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ON GETTING MEANINGFUL "BOCR" RESULTS WITH ANP'S SUPER DECISIONS SOFTWARE

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Abstract: *This note builds on a previous theoretical investigation of the validity of the synthesis of composite priorities on benefits, opportunities, costs and risks that were obtained from separate Analytic Hierarchy or Network models. More specifically, this note uses different monetary scenarios with known results that are all based on a common set of priorities on each of these four merits, and investigates whether or not the Super Decisions software of the Analytic Network Process actually reproduces these known results by formulaic synthesis. It appears that straightforward use of the software does not necessarily produce correct results. The note analyses the reasons for this and suggests ways of improvement.*

Keywords: ANP, benefits-opportunities-costs-risks analysis, BOCR, commensurateness, weights, formulaic synthesis

1. Introduction

It has already been shown (Wijnmalen, 2006) that combining composite results from separate benefits, opportunities, costs, and risks models can produce overall results that are ambiguous. Using an example with known monetary results, it was shown with different scenarios that, regardless of the mathematical formula for synthesis, the results can be meaningless or even deceiving (by incorrectly suggesting profitability of alternatives) and contradictory (showing rank reversals with results from more or less equivalent synthesis expressions). All this may lead to bad decisions.

That paper suggested, supported by numerical examples, that it is crucial to express priorities on benefits, opportunities, costs and risks in commensurate terms before synthesizing. It argued that although any set of weights reflecting relative importance can express priorities on a common priority scale, it is only a weighting scheme based on relative B, O, C and R magnitudes that will not only create commensurate priorities but also allow a sound profitability analysis, equivalent to monetary break-even analysis. These weights were called "rescaling weights".

In the ANP literature, for example (Saaty, 2001) and the ANP software package (Super Decisions, 2005) several mathematical formulas have been suggested for synthesis of separate B, O, C, and R priorities: multiplicative (unweighted, or with "personal" weights as powers), weighted additive using (normalized) inverse values ("reciprocals") of the C and R priorities, and weighted additive using negative values of the C and R priorities.

(Wijnmalen, 2006) argued however that in a return on investment ratio oriented analysis, a synthesis formula should be used that is a quotient of positives (B and O) to negatives (C and R) with the rescaling weights as coefficients, not powers. The revised synthesis formula comes in two variants: one where rescaled (and thus commensurate) benefit and opportunity priorities are *added* in the nominator (and the same with the rescaled cost and risk priorities in the denominator), and one where they are *multiplied*. Multiplication however produces a new unit that has no intuitive interpretation whereas addition has the advantage of keeping the original unit in both the nominator and denominator and should be recommended for this reason. Using a priority synthesis formula where negatives appear as *reciprocals* was not recommended, even if they are made commensurate by weights that take the magnitude of reciprocal values into account. This type of formula can reproduce monetary results, but does not help assess the profitability of alternatives based on B, O, C, and R. In a recent book (Saaty, 2005; section 3.3) Saaty recommends not using this formula.

Further, in a net value oriented analysis, an additive synthesis formula should be used where rescaled cost and risk priorities are *subtracted* from rescaled benefits and opportunities priorities.

Finally, if additional weights based on personal values are at all to be used to take account of feelings of relative importance of the four factors, (Wijnmalen, 2006) suggested that they should be applied to the rescaled priorities.

The results of all formulas mentioned above (with rescaling weights) produce final priorities of the alternatives that should *not* be normalized, in order to allow a profitability analysis. This is particularly important with multiplication, as the rescaling weights cancel out with normalization!

This note will focus on actually computing synthesized BOCR results with the ANP software package Super Decisions (2001). The same numerical (monetary) example as in the previous paper will be used to investigate whether or not the software reproduces the known results when applying three mathematical formulas that are pre-defined in the software.
However, the previously recommended quotient formula with addition of merits in the nominator and denominator does not yet exist in the software and will therefore not be considered in this note.

This note will not question a BOCR analysis per se, nor will it address the way in which composite alternative priorities on each of the four merit factors (B, O, C, R) are computed using network models. It is rather the computational method for getting meaningful synthesized results that is the note's subject. It is validation oriented and therefore investigates the way in which the composite priorities of alternatives on each of the four factors (B, O, C, R) are synthesized into a final BOCR value and the results thereof, in comparison with known monetary results. It turns out that reproduction of monetary results will, unfortunately, not necessarily occur.

