HOW TO WRITE A CONTRACT WITH THE AHP

ABSTRACT

In this paper we show how the Analytic Hierarchy Process could be used to develop a legal contract in the process of a negotiation. We illustrate the process with a well-known case used routinely in negotiation courses. We show that the AHP is particularly well suited for this type of applications where most of the dimensions and criteria are intangibles, and the scales used to measure the gains and costs of parties involved in the negotiation do not always exist.

Keywords: Negotiation, gain and loss ratios, value claim, value creation

1. Introduction

The dictionary definition of "contract" is "a binding agreement between two or more persons or parties" or "a document describing the terms of a contract." This implies that a contract has multiple dimensions and the parties must agree on each of the dimensions. For example, in the case of a recruiter trying to hire a candidate for a position in a company, the dimensions could be the signing bonus, salary, job assignment, company car, starting date, number of vacation days, percentage of moving expenses covered, the type of insurance coverage offered, and so on. Each dimension has a different impact on the parties.

There are two types of outcome at work when two parties negotiate: Value claim, and Value creation (see Figure 1).

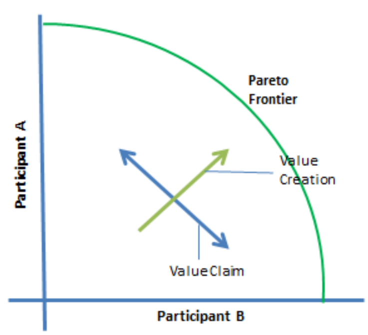

Figure 1. Value Claims, Value Creation and the Pareto Frontier

Value claim occurs when one party is able to capture value from the other party during the negotiation process. This is most prevalent among those dimensions of the negotiation that are distributive (i.e., what one party gains, the other party experiences as a comparable loss). However, it can also manifest itself for integrative elements (i.e., when multiple factors are negotiated – some of which are more important to one of the parties, and some of which are more important to the other party). However, for both integrative and compatible dimensions (i.e., factors where the same element is perceived as a gain for both parties), there are also opportunities for exchange that leads to value creation. Thus, value creation takes place when both parties are made better off during the negotiation. When value creation occurs, the parties move closer towards the Pareto frontier – the point at which neither party can be made better off without the counterparty being made worse off.

2. A Simple Example

We mentioned above that a contract has multiple dimensions and the parties must agree on each of the dimensions. Thus, a negotiation to arrive a mutually agreed contract needs to consider the gains and losses of the parties in each of the dimensions. For example, a recruiter is negotiating with a prospective employee for a position. They need to agree on the conditions of employment. The negotiation involves agreement on a number of dimensions. Each dimension can be considered a benefit or a cost. Table 1 shows an example of dimensions of a negotiation and their type.

Table 1. Dimensions and their type

Dimensions	Type
SIGNING BONUS (SB)	Benefit
SALARY (S)	Cost
JOB ASSIGNMENT (JA)	Cost
COMPANY CAR (CC)	Benefit
STARTING DATE (SD)	Benefit
VACATION DAYS (VD)	Benefit
MOVING EXPENSES REIMB (MER)	Benefit
INSURANCE COVERAGE (IC)	Benefit

In addition, within each type, the dimensions are not equally important. Table 2 shows the importance of the dimensions from both, the recruiter's and the employee's perspective.

To develop the contract, the parties need to agree the level at which each of the dimensions must be set. Let us assume that the dimensions have intensities as in Table 3, and that each of those alternative intensities accrues a benefit or a cost, also given in Table 3.

Table 2. Recruiter/Employee Priorities for Benefits and Costs

	Priorities	
Benefits	Recruiter	Employee
SIGNING BONUS (SB)	0.270	0.270
COMPANY CAR (CC)	0.081	0.081
STARTING DATE (SD)	0.108	0.270
VACATION DAYS (VD)	0.270	0.108
MOVING EXPENSES (MER)	0.054	0.216
INSURANCE COVERAGE (IC)	0.216	0.054
	Priorities	
Costs	Recruiter	Employee
SALARY (S)	0.75	0.75
JOB ASSIGNMENT (JA)	0.25	0.25

In many real life contract negotiations, neither the dimensions of the contract nor the intensity scales may be known. To make tradeoffs we need to identify the dimensions and the intensity scales. The intensity scales in Table 3 are expressed in relative terms in Table 4. The priorities of the dimensions are obtained by normalizing the sum total of each scale (see Table 4). In this example the scale values are all equispaced, i.e., they form a linear scale. However, in practice these values would be obtained through prioritization and they do not need to be linear.

