THE ART OF STRUCTURING AHP AND ANP MODELS

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ISAHP 2018
HONG KONG, HK

JULY 12 - JULY 15, 2018 / HONG KONG, HK

International Symposium on the Analytic Hierarchy Process

WELCOME

Welcome to this workshop on structuring AHP and ANP models

Tom Saaty worked for 40 years on this so I apologize for attempting to do it in an hour!

So the best thing for me to do is tell you what insights I learned about structuring models by working with him, his students and the AHP/ANP software during this time

BIRTH OF AHP

- AHP was developed by Thomas Saaty when he was consulting for the Navy in 1972 (in Appendix 3 of the final report)
- The Navy needed to prioritize their matériel –supplies and equipment – from toilet seats to tanks

BIRTH OF AHP

Tom developed these two important concepts:

- The pairwise comparison matrix of judgments used to derive priorities – the basic building block of the AHP and ANP

and

- How to combine the priorities for several properties with a weight and add process – a simple hierarchy

JUDGEMENTS

Our minds can use feelings and knowledge to express judgments – we are genetically endowed to do this:

1) We can compare two things with each other as to which has more of a certain property

or

2) We can compare things one at a time against standards we have in our memory called a rating process

AHP models can be work in either way

VALIDATION EXERCISE

Pairwise compare the 5 figures

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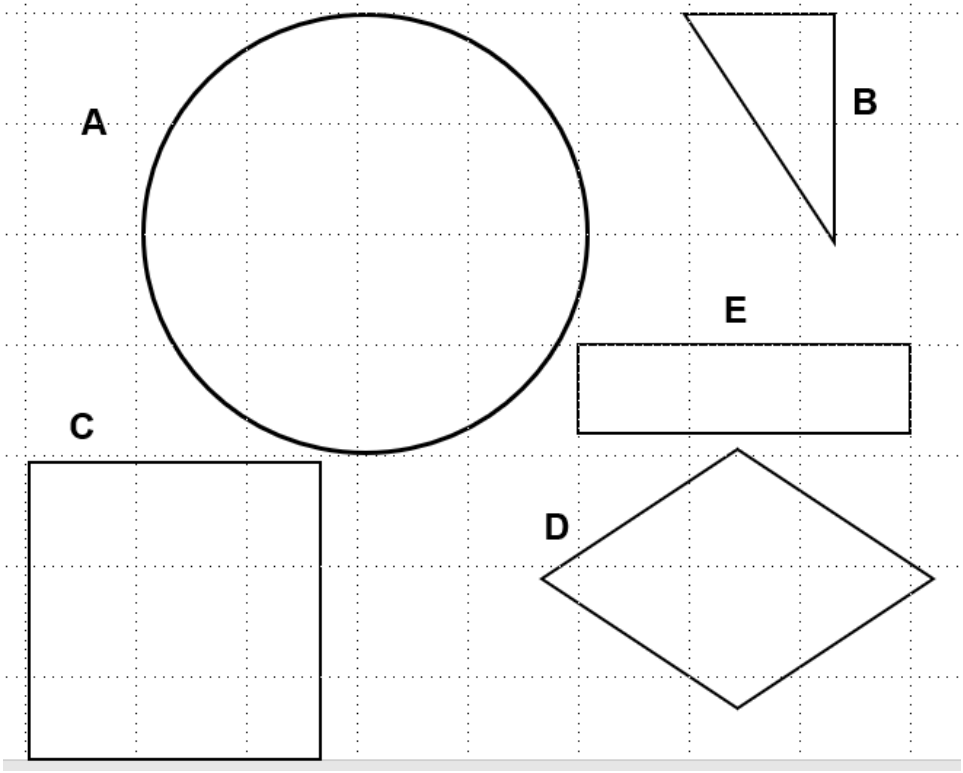
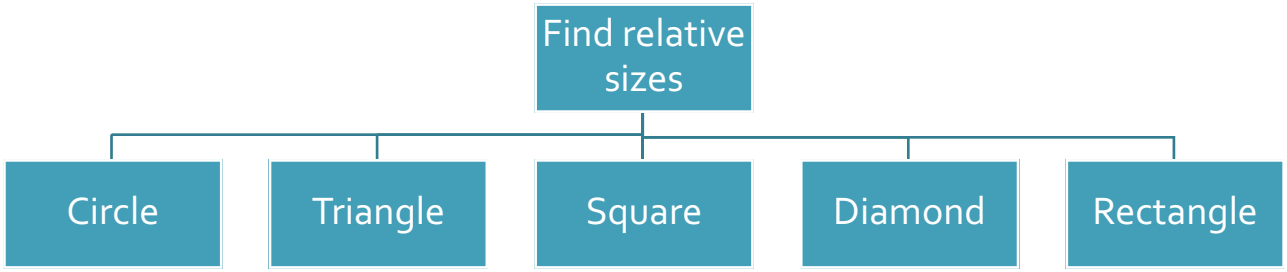
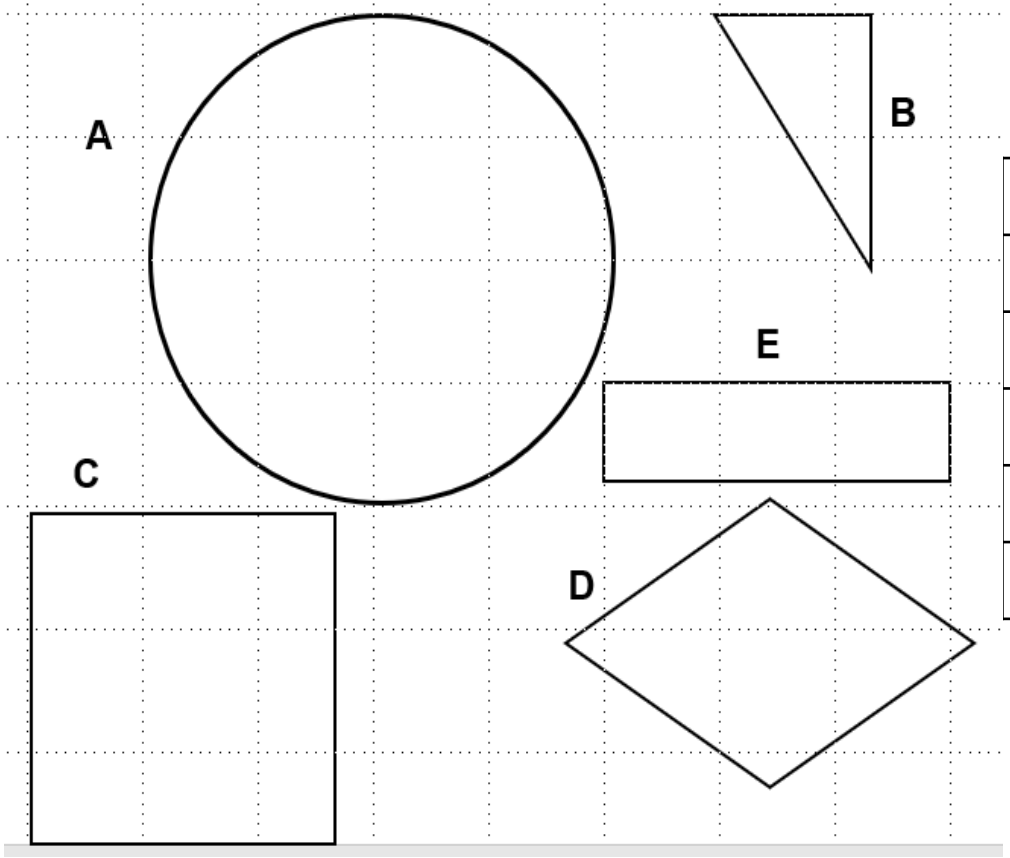