The remainder of this note is organized as follows. First, the reference example is introduced, next a straightforward application of the software is reported showing that the known monetary results are not reproduced. This is followed by a suggestion of how to proceed correctly regarding the BOCR weights and how to reproduce the referent results by an additional calculation or by editing the software's pre-defined formula. The note ends with a final discussion and a recommendation to adjust the Super Decisions software.

2. Reference example of BOCR analysis

In order to investigate the methods for synthesizing the composite priorities on each of the four BOCR factors, the same example is taken as in (Wijnmalen, 2006) which, in turn, was drawn from (Saaty, 2001; section 5-7] involving the development of a condominium.

The computations that produce the final priorities on each of the BOCR factors are of no concern here. Unlike the original example, in this note the weights of the BOCR factors proper will not be weighted by "personal values" implied by strategic criteria. The normalized composite priorities of the alternatives on each of the factors, and the normalized inverse priorities on costs and risks, are shown in Table 1. This table is based on Tables 5-26 and 5-27 in (Saaty, 2001).

Table 1: Alternative priorities (from underlying models) on BOCR factors and equal BOCR factor weights

Composite priorities on factors	Benefits (0.25) B_p	Opportunities (0.25) O_p	Costs (0.25) C_p	Risks (0.25) R_p	1/Costs (0.25) $1/C_p^*$	1/Risks (0.25) $1/R_p^*$
Alt. A1	0.097	0.112	0.191	0.167	0.514	0.552
Alt. A2	0.461	0.356	0.391	0.374	0.251	0.247
Alt. A3	0.442	0.532	0.418	0.459	0.235	0.201
sum	1.000	1.000	1.000	1.000	1.000	1.000

Table 2: Monetary scenarios represent different situations, but all generate Table 1's priorities; best alternative value shown in bold, normalized values shown between brackets

	Alt. A1	Alt. A2	Alt. A3	Total
Benefits in \$ (B_m)	388	1844	1768	4000
Opportunities in \$ (O_m)	224	712	1064	2000
Scenario 1: Costs in \$ (C_m)	1528	3128	3344	8000
Risks in \$ (R_m)	334	748	918	2000
Ratio $(B_m*O_m)/(C_m*R_m)$ *Unprofitable alternatives*	0.170 (0.126)	0.561 (0.417)	**0.613 (0.456)**	
Net value $(B_m+O_m-C_m-R_m)$ *Unprofitable alternatives*	**-1250 (-0.313)**	-1320 (-0.33)	-1430 (-0.357)	
Add. recipr. $(B_m+O_m+1/C_m+1/R_m)$ *No profitability indication*	612.004 (0.102)	2556.002 (0.426)	**2832.001 (0.472)**	
Scenario 2: Costs in \$ (C_m)	1528	3128	3344	8000
Risks in \$ (R_m)	167	374	459	1000
Ratio $(B_m*O_m)/(C_m*R_m)$ *Mixed profitability of alternatives*	0.341 (0.127)	1.122 (0.417)	**1.226 (0.456)**	
Net value $(B_m+O_m-C_m-R_m)$ *Unprofitable alternatives*	-1083 (-0.361)	**-946 (-0.315)**	-971 (-0.324)	
Add. recipr. $(B_m+O_m+1/C_m+1/R_m)$ *No profitability indication*	612.007 (0.102)	2556.003 (0.426)	**2832.002 (0.472)**	
Scenario 3: Costs in \$ (C_m)	1146	2346	2508	6000
Risks in \$ (R_m)	16.7	37.4	45.9	100
Ratio $(B_m*O_m)/(C_m*R_m)$ *Profitable alternatives*	4.541 (0.127)	14.964 (0.417)	**16.341 (0.456)**	
Net value $(B_m+O_m-C_m-R_m)$ *Mixed profitability of alternatives*	-550.7 (-0.55)	172.6 (0.172)	**278.1 (0.278)**	
Add. recipr. $(B_m+O_m+1/C_m+1/R_m)$ *No profitability indication*	612.06 (0.102)	2556.027 (0.426)	**2832.022 (0.472)**	
Scenario 4: Benefits in \$ (B_m)	388	1844	1768	4000
Opportunities in \$ (O_m)	560	1780	2660	5000
Costs in \$ (C_m)	1528	3128	3344	8000
Risks in \$ (R_m)	334	748	918	2000
Ratio $(B_m*O_m)/(C_m*R_m)$ *Mixed profitability of alternatives*	0.426 (0.127)	1.403 (0.417)	**1.532 (0.456)**	
Net value $(B_m+O_m-C_m-R_m)$ *Mixed profitability of alternatives*	-914 (-0.686)	-252 (-0.189)	**166 (0.125)**	
Add. recipr. $(B_m+O_m+1/C_m+1/R_m)$ *No profitability indication*	948.004 (0.105)	3624.002 (0.403)	**4428.001 (0.492)**	

