111Equation Chapter 1 Section 1A GROUP CONSENSUS MODEL WITH AHP

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ABSTRACT

Consensus reaching models aiming at helping a group to reach a certain level of consensus are crucial in group decision making process. The Analytic Hierarchy Process(AHP) is an effective tool and has been widely used in group decision making. In this paper a new consensus reaching model based on the AHP is proposed, which considers both individual and aggregated opinions. The compatibility index can be used to determine both the individual consensus level(ICI) and the central consensus level(CCI). Then this model provides feedback suggestions to the most incompatible decision makers so they can adjust their opinions adaptively depending on their ICI and CCI in each round. The integrated adaptive consensus reaching model is constructed. Finally, a numerical example is given to verify the feasibility and effectiveness of the model.

Keywords: Group decision making, Consensus reaching, The Analytic Hierarchy Process (AHP).

1. Introduction

The complexity of decision-making problems makes it difficult to obtain an accurate and reliable solution from one expert. Therefore, group decision-making(GDM) has received widespread attention from both academia and industry. GDM is a process that aggregates individual opinions of experts into collective opinions on behalf of the entire group. To express individual opinions, the Analytic Hierarchy Process(AHP) [1] is an effective tool because people are good at comparing two alternatives at one time. Due to the diversity of individual opinions, it is difficult to reach full consensus. Moreover, there are some experts give incompatible judgements in real life, Therefore, the crucial problem in GDM is how to reach a certain level of consensus for a valid solution.

There exist many researches [2-3] about the GDM consensus reaching model which are mainly focused on two aspects: (1) Approaches to measure the consensus level; (2) Mechanisms to improve the consensus level. In the traditional consensus reaching model, whether the approach to measure the consensus level or the mechanism to improve the consensus level, they use the aggregated opinions as the reference point. Dong [4] proposed a peer-to-peer dynamic adaptive consensus reaching model, which used individual opinions as the reference point both in measuring the individual consensus level and improving the consensus level. The above researches have their own meanings, the former used the aggregated opinions and the latter used the individual opinions as their reference point, but there is no corresponding research about the GDM consensus reaching model which uses both the aggregated opinions and the individual opinions as the reference point.

2. Objectives

Based on the above related researches, this paper proposes an AHP consensus reaching model which considers both individual and aggregated opinions. The purpose is to unify the forms of current consensus reaching models. Through the new algorithm, the decision makers can adjust their opinions adaptively in different cases, particularly, the decision makers have the opportunity to reject those who have a huge difference with their own opinions without any penalties. In addition, the convergence of the proposed algorithm should be proved, which verifies the feasibility and effectiveness of the model.

3. Methodology

The details of the AHP consensus reaching model with both individual and aggregated Opinions are depicted through following algorithm.

Input: Initial pairwise compare matrixes(PCMs) $A_1, A_2 \ldots A_m, A_m = \left(a_{ij(m)} \right)_{n \times n}$ with acceptable consistency level, the threshold value λ of the individual consensus index and the central consensus index, the refusal threshold value $\varphi(\varphi > \lambda)$ of the individual consensus index, the parameters η_p η_q, and the maximum number of iterations T.

Output: group PCM G^*

Step 1. Calculate the individual consensus level $ICI_{pq}^t, (p = 1, 2, \ldots m)$ and the central consensus level $CCI_i^t, (i = 1, 2, \ldots m)$ in iteration t, if each $ICI_{pq}^t \le \lambda$, $p = 1, 2, \ldots m$, $q = 1, 2, \ldots m$, then go to **step 4**; otherwise, continue.

Step 2. Identification process. Select the most incompatible decision makers $MD_{pq} = \left\{ A_p, A_q \mid \max_{p<q;p,q=1,2,\ldots m} \left\{ ICI_{pq} \middle| ICI_{pq} \geq \lambda \right\} \right\}$, then,continue.

Step 3. Adjustment process. Adjust the MD_{pq} opinions according to the following formula:

$$\begin{cases} a_{ij(p)}^{t+1} = \left[\left(a_{ij(p)}^{t} \right)^{\theta_p^t} \left(a_{ij(q)}^{t} \right)^{1-\theta_p^t} \right] \left[\left(a_{ij(p)}^{t} \right)^{\mu_p^t} \left(g_{ij}^{t} \right)^{1-\mu_p^t} \right]^{(1-\eta_p)} \\ a_{ij(q)}^{t+1} = \left[\left(a_{ij(q)}^{t} \right)^{\theta_q^t} \left(a_{ij(p)}^{t} \right)^{1-\theta_q^t} \right] \left[\left(a_{ij(q)}^{t} \right)^{\mu_q^t} \left(g_{ij}^{t} \right)^{1-\mu_q^t} \right]^{(1-\eta_q)} \end{cases} \quad 22\text{* MERGEFORMAT ()}$$

Which

$$\theta_p^t = \frac{ICI_{pq}^t}{\sum_{i=1}^{m} ICI_{pi}^t}, \theta_q^t = \frac{ICI_{pq}^t}{\sum_{i=1}^{m} ICI_{qi}^t} \quad 33\text{* MERGEFORMAT ()}$$

$$\mu_p^t = \frac{GCI_p^t}{\sum_{i=1}^{m} GCI_i^t}, \mu_q^t = \frac{GCI_q^t}{\sum_{i=1}^{m} GCI_i^t} \quad 44\text{* MERGEFORMAT ()}$$