A hierarchy

AHP model for validation exercise on areas



PAIRWISE COMPARE THE AREAS



	A	B	C	D	E
A	1				
B		1			
C			1		
D				1	
E					1

PAIRWISE COMPARISON RESULTS

	A	B	C	D	E	AHP Priorities
A	1	8	2	3	5	0.469
B	1/9	1	1/4.5	1/3	1/2	0.052
C	1/2	4.5	1	1.8	2.5	0.243
D	1/3	3	1/1.8	1	1.3	0.144
E	1/5	2	1/2.5	1/1.3	1	0.100

PAIRWISE COMPARISON RESULTS

	AHP (Eigen-vector)	Shortcut - Sum rows & normalize	Ratios of measured areas
Circle	0.459	0.452	0.479
Triangle	0.052	0.052	0.050
Square	0.244	0.245	0.240
Diamond	0.144	0.147	0.150
Rectangle	0.101	0.104	0.090

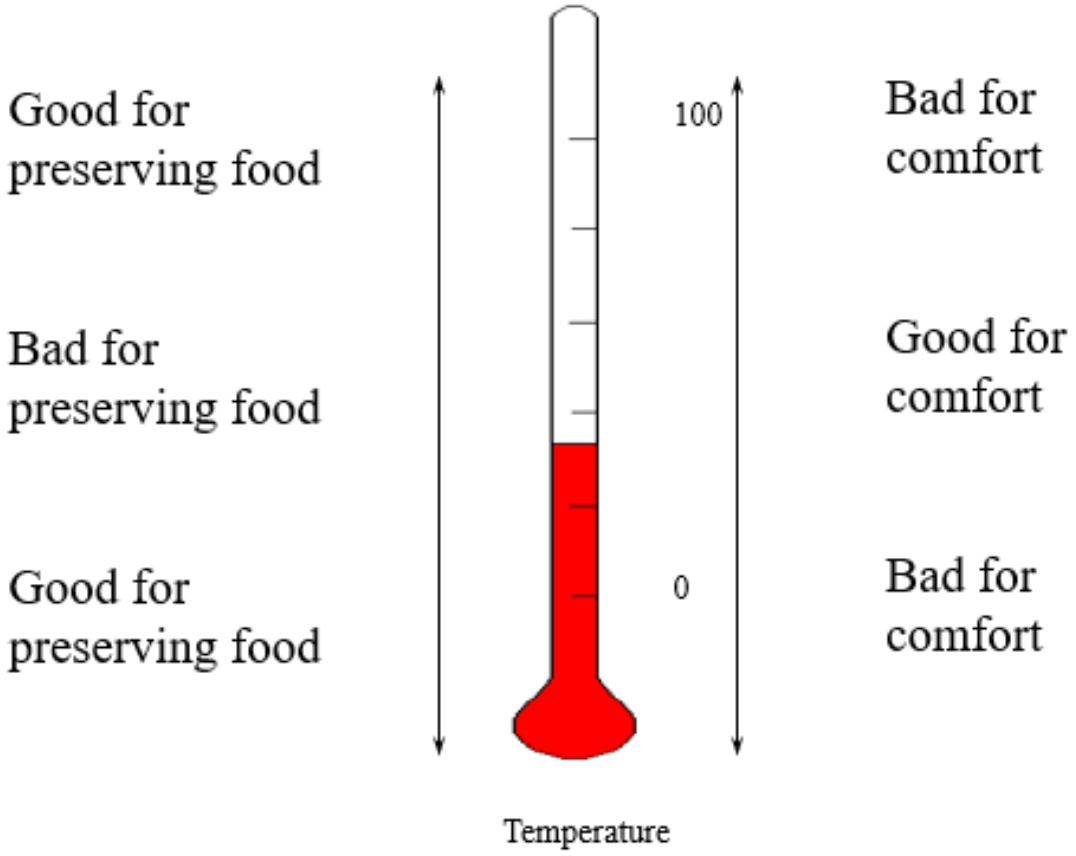
Fundamental Scale of Absolute Numbers

1	Equal importance
3	Moderate importance of one over another
5	Strong or essential importance
7	Very strong or demonstrated importance
9	Extreme importance
Use	2,4,6,8 for intermediate values

Use decimals for finer discrimination: 1.1, 1.2, 1.3, 5.2,5.3, …9.0

Use the inverse for the reverse comparison in the matrix

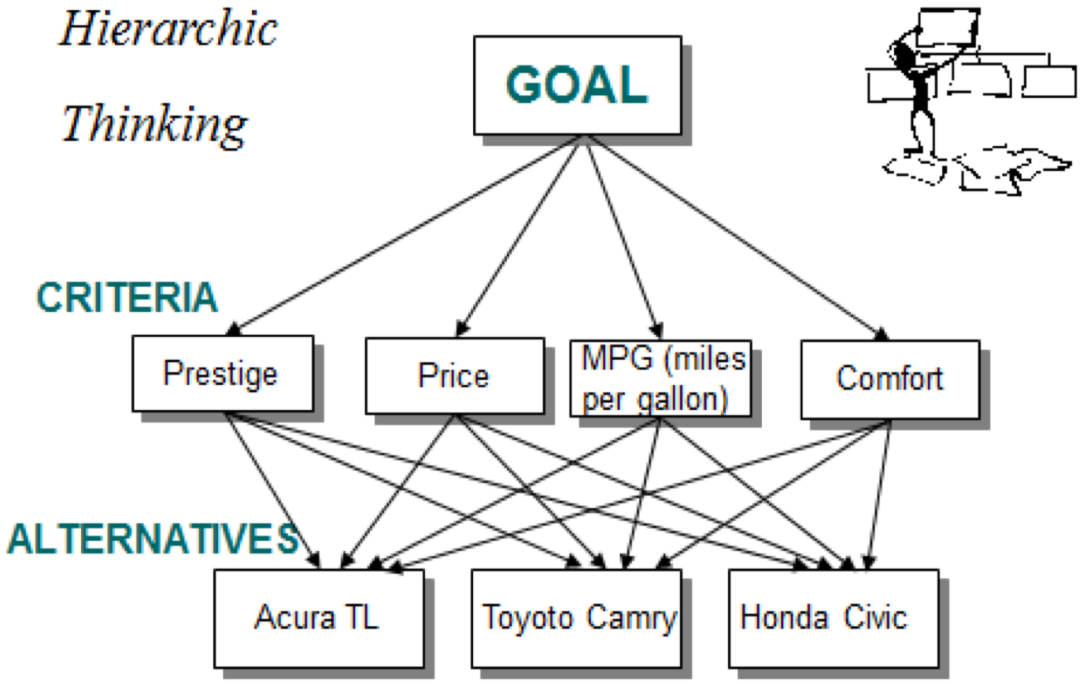
INTERPRETING NUMBERS



Measuring versus Pairwise Comparing

- With an existing scale first one measures, then interprets it ; the same number can mean different things depending on your purpose
- With AHP one first judges then derives the relative measurement scale
- Both approaches require experience and knowledge, but with the constantly changing world AHP works better