There are many sets of monetary values that generate the priorities in Table 1. In Table 2 four monetary scenarios are shown. The first three are characterized by the same benefit and opportunity amounts but different amounts for costs and risks; the fourth is equal to the first scenario except for the opportunity values that are two-and-a-half times as much. All scenario sets of monetary values produce the (unweighted) priorities shown in Table 1. Table 2 also shows the results of a monetary ratio, a monetary net value, and a monetary additive value for each alternative. The latter one is the additive formula with reciprocals. Although even criticized by (Saaty, 2005), it is included as it is still one of the synthesis formulas in the available software. Moreover, it is shown that even this formula can reproduce known monetary results (provided that these are based on reciprocals as well) if the correct procedure is followed as suggested in this note.

Scenario 1 represents a situation where all alternatives are unprofitable both in a return on investment ratio analysis (with the unity value as the break-even point) and a net value oriented analysis (with the zero value as the break-even point). Scenarios 2 and 4 represent a mixed situation where all alternatives are unprofitable except for A3 in a return on investment ratio analysis (their values are larger than 1.0) and A2 in both types of analysis. In scenario 3 all alternatives are profitable except A1 in a net value analysis (negative value). Which is the best alternative depends therefore on the scenario and the type of analysis conducted (see the bold-faced values in Table 2). A3 appears to be the best in a ratio analysis independently of the scenario used, but this may not be generally valid. It is only in scenarios 3 and 4 that all synthesis results point at the same best alternative A3.

In the previous paper the purpose of the first three scenarios was to demonstrate that although different as regards the profitability of the alternatives in monetary terms, the priorities based on them are equal and the various synthesis formulas could not reproduce the monetary results and their differences across the scenarios without very specific rescaling weights. That investigation was done without the Super Decisions software. In this note it is investigated how the Super Decisions software should be used computationally to actually reproduce the monetary results.

3. Straightforward use of Super Decisions

In Super Decisions we first create a top level network which consists of a cluster containing the goal node and a second cluster containing the four merit nodes: Benefits (B), Opportunities (O), Costs (C) and Risks (R). There are no strategic criteria in the top level network, which means that any weighting of the four merit nodes should be done directly with respect to the goal node. The BOCR nodes have sub-networks attached to them. Each sub-network is a decision network: it contains a cluster with the specific merit goal and a cluster with the three alternatives. Figure 1 shows the complete model. Notice that there are no inner or outer dependence or feedback links in the model, in conformity with the narrow focus of this note. This is one of the types of network model where "formulaic synthesis" is allowed using the pre-defined formulas.

The next step is evaluating the alternatives on each of the merits. This is relatively easy: we take the priority values from Table 1 and use the software's "Direct data entry" option to input the data.

Next the BOCR nodes should be weighted. We do not use "personal values" based on strategic criteria as there are not any. The straightforward way in which to proceed would be to keep the weights equal. We know however from (Wijnmalen, 2006) that this will not likely reproduce the monetary results due to the merit-confined normalization procedure which does not account for differences in magnitude between the merits. In order to re-establish commensurateness, rescaling weights should be determined based on BOCR magnitude comparisons.

We take scenarios 1 and 3 as an example. Magnitude comparisons of the merits involve asking questions such as "In the context of the scenario, which is more: total benefits from the alternatives or their total opportunities and by how much?", and can be done using the known values in the last column of Table 2. Note that in real-life situations these monetary values are not known as we would most likely be dealing with a mixture of tangibles and intangibles and would then have to think very hard. The comparison based on Table 2 however yields:

B : O : C : R = 4000 : 2000 : 8000 : 2000 for scenario 1, and
B : O : C : R = 4000 : 2000 : 6000 : 100 for scenario 3.

