CONSISTENCY IN THE CONTEXT OF AHP:

HALF FRIEND, HALF FOE

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ABSTRACT

A general measure of the comparative judgments' goodness in building up decision matrices within the AHP is consistency, measured through the random index. Indeed, for a consistent, positive, reciprocal matrix, every element is the ratio of the correspondent components in the eigenvector associated with the highest correspondent eigenvalue, thus allowing a perfect match for the interpretation of each element in a decision matrix as being a comparison-expressed as a ratio- of the criterions' weights in the associated priority vector. Starting from Saaty's famous examples in which the decision matrices with small random indices were showing an almost perfect fit between the estimated priority vector and the real ones, this algebraically general construction was almost unanimously recognized as an ideal for every decision matrix. A quest for improving consistency in every decision matrix, either through the achievement of consensus or through other methods, led to the development of two main directions, linguistic and bold consistency. This paper is raising two questions. First is about how relevant are Saaty's famous examples. It is thus illustrated that in term of the relationship between the consistency ratio and the standard deviation of the associated priority vector, achieving consensus in these consecrated examples were highly improbable events. Second, following the spirit of Saaty's famous examples, some experiments in which the true priority vectors are known are replicated on a larger scale, in order to see whether improving on the consistency is indeed conducing to a closer match between the priority vector and the true one. Results achieved so far revealed the fact that only in approximate one half of the decisions matrices, the effort of improving consistency (either linguistic or bold) led to a priority vector closer to the true one.

Keywords: consistent decision matrices, priority vector, linguistic consistency

1.Linguistic versus Bold Consistency

Improving consistency is n constant focus in order to ensure that the interpretation given to each of the elements in the DM matches the interpretation of their correspondent elements in the priority vector. Whether improving consistency is shifting closer in the discovery of the true priority vector is a question with some partial answers. Out of these, the most important are Saaty's famous examples (see Saaty,2010). This paper is looking at the relationship between the consistency and the priority vector. In this context, Saaty's famous examples appear isolated. Therefore, following the spirit of these examples, an experiment is designed in which five geometrical areas have to be compared. For each decision matrix two more are derived, having their consistency improved. The methods of improving consistency follow the two main directions of research, namely linguistic consistency (LC) and bold consistency (BC).Linguistic consistency, as introduced in Dong,Y,,Xu,Y.,Li,H,Dai,M (2008) is improved using simulated Annealing algorithm, as in Agapie,Ad. (2012). Bold consistency follows Benitez et. all (2011). In section 2 the relationship between the consistency ratio and the priority vectors is introduced together with the probability of consent. Section 3 illustrates the relevance of one of Saatys' examples. Section 4 introduces and experiment to check more the relationship between the consistency index and the root square error of the current priority vector with respect to the true. Section 5 conclude the paper.

2. The relationship between the consistency ratio and the standard deviation of the associated priority vector

In the following the experiment described in Saaty (2010) for estimating the R.I. is replicated 1,000 . The addition is that for each decision along with the computation of the correspondent CR it was also computed the associated priority vector(P.V).The figure 2.1. below match 1,000 replications for the R.I. as in Saaty (2010) and show the plot the CR versus the correspondent PV's standard deviation, for DM's whose dimension vary from 3 to 8.

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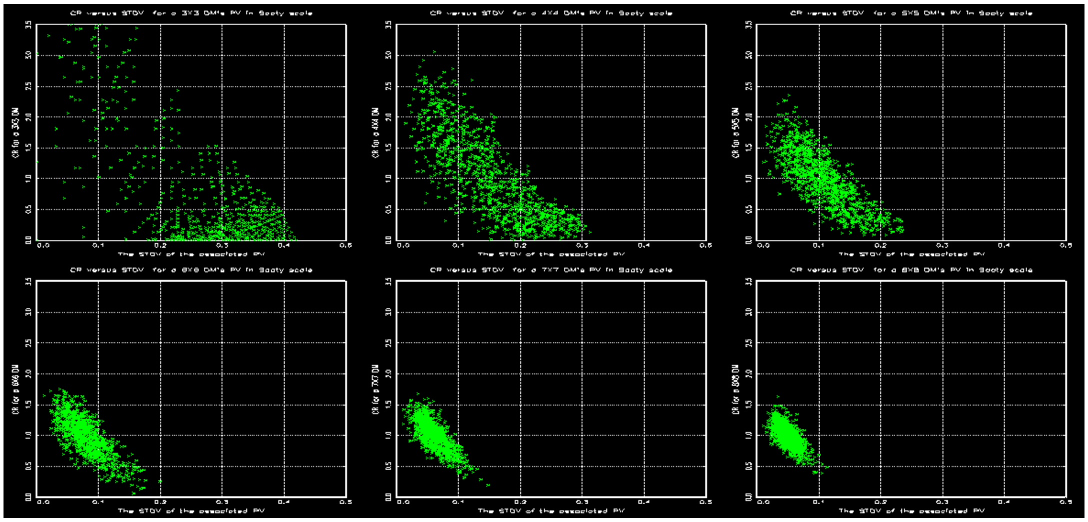