THREE LEVEL BASIC HIERARCHY



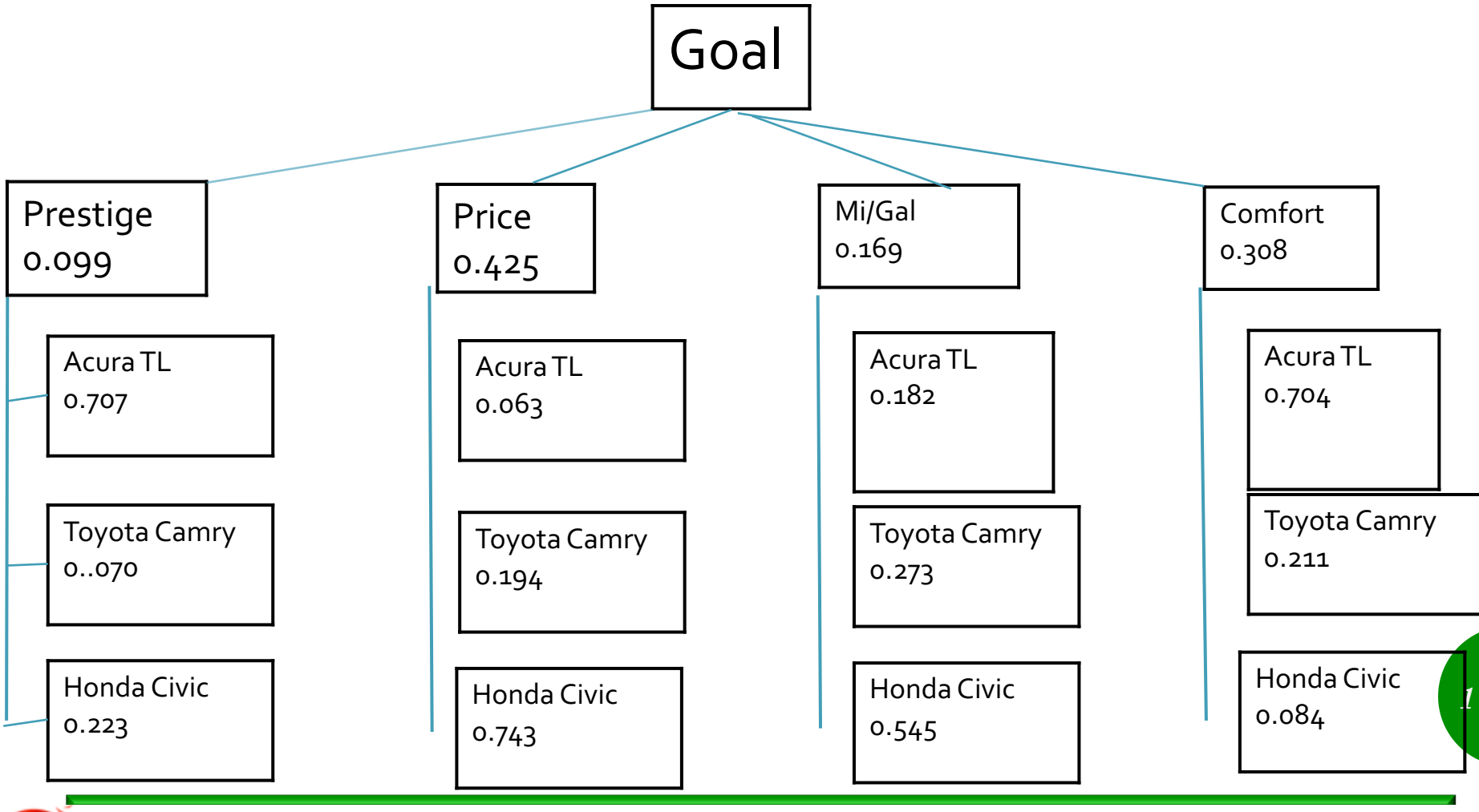
POSING THE PAIRWISE QUESTION

There are three generic terms one can use in asking the pairwise comparison question:

- IMPORTANCE (for criteria, subcriteria…)
- LIKELIHOOD (for criteria or alternatives)
- PREFERENCE (for alternatives)

Wall Street Journal cartoon July 3, 2018.

HIERARCHY WITH JUDGMENTS



Coming up with a Decision Structure