From which follow the normalized weights:

$w_B = 0.25$; $w_O = 0.125$; $w_C = 0.5$; $w_R = 0.125$ for scenario 1, and

$w_B = 0.331$; $w_O = 0.165$; $w_C = 0.496$; $w_R = 0.008$ for scenario 3.

After entering these weights using the software's "Direct data entry" module and successively selecting the three formulas considered in this note, we get Table 3.

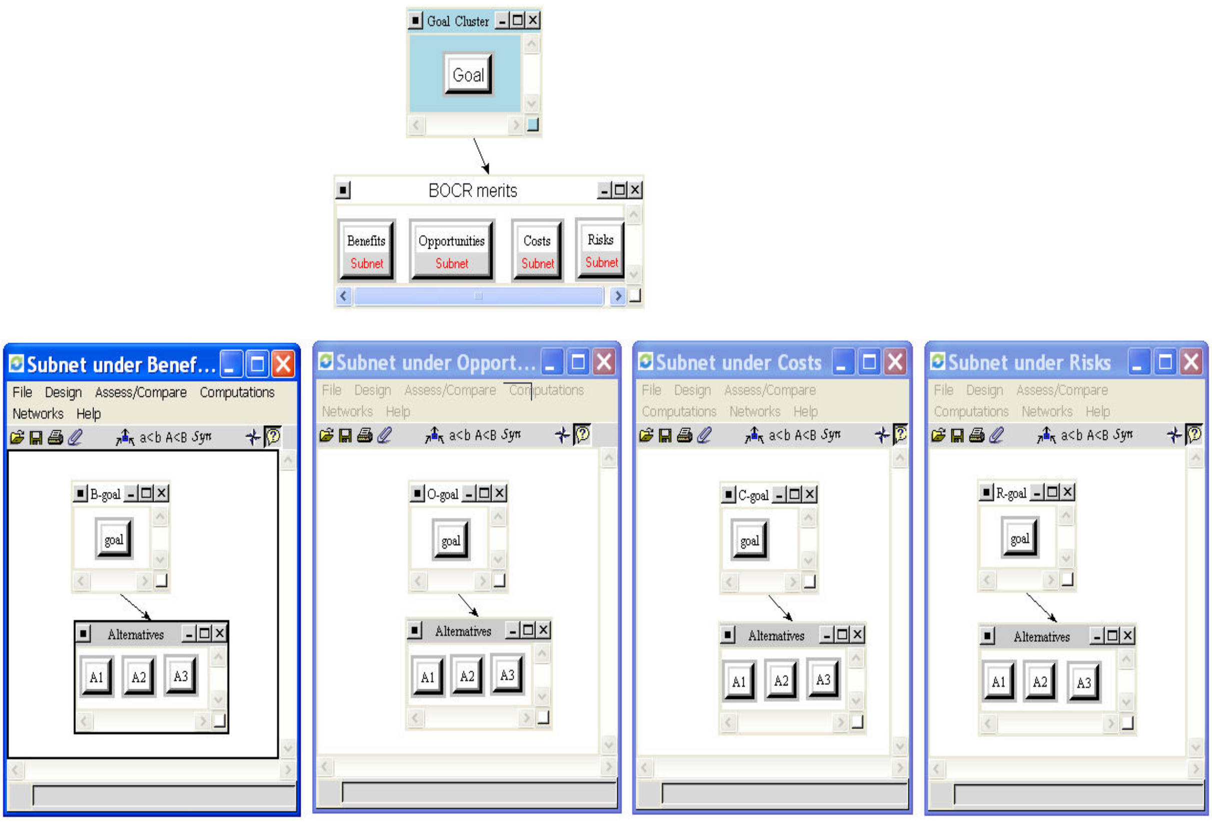

Fig. 1: Simple BOCR model with merit nodes and alternatives

Table 3: Normalized and raw final priorities according to three synthesis formulas, for scenarios 1 and 3, with BOCR factors weighted according to their magnitudes

