of relative efficiency of decision making units characterized by multiple inputs and outputs. Let us suppose that the set of decision making units (DMUs) contains $n$ elements. The DMUs are evaluated by $m$ inputs and $r$ outputs with input and output values $x_{ij}$, $i = 1, 2, \ldots, m$, $j = 1, 2,\ldots, n$ and $y_{kj}$, $k = 1, 2,\ldots, r$, $j = 1, 2,\ldots, n$, respectively. The efficiency of the $q$-th DMU can be expressed as the weighted sum of outputs divided by the weighted sum of inputs with weights that reflect the importance of single inputs $v_i$, $i = 1, 2,\ldots, m$, and outputs $u_k$, $k = 1, 2,\ldots, r$ as follows:

$$\theta_q = \frac{\sum_{k=1}^{r} u_k y_{kq}}{\sum_{i=1}^{m} v_i x_{iq}}.$$

Standard CCR input oriented DEA model formulated in (Charnes et al., 1978) consists in maximization of efficiency score of the DMU$_q$ subject to constraints that efficiency scores of all other DMUs are lower or equal than 1. The linearized form of this model is as follows:

Maximize

$$\theta_q = \sum_{k=1}^{r} u_k y_{kq}$$

subject to (1)

$$\sum_{i=1}^{m} v_i x_{iq} = 1,$$

$$\sum_{k=1}^{r} u_k y_{kj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \ j = 1,2,\ldots,n,$$

$$u_k, v_i \ge \varepsilon, \qquad k = 1,2,\ldots,r, i = 1,2,\ldots,m.$$

Model (1) identifies some of the units as efficient with their efficiency scores 1 (100%), the remaining ones are inefficient with lower values of efficiency scores. In order to rank efficient units (not only them) we suggest the following AHP model – see Figure 1. The hierarchy of the models consists of four levels. The topmost level is the goal of evaluation which is efficiency analysis of DMUs. The second level contain decision makers. We suppose that theoretically *s* decision makers with different priorities and suppose that all of them have identical importance. The third level contains criteria, i.e. inputs and outputs in terminology of DEA. The criteria are not evaluated directly by decision makers but they are shaped into pairs, i.e. all pairs financial ratios output divided by input. So, the number of criteria $p = m.r$, where $m$ is the number of inputs and $r$ the number of outputs. The importance of criteria according to the DMs preferences are given by pairwise comparisons – let us denote them $v_{il}$,$i = 1,2,\ldots,p$, $l = 1,2,\ldots,s$. The number of criteria is a limit for efficient application of the proposed model – the total number of inputs and outputs cannot exceed 6 in order the pairwise comparison matrix is of an acceptable size. This limit is not too restricting because the total number of inputs and outputs in real studies usually does not exceed the given value.

Figure 1 AHP model – efficiency analysis.

The last level of hierarchy contains the DMUs. As the number of DMUs can be quite high it is not possible to use the AHP model with relative measurement but the absolute measurement must be used. Let us suppose that all criteria (ratios) are evaluated in identical five-element scale – excellent, above average, average, below average and poor. According to the pairwise comparisons they all have the following vector of preference indices (0.510, 0.264, 0.130, 0.064, 0.033). The partial preference index of the *i*-th DMUs according to the *j*-th criterion and the *l*-th DM is denoted as $w_{ijl}$. Global preference indices of the *i*-th DMUs is given as a simple sum of preference indices $w_{ijl}$:

$$u(DMU_i) = \sum_{j=1}^{p} \sum_{l=1}^{s} w_{ijl}, \quad i = 1,2,\ldots,n. \tag{2}$$

The indices (2) allow final ranking of all DMUs under evaluation.