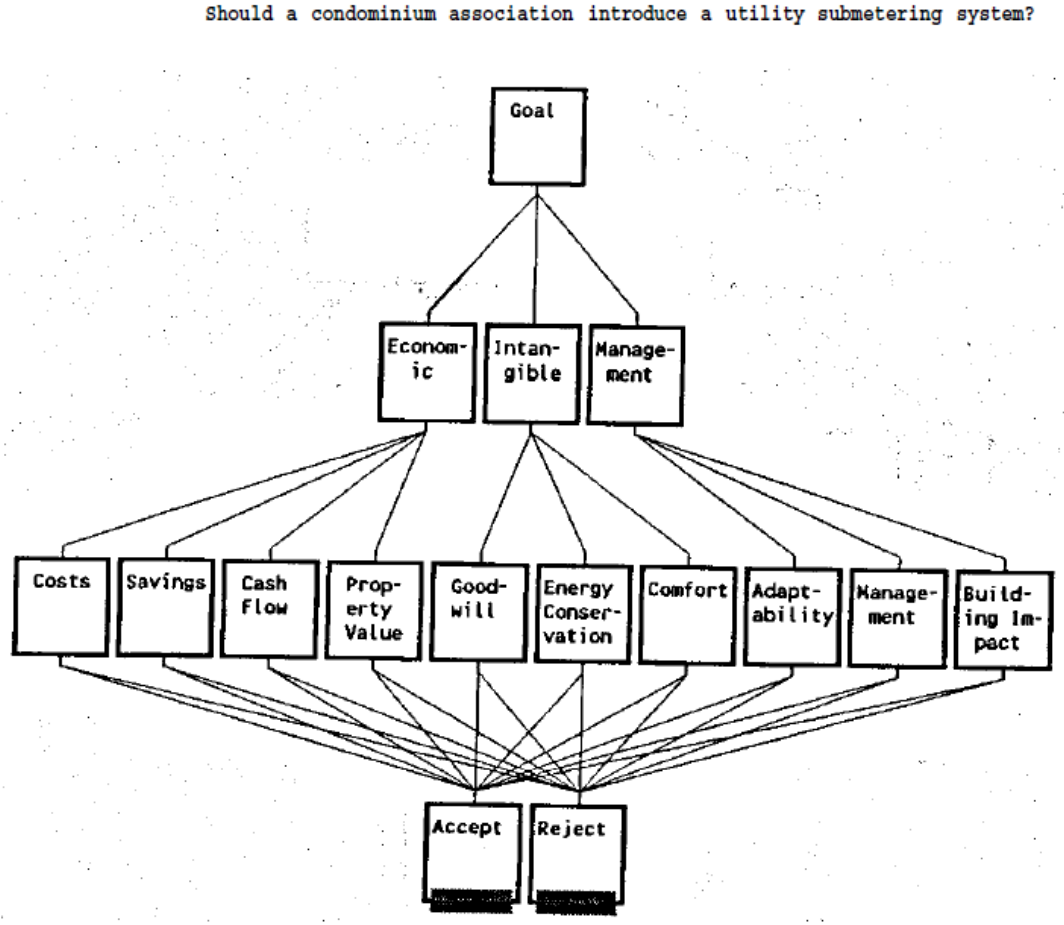
- Determine the goal of your decision
- Who is making the decision?
- Find the alternatives or construct some. Don't make them widely disparate. They should be appropriate for the goal.
- Will you pairwise compare the alternatives or rate them one at a time – useful for large numbers of alternatives? (Criteria are always pairwise compared)