		Alt. A1	Alt. A2	Alt. A3
Scenario 1: Ratio (multiplicative formula):	*normal.*	0.127	0.417	**0.456**
	raw	0.2664	0.8780	**0.9588**
Net value (additive formula with negatives):	*normal.*	**-0.282**	-0.341	-0.377
	raw	**-0.1950**	-0.2359	-0.2603
Add. value (add. formula with reciprocals):	*normal.*	**0.356**	0.321	0.323
	raw	**0.7039**	0.6337	0.6386
Scenario 3: Ratio (multiplicative formula):	*normal.*	0.127	0.417	**0.456**
	raw	0.2664	0.8780	**0.9588**
Net value (additive formula with negatives):	*normal.*	-0.709	-0.166	**-0.124**
	raw	-0.1252	-0.0294	**-0.0219**
Add. value (add. formula with reciprocals):	*normal.*	0.303	0.342	**0.355**
	raw	0.6085	0.6871	**0.7118**

Examination of the results and comparison with those of scenarios 1 and 3 in Table 2 reveal that the ratios of the final priorities are different from those in Table 2 (not shown, but compare the normalized values and priorities in Tables 2 and 3 respectively). This is a matter of concern as ANP is a ratio-based process and we would have hoped that at least the ratios of the priorities were reproduced. From the results of the reciprocal formula for scenario 1 we see that, if ratios are too distorted, even rank reversal can occur: A1 appears to be the best priority-based alternative whereas in monetary terms it is A3.

Another matter of concern is the fact that indications of profitability from the priority-based results in Table 3 seem to be different from those in monetary terms for scenario 3: Table 3 suggests that all alternatives are unprofitable in both types of analysis, whereas in monetary terms all alternatives are profitable except for A1 in a net value analysis.

Notice further that the multiplicative results do not change across the scenarios. This is due to the multiplicative nature of the formula which cancels out weights when re-normalizing the outcomes. At this point it is not clear why the raw results should be identical as well across the scenarios (but see section 5).

In conclusion, it would appear that the distorted final priority ratios and the differences in sign do not allow for any way of reproducing the original monetary results.

4. Correctly weighting the BOCR merit factors

It is necessary to dive into the Help information of the software, more specifically into the "Formulas" and the "Rules for Constructing Formulas" texts. There one can read that in the formulas the terms "SmartAlt" and "SmartInvAlt" are used for feeding up alternative priorities from sub-networks to the nodes in the top level network and combining them. This denotes the form in which the alternative priorities are passed up: as ideals (normalized with the maximum getting 1.0), as normals (normalized with the sum getting 1.0), as totals (the raw values are passed up with no normalization), or as smarts (the best of the three depending on the situation).

Here we have the "smart" indication which means that the best of the three is used. The ideal is used if the subnet is a bottom level one; in most other situations however the raw values are used. Our model (Fig. 1) shows subnets that are bottom level ones, and therefore apparently ideal values will be passed up[1]. In more complex network models than in this note's, it may be more difficult to ascertain what is passed up.

Following the argument in (Wijnmalen, 2006), the unit of measure of each of the BOCR nodes is therefore the ideal on each respective node. The BOCR rescaling weights should therefore be established by *comparing the magnitudes of the ideals* and not the magnitudes of the totals of the merits as was done in section 3.

Table 4 shows the BOCR rescaling weights for all scenarios, based on comparison of ideals, and Table 5 the recalculated results from the three formulas, presented both in raw (directly from the limit super matrix) and normalized form. Each scenario in Table 4 consists of two parts: one which pertains to the multiplicative synthesis and additive synthesis using negatives, and the other which pertains to additive synthesis using reciprocals. Costs and Risks are inverted in the reciprocal formula; their ideals should therefore be based on reciprocal values in that formula and are identified as the maximum inverted value of the alternative priorities.

Validation should begin with verifying that priority ratios are maintained, considering the fact that the ANP is a ratio-based process. It can be verified that the final priority ratios of the alternatives in Table 5 are now identical (allowing for rounding errors) to those of the monetary results in Table 2, which was the first aim of the exercise.

Notice further in Table 5 that the reciprocal results do not seem to differ across the first three scenarios. This is due to the very small reciprocal values of C and R and the equal values of B and O.

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[1] The author verified this to be the case. The output text and values in the "Full Report" of Super Decisions however suggest that it is the raw values ("Totals") that are passed up!

The second aim was to find out how a profitability analysis based on priorities can be conducted correctly. This will be addressed in section 5.

