Optimal strategy for local development in the presence of intangible resources and outcomes

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1. Introduction

Economic and social development is a complex process. Its effectiveness depends on numerous factors of diverse nature. To ensure an optimal development path, an appropriate resource allocation strategy is required. Dynamic programming (DP) is a methodology frequently applied for the analysis of the strategy. However, since PD is a kind of mathematical programming approach, tangible data are required for its implementation. Unfortunately, local development depends on many conflicting issues. The issues can be both of tangible and intangible nature. For example, available resources, governing criteria and even outcomes of an implemented strategy can be of intangible nature. To cope with such difficulties effectively, a special methodology should be applied. A unified approach, based on analytic hierarchy process (AHP) is utilised in the paper for joint tangible-intangible issues analysis. That way, PD can be successfully applied for the development strategy arrangement. The proposed approach is illustrated using the results of a calculational example.

2. Dynamic Programming and Resource Allocation Strategy Analysis

DP approach constitutes a widely used mathematical programming method [1]. It has been successfully applied for the solution of many decision-making problems of different nature. DP can be used for the solution of problems of appropriate structure. The term *appropriate structure* means that:

1. there is a way of problem's division into a number of inter-related and simpler subproblems;
2. solution of the problem can be efficiently obtained on a basis of the subproblems solutions.

Therefore, DP is useful in case of problems which are dividable into separate stages.

Sustainable economic and social development requires allocation of (usually scarce) assets amongst undertaken activities. Usually, the activities are divided into several time stages because of resource availability constraints. Thus, DP is suitable for regional development planning as well.

DP foundation is given by the principle of optimality by Richard Bellman. It says that *an optimal path has the property that whatever the initial conditions and choices over some initial period are, the choices over the remaining period must be optimal to achieve the best result*. The principle results in a recursive definition of a goal function g (1):

$$g_i = g_{i-1} + f_i(\mathbf{x}_i, \mathbf{s}_{i-1}) \,, \tag{1}$$

where: g_i and g_{i-1} denote values of goal function with regard to all periods (stages) preceeding the present (the i-th) stage and the previous one respectively; f_i is a value of the present stage effect function; vector $\mathbf{x}_i$ contains values of decision variables which express choices (decisions) made at the present stage and vector $\mathbf{s}$ contains values of variables called *state variables* which identify a state of an analysed system.

Valid values of decision variables $\mathbf{x}_i$ depend on a chain of decisions made (values assigned to decision variables during previous stages: $\mathbf{x}_1$, $\mathbf{x}_2$, ..., $\mathbf{x}_{i-2}$, $\mathbf{x}_{i-1}$). The decisions result in vector of state variables $\mathbf{s}_{i-1}$. Therefore, vector $\mathbf{s}_{i-1}$ limits a set of possible decisions $\mathbf{x}_i$ made at the present stage. On the other hand, decisions made at each state cause the change in state vector components. The change is governed by so called *state transition function h*:

$$\mathbf{s}_i = h(\mathbf{x}_i, \mathbf{s}_{i-1}) \tag{2}$$

Values of state variables $\mathbf{s_i}$ are constrained. Due to the interdependability of decision variables and state variables, the constraints define acceptable value ranges for both sets of variables.

The best strategy is described in terms of maximisation of goal function value for the last (N-th) stage:

$$g_N \rightarrow \max \tag{3}$$

Local development strategy implementation is based on realisation of a set of tasks. Each task makes it possible to achieve a partial goal. Moreover, satisfaction of partial goals can lead (through the synergy effect) to the achievement of the overall development goal. To obtain the best possible outcome of the strategy implementation common, usually scarce, resources should be adequately divided among competing tasks. The division should maximise the overall outcome of the strategy implementation.

There are some reasons for good suitability of stage-like nature of PD in case of application for resource allocation during development strategy implementation. First of all, implementation tasks are usually long-lasting. Secondly, they are asset-demanding. Therefore, each stage of computations can be devoted to a unique combination of a task and time period. The state of allocation process can be identified using a set of variables — so called *state variables.* Each state variable describes the amount of the devoted resource available for allocation during next stages. Finally, transition function *h* defines valid changes of asset amount (and consequently — the decision made with regard to allocation of resources to considered tasks) at each stage. Constraints of an inter-stage transition process result from the stage and overall availability of resources as well as the needs of considered tasks.

3. Intangible Issues in Allocation Strategy Analysis

Considering joint effects of both tangible and intangible issues constitutes one of the most demanding problems in case of optimal resource allocation during local development implementation. To cope with the problem of mixed-nature issues effectively, some aproaches have been proposed. For example, an approach including AHP and linear programming (LP) combination is proposed in [3].

To include joint influence of tangible and intangible assets, their amount and influence on effects of tasks completion can be expressed using unrated values of common scale. Mixed nature of tasks outcomes and their influence on the overall goal achivement level can be addressed similarly. AHP is applied in this regard in the paper.