The negotiation process consists in finding out what value each dimension should take for the recruiter and the candidate so that the total amount they get (benefit/cost ratio) is maximized satisfying the constraint that neither party gets more than the other, i.e, the contract is fair and equitable (Fisher and Ury 1981).

3. The Trading Model

To find the solution of this problem we model it with integer programming. A solution is represented by an 8-by-5 matrix $\left(x_{ij}\right)$ of 0's and 1's. Each row corresponds to a dimension and each column corresponds to an intensity of the scale corresponding to that dimension. $x_{ij} = 1$ if the ith dimensions takes the jth intensity value. Let b_{ij}^{R} (b_{ij}^{C}) and

c_{ij}^R (c_{ij}^C) the benefit and cost corresponding to the j[th] intensity of the i[th] dimension for the recruiter (candidate).

The benefits/costs ratios of the recruiter and the candidate are given by

$$r_R(x) = \frac{\text{benefits}}{\text{costs}} = \frac{\sum_i w_i^R \sum_j x_{ij} b_{ij}^R}{\sum_i v_i^R \sum_j x_{ij} c_{ij}^R} \text{ and } r_C(x) = \frac{\text{benefits}}{\text{costs}} = \frac{\sum_i w_i^C \sum_j x_{ij} b_{ij}^C}{\sum_i v_i^C \sum_j x_{ij} c_{ij}^C},$$

respectively.

The objective is to find a solution $x*$ such that the parties gain as much as possible,

$$r_R(x^*) = r_C(x^*) = \underset{x \in X_S}{Max}\{\text{Min}\{r_A(x), r_B(x)\}\},$$

where X_S is the solution space defined as the set of matrices (x_{ij}) that satisfy the conditions $\sum_{j=1}^{5} x_{ij} = 1$, for all *i*, $x_{ij} = 0,1$, for all *i* and *j*, and the two parties gain the same, i.e., their ratios are equal.

A given solution has benefits/costs ratios that are different for the parties. For example, in Table 5 we give a solution. In this solution, the recruiter has a lower benefits/costs ratio than the candidate, so the recruiter will try to change to another solution where he will get a greater benefits/costs ratio. Table 6 shows the solution in matrix form.

Table 3. Intensities and the benefits/costs accrued by the recruiter and the employee

	INTENSITY	RECRUITER	CANDIDATE
SIGNING BONUS (SB)	10%	0	4000
	8%	1000	3000
	6%	2000	2000
	4%	3000	1000
	2%	4000	0
SALARY (S)	$ 60,000.00	-6000	0
	$ 58,000.00	-4500	-1500
	$ 56,000.00	-3000	-3000
	$ 54,000.00	-1500	-4500
	$ 52,000.00	0	-6000
JOB ASSIGNMENT (JA)	Division A	0	0
	Division B	-600	-600
	Division C	-1200	-1200
	Division D	-1800	-1800
	Division E	-2400	-2400
COMPANY CAR (CC)	LUX EX2	1200	1200
	MOD 250	900	900
	RAND XTR	600	600
	DEPAS 450	300	300
	PALO LSR	0	0
STARTING DATE (SD)	1-Jun	1600	0
	15-Jun	1200	1000
	1-Jul	800	2000
	15-Jul	400	3000
	1-Aug	0	4000
VACATION DAYS (VD)	30 days	0	1600
	25 days	1000	1200
	20 days	2000	800
	15 days	3000	400
	10 days	4000	0
MOVING EXPENSES	100%	0	3200
REIMBURSEMENT (MER)	90%	200	2400
	80%	400	1600
	70%	600	800
	60%	800	0
INSURANCE COVERAGE (IC)	Allen Insurance	0	800
	ABC Insurance	800	600
	Good Health Insurance	1600	400
	Best Insurance Co.	2400	200
	Insure Alba	3200	0