Table 4: Ideal alternative values on each merit factor, depending on the synthesis formula, per scenario

	Benefits	Opportunities	Costs	Risks
Scenario 1				
ratio & net: ideals per merit factor	1844	1064	3344	918
normalized weights	0.2572	0.1484	0.4664	0.128
reciprocal: ideals per merit factor	1844	1064	0.0006545	0.002994
normalized weights	0.6341	0.3659	$2*10^{-7}$	$1*10^{-6}$
Scenario 2				
ratio & net: ideals per merit factor	1844	1064	3344	459
normalized weights	0.2747	0.1585	0.4983	0.0684
reciprocal: ideals per merit factor	1844	1064	0.0006545	0.005988
normalized weights	0.6341	0.3659	$2*10^{-7}$	$2*10^{-6}$
Scenario 3				
ratio & net: ideals per merit factor	1844	1064	2508	45.9
normalized weights	0.3376	0.1948	0.4592	0.0084
reciprocal: ideals per merit factor	1844	1064	0.0008726	0.05988
normalized weights	0.6341	0.3659	$3*10^{-7}$	$2*10^{-6}$
Scenario 4				
ratio & net: ideals per merit factor	1844	2660	3344	918
normalized weights	0.2104	0.3034	0.3815	0.1047
reciprocal: ideals per merit factor	1844	2660	0.0008726	0.05988
normalized weights	0.4094	0.5906	$2*10^{-7}$	$1.33*10^{-5}$

Table 5: Normalized and raw final priorities from Super Decisions, per scenario

		Alt. A1	Alt. A2	Alt. A3
Scenario 1: Ratio (multiplicative formula):	*normal.*	0.127	0.417	0.456
	raw	0.2664	0.8780	0.9588
Net value (additive formula with negatives):	*normal.*	-0.313	-0.33	-0.357
	raw	-0.1743	-0.1840	-0.1994
Add. value (add. formula with reciprocals):	*normal.*	0.102	0.426	0.472
	raw	0.2105	0.879	0.9739
Scenario 2: Ratio (multiplicative formula):	*normal.*	0.127	0.417	0.456
	raw	0.2664	0.8780	0.9588
Net value (additive formula with negatives):	*normal.*	-0.361	-0.315	-0.324
	raw	-0.1614	-0.1411	-0.1448
Add. value (add. formula with reciprocals):	*normal.*	0.102	0.426	0.472
	raw	0.2105	0.879	0.9739
Scenario 3: Ratio (multiplicative formula):	*normal.*	0.127	0.417	0.456
	raw	0.2664	0.8780	0.9588
Net value (additive formula with negatives):	*normal.*	-0.550	0.172	0.278
	raw	-0.1008	0.0316	0.0509
Add. value (add. formula with reciprocals):	*normal.*	0.102	0.426	0.472
	raw	0.2105	0.879	0.9738
Scenario 4: Ratio (multiplicative formula):	*normal.*	0.127	0.417	0.456
	raw	0.2664	0.8780	0.9588
Net value (additive formula with negatives):	*normal.*	-0.686	- 0.189	0.125
	raw	-0.1043	- 0.0287	0.0189
Add. value (add. formula with reciprocals):	*normal.*	0.105	0.403	0.492
	raw	0.2105	0.8046	0.9831

The conclusion of this section is that the weighing of the BOCR merits should be done with reference to the unit of measure of each of the merits. It would seem that this has already been known since using a ratings model for BOCR control criteria: in his latest book Saaty (Saaty, 2005) writes, when describing examples of BOCR analysis, that one must keep in mind the alternative that has the highest priority in the synthesized results for each of the merits when selecting the ratings for the merits on each of the strategic criteria. However, on the one hand it is not made clear why this should be so and, on the other, how one should proceed when there are no strategic criteria. Moreover, it is *not always* the highest priority that should be kept in mind. What should be kept in mind depends on the unit of measurement: the totality of alternatives when normalizing to the unity sum, the ideal alternative when normalizing to the maximum priority. However, when raw values are passed up it is not clear what the unit of measurement is, and therefore what referent should be kept in mind; it may therefore be best to never use raw values for passing up.

5. Profitability validation

Whereas the original ratios of the alternatives in our example are recaptured by using weights based on comparing the magnitudes of the best alternative on each merit factor (see previous section), the priorities' synthesis results do still, at a first glance, not seem to reflect the monetary values of the alternatives.