Coming up with a decision structure

- How do you start thinking about a decision? Make a list of the factors you think belong in the model.
- Group them into high level factors, then sub-factors. Limit size of groups to about 7 or, preferably even less – around 5.
- Criteria may be goals or purposes you have in mind; or may be based on the properties of your alternatives.

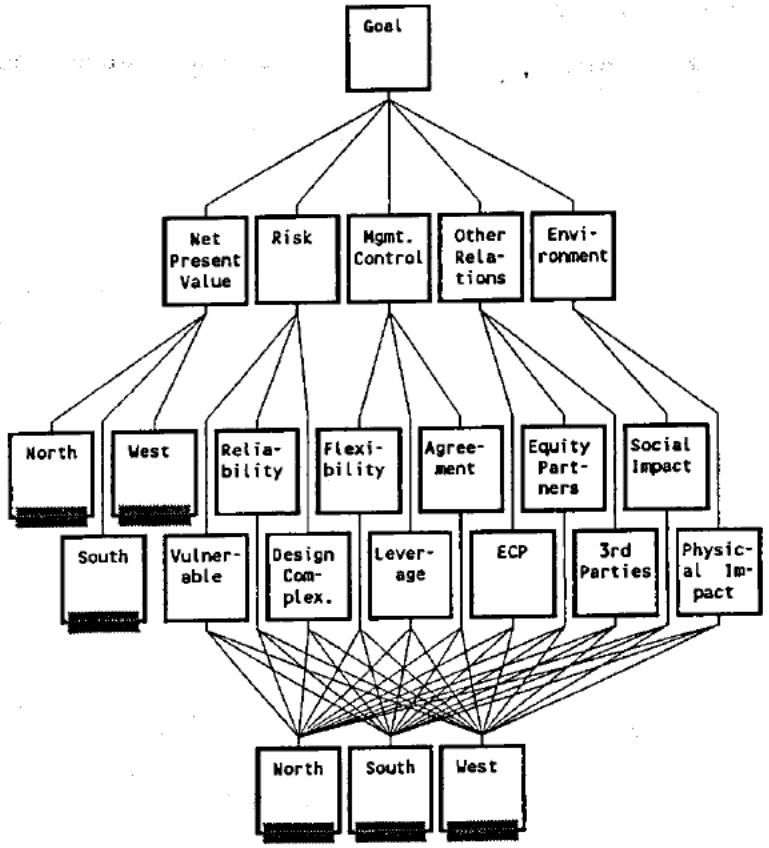
FOUR LEVEL BALANCED HIERARCHY

INTRODUCE UTILITY SUBMETERING SYSTEM IN CONDOMINIUMS?