As D.M.'s dimensions are increasing from 3 to 8, the striking shrinkage in terms of the range of standard deviation point to the fact that priority vectors having, for instance, a standard deviation greater than 0.1 and corresponding to D.M.'s of dimensions greater than 6 are quite a rare eventMore, it can be observed that for D.M.s of dimensions like 6,7 or 8 the consistency ratio is hardly becoming smaller than 0.1.So, if one has to compare 6 or more alternatives in order to derive a priority vector it is almost impossible to come to a consistent consent in the sense that the C.R. for the DM is less than 0.1. To partially conclude the findings in terms of standard deviation and CR, it is advisable if six or more alternatives are to be compared, to form clusters of 3, 4 or at maximum 5 alternatives and afterwards eventually apply the cluster method as in Saaty (2011).

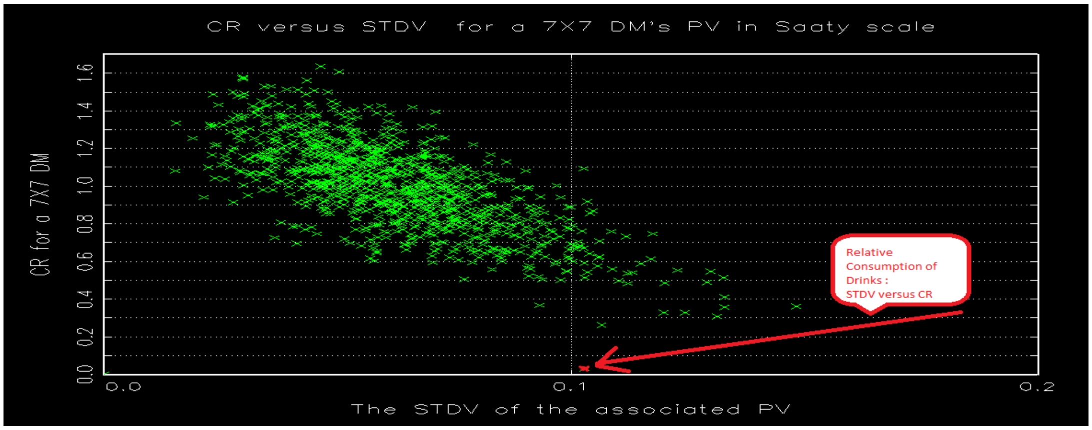
In the context of the experiment designed initially for estimating R.I. , the "*probability of consent*" (P.$Cs_{.(n,n)}$) for a (n,n) DM is introduced as the ratio of the number of (n,n) matrices having a C.R. less than 0.1 over the total number of generated (n,n) D.M.'s.
In table 2.2. are estimated the probabilities of consent for DM's whose dimensions vary from 3 to 8, corresponding to 1000 and respectively 10,000 replications.
Table 2.2.

Probability of consent $PCs_{(n,n)}$	1,000 replications	10,000 replications
(3,3) DM	0.2100	0.2047
(4,4) DM	0.0320	0.0280
(5,5) DM	0.0030	0.0019
(6,6) DM	0.0010	0.0000
(7,7) DM	0.0000	0.0000
(8,8) DM	0.0000	0.0000

3. Consistent Consent: how hard can it be? Saaty's most famous examples revisited.

The relative consumption of drinks example revisited

For the example regarding "The Relative Consumption of Drinks" as in Saaty (2010, page 131), for a 7x7 decision matrix, although the standard deviation of both true relative consumption (from statistical sources) and the priority vector corresponding to the associated DM are respectively 0.1053 and 0.1029 , which point to a priority vector in the range of Saaty's scale, the consistency ratio is 0.029 and this is pointing to a position quite isolated on the corresponding above graph.