Reciprocal-based results do not allow for a profitability analysis as there is no break-even point. Results are always positive, and larger outcomes indicate more attractive alternatives without, however, revealing whether or not the positives outweigh the negatives.

In order to reproduce the actual monetary results of the additive net-value analysis, an additional multiplication with a normalization constant would be necessary: this constant equals the grand total of the merit ideals. In our validation example, this total is known for each scenario, but in reality this is not likely to happen and other ways have to be found instead. It is sufficient to check the sign of the priority-based results: negative outcomes indicate unprofitable alternatives whereas positive outcomes indicate profitable ones, which will not change when multiplying with a normalization constant. After examination of Table 5 it can be concluded that the sign of the outcomes always coincides with that of the monetary results in Table 2. But, it can be verified that multiplication with the monetary totals would indeed exactly reproduce the original monetary values of the alternatives.

In (Wijnmalen, 2006) it was shown that applying the formulas to the composite alternative priorities on each of the (correctly rescaled) BOCR factors, without renormalization, reproduced the original monetary values of the alternatives. One would think that this was what the ANP software would do with the input from Tables 1 and 4. Examination of the results of the multiplicative formula however reveals a problem. Table 5 does not contain the correct outcomes of that formula according to the following expression (assuming perfect consistency):

$$\frac{B_m^i * O_m^i}{C_m^i * R_m^i} = \frac{\left(B_m^I/\Sigma\right) * \left(B_m^i/B_m^I\right) * \left(O_m^I/\Sigma\right) * \left(O_m^i/O_m^I\right)}{\left(C_m^I/\Sigma\right) * \left(C_m^i/C_m^I\right) * \left(R_m^I/\Sigma\right) * \left(R_m^i/R_m^I\right)} = \frac{(s_B^I * B_p^{iI}) * (s_O^I * O_p^{iI})}{(s_C^I * C_p^{iI}) * (s_R^I * R_p^{iI})} = \alpha * \frac{B_p^{iI} * O_p^{iI}}{B_p^{iI} * O_p^{iI}}$$

where:

$X \in \{B, O, C, R\}$

$X_m^i =$ monetary value of alternative *i* on merit *X*

$X_m^I =$ monetary value of ideal alternative on merit *X* (i.e. maximum)

$X_p^{iI} =$ alternative *i*'s priority value after normalization to the ideal

$s_X^I =$ normalized weight of merit *X* based on ideals

Σ = sum of monetary ideal values (normalization constant)

α = multiplicative factor.

The expression above shows that the quotient of the monetary values is equal to the product of a quotient of BOCR priorities (normalized to the ideal) and a multiplication factor which is the quotient of multiplied BO weights and multiplied CR weights. This latter factor is in fact the ratio of the units of measurement of the merit nodes and should be incorporated in the multiplicative synthesis formula which, as one might expect, combines the alternative priorities on the BOCR nodes with their weights.

After, again, diving into the software and manual, it appeared that in the software's multiplicative formula the BOCR weights are not used at all. We could do one of two as a way out: either reprogramming the multiplicative formula to incorporate the BOCR weights (which is possible in a "edit formula" module) or compute an adjustment factor based on these weights and then multiply the raw outcomes with it. Table 6 shows the values of that factor α (quotient of multiplied BO weights and multiplied CR weights).
Both procedures exactly reproduced the original monetary ratios of Table 2 allowing a proper profitability analysis based on priorities.

Table 6: Adjustment factors, derived from ideal-based BOCR weights according to multiplicative formula

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Adjustment factor α	0.6391	1.2783	17.0436	1.5978

6. Final remarks and recommendations

Taking a tableau of normalized composite priorities of three alternatives on each of the four BOCR merit factors, we have created four monetary scenarios that exactly reproduce those priorities for validation purposes. The scenarios are different, not only in their monetary values but also in terms of profitability of the alternatives. In order to derive profitability conclusions we have been considering two types of analysis: a ratio-based return on investment analysis and an addition-subtraction-based net value analysis. In addition to that, merely for the purpose of comparing the known monetary results with the priority-based results that were obtained using the Super Decisions software, we have used an additive formula where the reciprocals of costs and risks are taken rather than their negatives. This third type however does not allow for a profitability analysis as there is not an obvious break-even point, and should therefore be avoided in the first place.