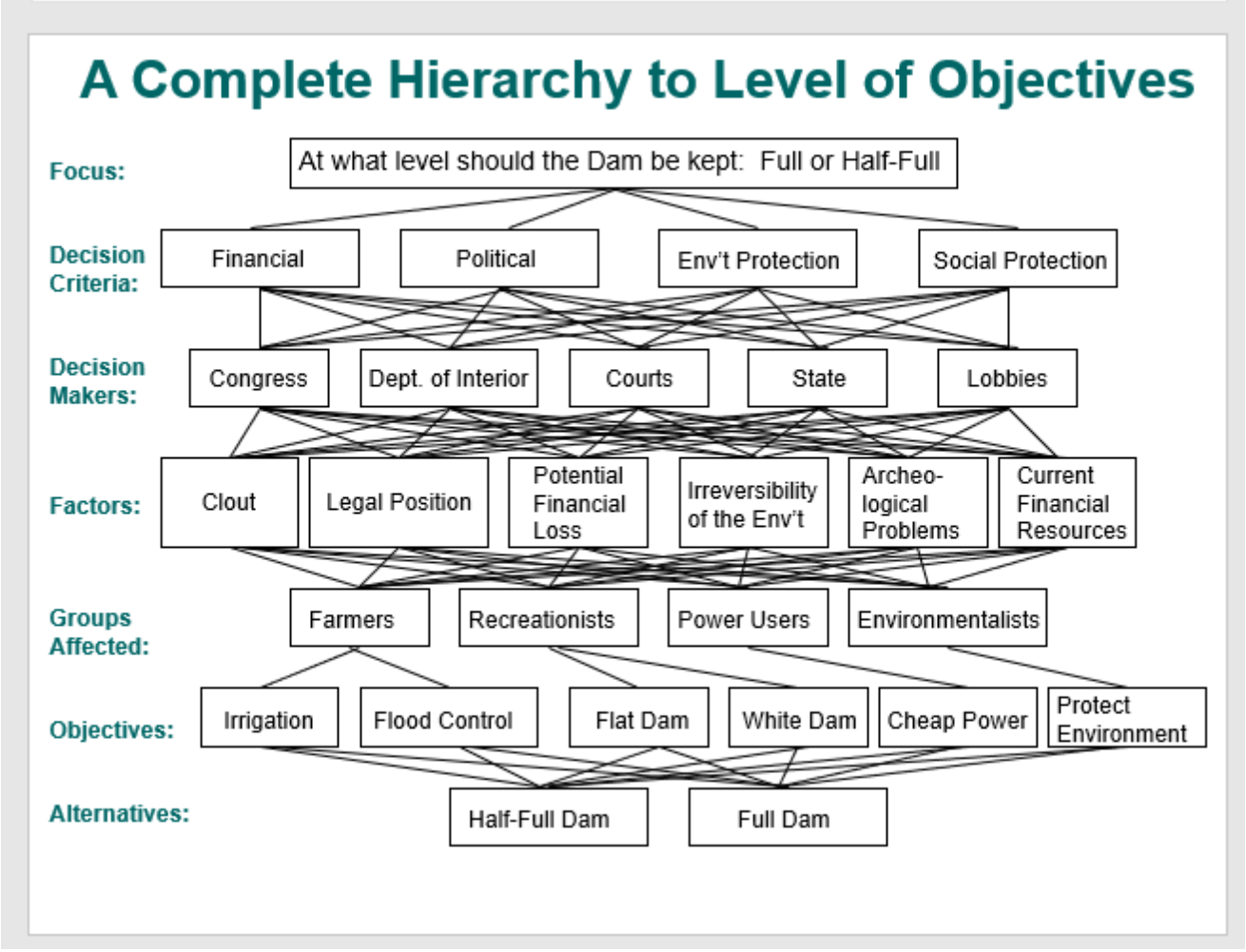


Unbalanced Hierarchy – 4 Levels

Choose best route for a pipeline

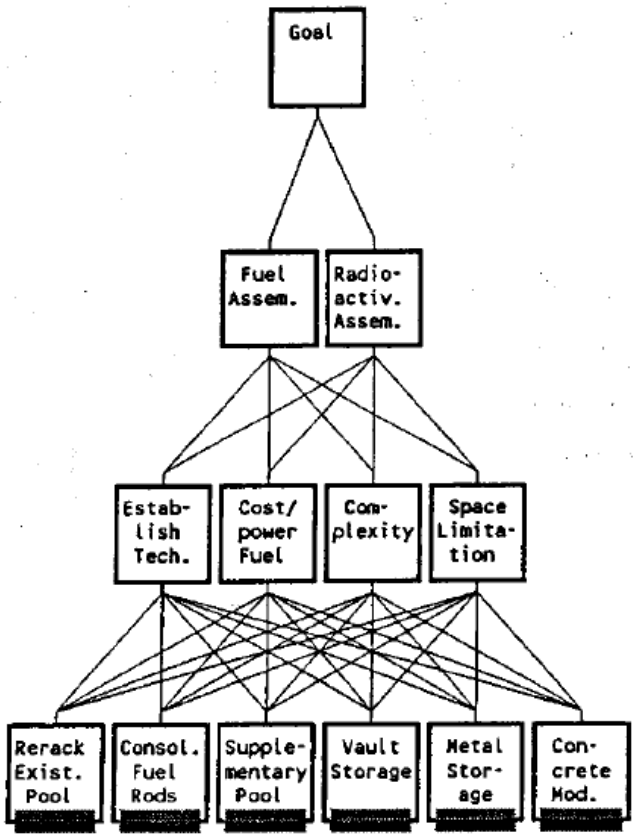


A Hierarchy of Many Levels



FOUR LEVEL COMPLETE HIERARCHY

Best storage for nuclear waste?



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PRIORITY FLOWS IN HIERARCHIES

Think of the criteria as "owning" a particular resource

- There can be a fixed amount that is being distributing downward through the structure (top-down)

or

- The amount of a resource a criterion owns is being collected from the alternatives. (bottom-up)

DATA MODEL

Which farm has the highest income and is the best one to buy? Clearly Farm 2 with $10,650 income.