4. Sample analysis

To illustrate the proposed approach, assumptions and results obtained for sample calculational analysis are presented. Mid-term local development strategy is considered. The overall goal of the strategy depends on three tasks. The first task (T1) is devoted to the improvement of local drinking water supply. The second (T2) deals with the development of co-operation between academia and economy. The third one (T3) ensures the improvement of local transportation infrastructure. Outcomes of tasks realisation depend on the amount of applied resources.

Financial assets comprise the most important resource in case of considered tasks. Therefore, PD analysis deals with their allocation exclusively. Relations between outcomes of tasks and applied resource amount are presented in fig.1.

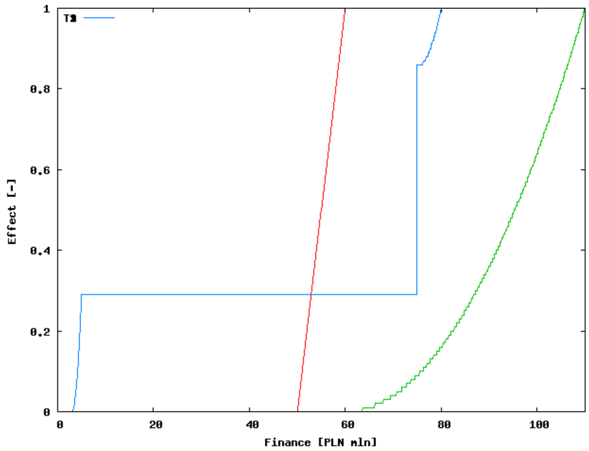

Fig.1. Functions of resource allocation effects for tasks
T1 (red line), T2 (blue line) and T3 (green line)

Distribution of considered tasks influence on overall goal achievement is expressed using normalised weights. AHP is applied for weights estimation. Expert opinions take into account the overall effects of the tasks. 9-point Saaty's scale and additive normalisation method (AN) are applied during weight estimation. Obtained values of weights for tasks effects along with considered judgments (matrix **W**) are presented in tab.1.

Due to resource limitations, implementation of the strategy is divided into three distinct periods. The amount of financial resources available at each period is limited. However, it is possible to transfer the unused resource amount to next stages.

Table 1. Task effect influence estimation (*c.r.* = 0,01)

W	T1	T2	T3	w_j
T1	1	2	3	0,540
T2	1/2	1	2	0,300
T3	1/3	1/2	1	0,160
Sum:	1+2/3	3+1/2	6	1

The calculations have been made using open source software system for numerical computations called *GNU Octave*, available at http://www.octave.org. Tab.2–3 contain the results of calculations with regard to stage resource allocation limit for the first and the third allocation stage (resource amount is measured in Polish currency PLN). The influence of the allocation limits structure on the overall effect is illustrated in tab.4.

Table 2. Effects of resource allocation — influence of the first stage resource allocation limit

	State I				State II				State II				Overall
No.	Limit	T1	T2	T3	Limit	T1	T2	T3	Limit	T1	T2	T3	Effect
1	40	0	0	10	80	60	0	0	60	0	110	0	0.886
2	50	50	0	0	80	10	70	0	60	0	40	20	0.886
3	60	60	0	0	80	0	80	0	60	0	30	30	0.886

Table 3. Effects of resource allocation — influence of the third stage resource allocation limit

	State I				State II				State II				Overall
No.	Limit	T1	T2	T3	Limit	T1	T2	T3	Limit	T1	T2	T3	Effect
1	60	60	0	0	80	0	80	0	110	0	30	80	1
2	60	60	0	0	80	0	80	0	100	0	20	80	0.892
3	60	60	0	0	80	0	80	0	90	0	30	60	0.886
4	60	60	0	0	80	0	80	0	80	0	30	50	0.886
6	60	60	0	0	80	0	80	0	70	0	30	40	0.886
7	60	60	0	0	80	0	80	0	60	0	30	30	0.886

Table 4. Effects of resource allocation — comparison of structure of stage resource allocation limits (the overall allocation limit at 190,000,000 PLN)

	State I				State II				State II				Overall
No.	Limit	T1	T2	T3	Limit	T1	T2	T3	Limit	T1	T2	T3	Effect
1	40	0	0	20	70	60	0	0	80	0	110	0	0.886
2	40	0	0	20	80	60	0	0	70	0	110	0	0.886
3	50	50	0	0	80	10	70	0	60	0	40	20	0.886
4	60	60	0	0	80	0	80	0	40	0	30	10	0.886

Included results of calculations provide an idea about the possible influence of financial resources limitation on an allocation structure and finally — on the overall effect of the implemented allocation strategy. Thus, in case of resource availability uncertainty, decision makers are supplied with the knowledge of the effectiveness of the considered development paths.

5. Conclusions

Presented approach is still under development. Test analyses identify the potential of the DP-AHP approach to address effectively the influence of both intangible and tangible issues. Thanks to the application of the proposed methodology, the influence of some important factors (e.g. inherited uncertainty governing local development process) can be considered adequately. Thus, decision makers obtain a valuable tool for development strategy paths evaluation.

Bibliography

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