| | Prof/Eq. | Prof/FTE | Dep/Eq. | Dep/FTE | AHP | DEA-CCR |
|---|---|---|---|---|---|---|
| $DMU_1$ | 1.02 | 1.41 | 0.68 | 1.41 | 0.033 (20) | 0.128 (19) |
| $DMU_2$ | 21.26 | 15.90 | 14.07 | 15.91 | 0.361 (3) | 2.354 (1) |
| $DMU_3$ | 3.73 | 2.11 | 2.61 | 2.23 | 0.097 (14) | 0.216 (16) |
| $DMU_4$ | 4.97 | 1.68 | 12.22 | 6.25 | 0.085 (16) | 0.260 (15) |
| $DMU_5$ | 14.57 | 4.81 | 60.32 | 30.08 | 0.328 (4) | 0.790 (7) |
| $DMU_6$ | 9.77 | 6.67 | 112.73 | 116.40 | 0.371 (2) | 1.206 (4) |
| $DMU_7$ | 2.20 | 1.83 | 1.44 | 1.82 | 0.054 (19) | 0.160 (18) |
| $DMU_8$ | 0.07 | 0.14 | 13.03 | 40.91 | 0.054 (18) | 0.110 (20) |
| $DMU_9$ | 2.35 | 4.39 | 9.57 | 27.12 | 0.119 (12) | 0.540 (10) |
| $DMU_{10}$ | 2.20 | 13.11 | 6.00 | 54.03 | 0.192 (11) | 1.634 (3) |
| $DMU_{11}$ | 1.85 | 0.48 | 50.91 | 19.72 | 0.055 (17) | 0.161 (17) |
| $DMU_{12}$ | 0.90 | 1.71 | 24.92 | 71.59 | 0.095 (15) | 0.347 (14) |
| $DMU_{13}$ | 8.17 | 0.92 | 225.47 | 38.59 | 0.222 (10) | 0.632 (9) |
| $DMU_{14}$ | 5.56 | 1.51 | 242.81 | 99.42 | 0.291 (6) | 1.649 (2) |
| $DMU_{15}$ | 6.16 | 4.83 | 55.38 | 65.67 | 0.264 (7) | 0.394 (12) |
| $DMU_{16}$ | 18.27 | 5.22 | 215.22 | 92.97 | 0.442 (1) | 1.010 (6) |
| $DMU_{17}$ | 2.57 | 2.88 | 23.73 | 40.22 | 0.104 (13) | 0.374 (13) |
| $DMU_{18}$ | 14.82 | 3.23 | 169.10 | 55.71 | 0.322 (5) | 1.087 (5) |
| $DMU_{19}$ | 3.49 | 5.86 | 22.55 | 57.27 | 0.256 (9) | 0.641 (8) |
| $DMU_{20}$ | 5.29 | 2.73 | 557.03 | 433.85 | 0.257 (8) | 0.509 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | (11) |

Table 1 AHP/DEA data set and results.

## 5. Data/Model Analysis

This section contains information about results of numerical experiments realized on the real data set of 20 Czech commercial banks. Each of the DMUs is described by two inputs – equity in millions of CZK (Czech crowns) and the number of full time employees (FTE) – and by two outputs – profit (millions of CZK) and the number of deposit and credit accounts. Due to the limited space for this abstract it is not possible to present the original data set. Instead of this we present in Table 1 all financial ratios outputs/inputs. So, we have together 4 criteria with the following vector of weights (0.388, 0.277, 0.175, 0.159). The elements of this vector are given as a simple average of three vectors estimated by three decision makers using pairwise comparisons. As the comparison matrix is 4x4 only, there was not any problem with its consistency. All three matrices were deeply under the recommended bound. Application of the AHP model presented in Figure 1 leads to global preference indices for all DMUs. They are presented in the pre-last column of Table 1 including the final ranking of DMUs. The results given by AHP model are compared with efficiency scores computed using the DEA CCR model (1). The efficiency scores for efficient DMUs are greater than 1 because a super-efficiency model that allows ranking of units was applied.

Comparison of results shows that there is a quite tight dependence between both rankings. It holds for all units with some exceptions but they can be easily explained. The strongest difference holds for $DMU_{10}$. This unit is identified as efficient by the CCR model and finally is on the third place when the super-efficiency model is applied. The AHP model ranks this unit much farther. What is the reason? $DMU_{10}$ has a second highest ratio profit per one FTE. Even the other ratios are below average this fact leads to high efficiency of this unit by CCR model. It is clear that the conclusion given by the AHP analysis is closer to the real expectations.

## 6. Limitations and conclusions

The study presented in this article is not a serious case study even it is based on the real data set published by the commercial banks. Its aim is showing the possibility for application of simple AHP model for efficiency analysis of the DMUs and refer to limits of both approaches. DEA results are based on objective data only and certain level of subjectivity is missing when they are applied and when the results are presented do DMs without a very deep analysis. The AHP model is based on subjective DMs' evaluations. On one hand it is a positive feature, on the other hand it places greater demands on DMs. In all cases both approaches – DEA and AHP – can usefully complement.

## 7. Key References

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