	Rent Income	Product Sales Income	Total Income	Normalize Totals to get priorities	Rank
Totals	*$15,000*	*$15,000*	*$30,000*		
Farm 1	$ 9,000.00	$ 1,200.00	$ 10,200.00	0.340	2
Farm 2	$ 1,500.00	$ 9,150.00	$ 10,650.00	0.355	1
Farm 3	$ 4,500.00	$ 4,650.00	$ 9,150.00	0.305	3

	Original overall priorities	Relative priorities Farm2/Farm 3 (normalized)
Farm 2	0.355	0.538
Farm 3	0.305	0.462
Sum	*0.66*	*1*

Original relative priorities

AHP VERSION OF DATA MODEL

Normalize the column data for the alternatives; normalize the totals to get the criterion priorities. Weight and add to synthesize results. The Overall AHP priorities are the same as for the data model.

	Rent Income	Product Sales Income	Overall synthesized priorities	Rank
	C1=0.5	*C2=0.5*	*1*	
Farm 1	0.6	0.08	0.34	2
Farm 2	0.1	0.61	0.355	1
Farm 3	0.3	0.31	0.305	3

THE RANK REVERSAL ISSUE

- A long standing criticism of the AHP has been that when alternatives are added or deleted the rank order of the original alternatives changes.

- Why? The criteria weights need to be adjusted as the amount of the resource each "owns" has now changed. The new overall priority vector will have different numbers in it, but the ratios of any two of the original alternatives will stay the same if the criteria are adjusted.

Back to the data version of the model to see the proper way to calculate by adjusting the criteria weights.

REMOVE THE FIRST ALTERNATIVE IN THE DATA MODEL

In the data model, sum the sources of income for the two remaining farms. Farm 2 is still the best and the relative priority of Farm1/Farm2 is the same.as with 3 alternatives.

	Rent Income	Product Sales Income	Overall synthesized priorities	Rank
Original criteria Wts	*C1=0.5*	*C2=0.5*	*1*	
Farm 2	$ 1,500.00	$ 9,150.00	$ 10,650.00	1
Farm 3	$ 4,500.00	$ 4,650.00	$ 9,150.00	3
Sums	*$ 6,000.00*	*$ 13,800.00*	*$ 19,800.00*	

	Rent Income	Product Sales Income	Overall Income	Normalize to get priorities	Rank
Farm 2	$ 1,500.00	$ 9,150.00	$ 10,650.00	0.538	1
Farm 3	$ 4,500.00	$ 4,650.00	$ 9,150.00	0.462	2
Sums	*$ 6,000.00*	*$ 13,800.00*	*$ 19,800.00*	1.000	

	Original overall priorities	Relative priorities Farm2/Farm 3 (normalized)
Farm 2	0.355	0.538
Farm 3	0.305	0.462
Sum	*0.66*	*1*

WHY CRITICS GET RANK REVERSALS

The same process should be followed as in the data model which essentially re-evaluates the criteria weights. But the critics leave their criteria weights the same, normalize the alternatives and weight and add.

	Rent Income	Product Sales Income	Overall Priorities Weight and add	Rank
	0.5	0.5		
Farm 2	0.25	0.66	0.46	2
Farm 3	0.75	0.34	0.54	1
Sums	1.00	1.00	1.00	

Rank reversal has occurred! Farm 2 is no longer the best and the ratio of the two original alternatives has changed too.

Cardinal Rule: If the ratios in the overall priority vector of any two of the original alternatives stay the same, the order of all the original alternatives will stay the same..

PREVENT RANK REVERSAL

In an AHP model of priorities, the process is the same as for the data model.

1) Remove an alternative by deleting its row in the table.
2) Sum the priorities of the remaining alternatives and subtract from 1 for each criterion.
3) Divide this number by the sum of the original alternatives and multiply times the respective original criterion weights to adjust them.
4) Multiply and add to synthesize as usual.

	Rent Income	Product Sales Income	Overall synthesized priorities
Orig. criteria	*0.5*	*0.5*	*1*
Orig. Priority	*1*	*1*	
Farm 2	0.1	0.61	
Farm 3	0.3	0.31	
Sums	0.4	0.92	

Multiply the sums of the priorities of the remaining alternatives times the original criteria weights.