As such we have 4 cases:

(1) When the most incompatible decision makers' individual consensus levels are greater than the threshold but do not exceed the threshold of rejection $(\lambda < ICI_{pq}^t \leq \varphi)$, at the same time the central consensus levels do not exceed the threshold $(GCI_p^t \leq \lambda, GCI_q^t \leq \lambda)$, the algorithm will only use another individual opinions as the reference point. Where

$$\begin{cases} a_{ij(p)}^{t+1} = \left(a_{ij(p)}^{t} \right)^{\theta_p^t} \left(a_{ij(q)}^{t} \right)^{1-\theta_p^t} \\ a_{ij(q)}^{t+1} = \left(a_{ij(p)}^{t} \right)^{\theta_p^t} \left(a_{ij(q)}^{t} \right)^{1-\theta_p^t} \end{cases} \quad 55\text{* MERGEFORMAT ()}$$

Then set $t = t+1$, and return to **step 2.**

(2) When the most incompatible decision makers' individual consensus levels exceed the threshold of rejection $(ICI_{pq}^t > \varphi)$, indicating that the two decision-makers will refuse each other's opinion, the algorithm will only use the aggregated opinions as the reference point. Where

$$\begin{cases} a_{ij(p)}^{t+1} = \left(a_{ij(p)}^{t} \right)^{\mu_p^t} \left(g_{ij}^{t} \right)^{1-\mu_p^t} \\ a_{ij(q)}^{t+1} = \left(a_{ij(q)}^{t} \right)^{\mu_q^t} \left(g_{ij}^{t} \right)^{1-\mu_q^t} \end{cases} \quad 66\text{* MERGEFORMAT ()}$$

Then set $t = t+1$, and return to **step 2.**

(3) When the most incompatible decision makers' individual consensus levels and the central consensus levels are greater than the threshold $(\lambda < ICI_{pq}^{t} \le \varphi$, $GCI_p^t > \lambda, GCI_q^t > \lambda)$,the algorithm will use both the individual and the aggregated opinions as the reference point. Where

$$\begin{cases} a_{ij(p)}^{t+1} = \left[\left(a_{ij(p)}^{t} \right)^{\theta_p^t} \left(a_{ij(q)}^{t} \right)^{1-\theta_p^t} \right] \left[\left(a_{ij(p)}^{t} \right)^{\mu_p^t} \left(g_{ij}^{t} \right)^{1-\mu_p^t} \right]^{(1-\eta_p)} \\ a_{ij(q)}^{t+1} = \left[\left(a_{ij(q)}^{t} \right)^{\theta_q^t} \left(a_{ij(p)}^{t} \right)^{1-\theta_q^t} \right] \left[\left(a_{ij(q)}^{t} \right)^{\mu_q^t} \left(g_{ij}^{t} \right)^{1-\mu_q^t} \right]^{(1-\eta_q)} \end{cases} \quad 77\text{* MERGEFORMAT ()}$$

Then set $t = t+1$, and return to **step 2.**

(4) When the most incompatible decision makers' individual consensus levels are greater than the threshold $(\lambda < ICI_{pq}^{t} \le \varphi)$,but only one central consensus level among the most incompatible decision maker is greater than the threshold $(GCI_p^t \le \lambda, GCI_q^t > \lambda)$, in this case, one will use both the individual and the aggregated opinions as the reference point, the other will only use another individual opinions as the reference point. Where

$$\begin{cases} a_{ij(p)}^{t+1} = \left(a_{ij(p)}^{t} \right)^{\theta_p^t} \left(a_{ij(q)}^{t} \right)^{1-\theta_p^t} \\ a_{ij(q)}^{t+1} = \left[\left(a_{ij(q)}^{t} \right)^{\theta_q^t} \left(a_{ij(p)}^{t} \right)^{1-\theta_q^t} \right] \left[\left(a_{ij(q)}^{t} \right)^{\mu_q^t} \left(g_{ij}^{t} \right)^{1-\mu_q^t} \right]^{(1-\eta_q)} \end{cases} \quad 88\text{* MERGEFORMAT ()}$$

Then set $t = t+1$, and return to **step 2.**

Step 4. The group PCM G^* can be calculated by aggregating with the weighted geometric mean. The consensus reaching process is stopped.

The convergence of the proposed model has been proved through mathematical proof, which guarantees the consensus level will be improved in the consensus reaching process.

4. Conclusions

The Analytic Hierarchy Process(AHP) is an effective tool and has been widely used in multi-criteria decision making because people are good at comparing two alternatives at one time. For improving the consensus level to reach consensus in the group decision making, an AHP consensus reaching model with both individual and aggregated opinions is proposed to unify the forms of current consensus reaching models. In this consensus reaching model, the decision makers can adjust their opinions adaptively in different cases. Moreover, the decision makers are not forced to follow those who have a huge difference with their own opinions, the consensus reaching process is closer to real life. The convergence of the proposed model has been proved through mathematical proof, which verifies the feasibility and effectiveness of the model.

The authors plan to make further researches in following aspects:

(1) The objective methods to determine the weights of decision makers and the size of thresholds. Because weights and thresholds in the context are given in advance, it is inevitable to be subjective.

(2) Develop a group decision support system based on the proposed consensus reaching model.

5. Key References

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