In an earlier paper it was demonstrated without using the ANP software that without carefully establishing BOCR weights, based on their relative magnitudes, the results from priorities can be deficient. This can lead to bad decisions. A BOCR decision is not only about identifying the best alternative by searching for the largest priority, it is also about deciding whether or not that best one is an attractive alternative where the positives exceed the negatives. In this note it was demonstrated that the straightforward way of using the software, even with magnitude-based weights, can produce incorrect results. This should be interpreted in the sense that the known monetary results are not reproduced, neither in terms of ratios of the final alternative priorities nor in terms of values that can be used for profitability conclusions.

It was shown that it is necessary to know precisely what kind of priorities (normalized, ideal, raw) is used by the software for synthesis; this appears to depend on the type of network model created. This information is necessary because the way in which the BOCR merit factors should be weighted depends on it. In the simple network model that was created for this note, ideal priorities were fed forward to the synthesis formula. The BOCR weights should therefore be based on magnitude comparisons taking their ideal alternative values into account. This produced the correct final priorities in terms of their ratios.

In more complex network models it may not be so easy to identify what the referent for weighting should be. It would even appear that at lower levels idealized priorities are fed up and at higher levels raw alternative values! The latter should be avoided as it is not clear what their unit of measurement is when the vector on each BOCR node is not in ideal form (i.e. the maximum priority does not get value 1). In that case one would in fact be weighting only parts of the maximum priority alternatives on each respective node depending on what their exact priorities are in the vector.

It was further discovered that when applying the multiplicative formula the BOCR weights are not used. This is not acceptable when computing the raw values, as it prevents one from drawing profitability conclusions in a return on investment analysis. Both an appropriate adjustment of the initial results and a correction of the formula to include the weights enabled reproduction of the original monetary quotient values.

The analysis in this note, and in fact in the previous paper as well, is in some ways similar to the example in (Saaty, 1986) and also (Saaty, 2005; section 1.10) where rank reversal is demonstrated after normalization of monetary criteria. There it is stated that normalization of tangible criteria with the same common scale should involve adjustment of the weights with the respective normalization constants. The reason for this is that normalization creates a new, local unit of measurement (usually the sum of the alternative values on the original scale) that is different for each criterion. Synthesis however requires combining priorities on a common scale. To re-establish commensurateness the additional weighting is needed based on the normalization constants, which is in fact a magnitude-based weighting. It is not clear why this should only apply to tangible criteria where results can be compared in a validation exercise. This should also be done with intangible criteria by pairwise comparing their units of measurement which are defined by the normalization method used. With intangible criteria there is not usually a common scale in the background. But, as the literature - for example (Schoner e.a., 1993), and many later validation examples thereof - of AHP linking pin technology shows, weighting should be done with respect to the amount of criteria the referent alternatives possess in order to maintain ratio stability and thus avoid rank reversal.

From the Super Decisions software, its manual, or the ANP theory for that matter, it is not at all clear what the unit of measurement is and on what basis correct rescaling weights should be established.

It is therefore recommended that:
- the software (manual) should draw a user's attention to the necessity of using rescaling weights in order to produce commensurate results fit for validation;
- advice should be given as to what kind of magnitude comparisons should be done and what the normalization referents are in order to establish correct rescaling weights;
- raw values should not be passed up to serve as input to any BOCR synthesis formula in order to avoid ambiguity with respect to the unit of measurement when weighting the BOCR merits; re-idealized vectors should be used (maximum priority set to unity) instead or vectors re-normalized to the priority sum;
- the software should compute correct formulaic values incorporating the weights of the BOCR nodes in order to produce final results on which a BOCR profitability analysis can be based; this is especially important when using the multiplicative formula where re-normalized final priorities are meaningless for that analysis;
- the additive formula with reciprocals should be removed from the software, and
- a quotient formula where the (weighted) positives B and O are added, and the same with the (weighted) negatives C and R, should be included as an additional synthesis formula (ref. section 1).

If a model of control criteria is to be used for establishing "personally weighted" BOCR merits, these should be applied after rescaling the BOCR merits. Even if, for some reason, there should be no wish to explicitly rescale the individual BOCR priorities based on magnitude comparison but BOCR nodes are weighted based on other ("personal") aspects, even then one should know what the normalization referents are that are to be used in order to meaningfully establish commensurate priorities on a common scale.

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