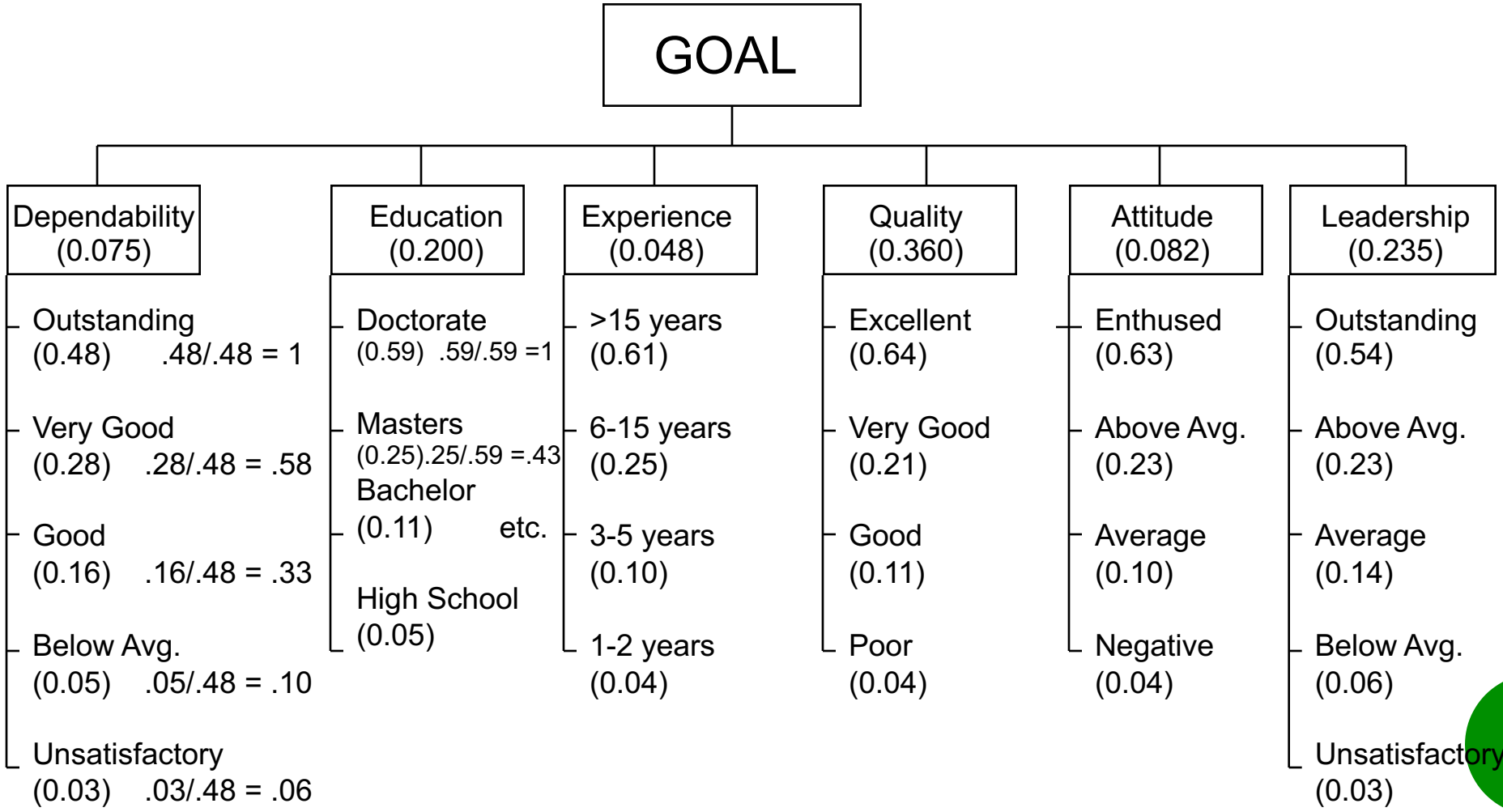
	Rent Income	Product Sales Income	Overall synthesized priorities
new criteria	*0.3030303*	*0.6969697*	*1*
Total Priority	*1*	*1*	
Farm 2	0.25	0.66304348	0.537878788
Farm 3	0.75	0.33695652	0.462121212

Normalize the alternative priorities and the criteria priorities and weight and add to get the results which are correct as they are validated by the original data model.

SUMMING UP

- Do we always want to preserve rank? It depends on your view of the world. If you expect alternatives to maintain their order is correct and should not change with new information there are two ways to do it: reweight the criteria to match the changed alternatives or use ratings to evaluate the alternatives one at a time.

Rating Employees for Performance



Final Step in Rating with Absolute Measurement

Rate each employee for dependability, education, experience, quality of work, attitude toward job, and leadership abilities.

	Dependability 0.0746	Education 0.2004	Experience 0.0482	Quality 0.3604	Attitude 0.0816	Leadership 0.2348	Total	Normalized
Esselman, T.	Outstand	Doctorate	>15 years	Excellent	Enthused	Outstand	1.000	0.153
Peters, T.	Outstand	Masters	>15 years	Excellent	Enthused	Abv. Avg.	0.752	0.115
Hayat, F.	Outstand	Masters	>15 years	V. Good	Enthused	Outstand	0.641	0.098
Becker, L.	Outstand	Bachelor	6-15 years	Excellent	Abv. Avg.	Average	0.580	0.089
Adams, V.	Good	Bachelor	1-2 years	Excellent	Enthused	Average	0.564	0.086
Kelly, S.	Good	Bachelor	3-5 years	Excellent	Average	Average	0.517	0.079
Joseph, M.	Blw Avg.	Hi School	3-5 years	Excellent	Average	Average	0.467	0.071
Tobias, K.	Outstand	Masters	3-5 years	V. Good	Enthused	Abv. Avg.	0.466	0.071
Washington, S.	V. Good	Masters	3-5 years	V. Good	Enthused	Abv. Avg.	0.435	0.066
O'Shea, K.	Outstand	Hi School	>15 years	V. Good	Enthused	Average	0.397	0.061
Williams, E.	Outstand	Masters	1-2 years	V. Good	Abv. Avg.	Average	0.368	0.056
Golden, B.	V. Good	Bachelor	.15 years	V. Good	Average	Abv. Avg.	0.354	0.054

The total score is the sum of the weighted scores of the ratings. Money can be allocated for raises according to the normalized total score. In practice different jobs need different hierarchies.