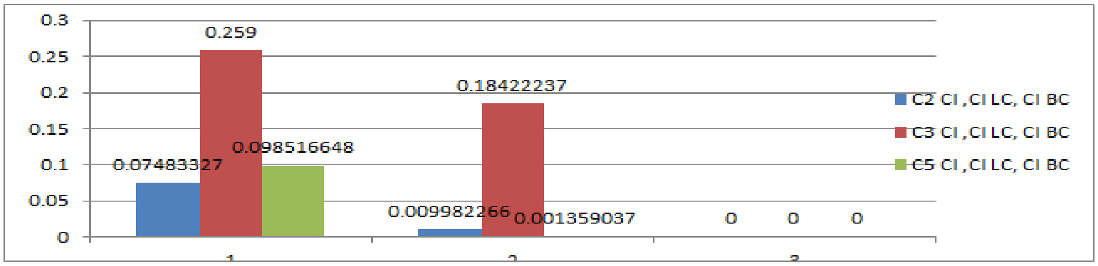


Improving consistency, either linguistic or bold is not producing a priority vector closer to the true than the initial one. More, linguistic consistency in this case is delivering a worse priority vector than one corresponding to the bold consistency.

4.Improving linguistic versus bold consistency in comparing geometrical areas: An experiment

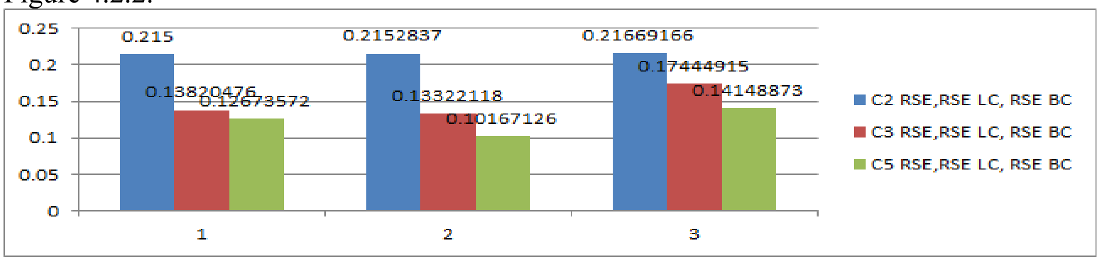
A hundred students were individually asked to compare the areas of five geometrical figures. There were 10 groups of five different figures ($C_1, C_2, \ldots C_{10}$) , so that 10 students answered individually to comparative questions regarding the area of the same set of five geometrical figures. For every set of five figures out of the ten constructed it was available the true priority vector of the corresponding set of the five areas. Below are shown the findings after processing one response for each of the set of five figures C_2, C_3 and C_5 above. In Figure 4.2.1 are the consistency indices for the decision matrix as it were derived out of three individual students' responses for each of the C2,C3 and C5 group of five geometrical figures , together with their linguistic and bold consistency versions. The last consistency index is zero, no matter the decision matrix.

Figure 4.2.1



In Figure 4.2.2 are shown the corresponding RSEs with respect to the true priority vectors. For the response associated with the five geometrical figures 'group C_2 there is almost no change in the RSE, indicating that the closeness to the true priority vector is decoupled from the improvements in consistency index. The individual response associated with group C_3, show a slight closure to the true priority vector when linguistic consistency is improved and a slight increase in the distance from the priority vector corresponding to the bold version to the true priority vector. As for the response corresponding to the five figure's C_5 it seems that an ideal zero consistency index is pushing further away the corresponding priority vector from the true one.

Figure 4.2.2.



The results above have to be completed for the whole 100 responses acquired during this experiment.

5. One look back, a glance ahead

The connection between the consistency and the priority vectors was traced by depicting the relationship CI versus stdv(PV) for decision matrices of dimensions 3 to 8. Some of Saaty's famous examples in which an almost perfect match with the true priority vector was put in correspondence to a small consistency index appear to be isolated. Further, probability of consent was introduced and estimated for decision matrices of dimensions in between 3 to 8.A preliminary conclusion after these estimations were performed was that it is rather indicated to apply the cluster method when more than 5 alternatives are to be compared. Following the line of Saaty's most famous examples, an experiment was designed and the relationship between the improvements in the consistency index and the closure to the true priority vector was reexamined. Results so far show that improving consistency is not a guarantee to coming closer to the true answer.

6. Key References

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