Table 4. Priorities of dimensions and relative scales for the recruiter – candidate case

							Relative Scales	
RECRUITER PRIORITIES		CANDIDATE PRIORITIES		Dimensions	RECRUITER	CANDIDATE	RECRUITER	CANDIDATE
Benefits	Costs	Benefits	Costs					
0.27027018		0.2702701		SIGNING BONUS				
				10%	0.01	4,000	1E-06	0.3999996
				8%	1,000	3,000	0.0999999	0.2999997
				6%	2,000	2,000	0.1999998	0.1999998
				4%	3,000	1000	0.2999997	0.0999999
				2%	4,000	0.01	0.3999996	1E-06
	0.714285918		0.714285918	SALARY	**10000.01**	**10000.01**		
				$60,000	-6,000	0.01	0.40000027	6.6667E-07
				$58,000	-4,500	-1,500	0.3000002	0.10000007
				$56,000	-3,000	-3,000	0.20000013	0.20000013
				$54,000	-1,500	-4,500	0.10000007	0.3000002
				$52,000	0.01	-6,000	6.6667E-07	0.40000027
	0.285714082		0.285714082	JOB ASSIGNMENT	**-14999.99**	**-14999.99**		
				Division A	0.01	0.01	1.6667E-06	1.6667E-06
				Division B	-600	-600	0.10000017	0.10000017
				Division C	-1,200	-1,200	0.20000033	0.20000033
				Division D	-1,800	-1,800	0.3000005	0.3000005
				Division E	-2,400	-2,400	0.40000067	0.40000067
0.08108124		0.08108122		COMPANY CAR	**-5999.99**	**-5999.99**		
				LUX EX2	1200	1200	0.39999867	0.39999867
				MOD 250	900	900	0.299999	0.299999
				RAND XTR	600	600	0.19999933	0.19999933
				DE PAS 450	300	300	0.09999967	0.09999967
				PALO LSR	0.01	0.01	3.3333E-06	3.3333E-06
0.10810823		0.2702701		STARTING DATE	**3000.01**	**3000.01**		
				1-Jun	1,600	0.01	0.399999	1E-06
				15-Jun	1,200	1,000	0.29999925	0.0999999
				1-Jul	800	2,000	0.1999995	0.1999998
				15-Jul	400	3,000	0.09999975	0.2999997
				1-Aug	0.01	4,000	2.5E-06	0.3999996
0.27026991		0.1081082		VACATION DAYS	**4000.01**	**10000.01**		
				30 days	0	1,600	0	0.399999
				25 days	1,000	1,200	0.1	0.29999925
				20 days	2,000	800	0.2	0.1999995
				15 days	3,000	400	0.3	0.09999975
				10 days	4,000	0.01	0.4	2.5E-06
				MOVING EXPENSES	**10000**	**4000.01**		
0.05405425		0.21621614		REIMBURSEMENT				
				100%	0.01	3,200	5E-06	0.3999995
				90%	200	2,400	0.0999995	0.29999963
				80%	400	1,600	0.199999	0.19999975
				70%	600	800	0.2999985	0.09999988
				60%	800	0.01	0.399998	1.25E-06
0.21621619		0.05405424		INSURANCE COVERAGE	**2000.01**	**8000.01**		
				Allen Insurance	0.01	800	1.25E-06	0.399998
				ABC Insurance	800	600	0.09999988	0.2999985
				Good Health Insurance	1,600	400	0.19999975	0.199999
				Best Insurance Co.	2,400	200	0.29999963	0.0999995
				Insure Alba	3,200	0.01	0.3999995	5E-06
					8000.01	**2000.01**		

Table 5. A solution with the Benefit/Cost Ratios

A Solution		Intensities				Recruiter		Candidate	
	1	2	3	4	5	Benefits	Costs	Benefits	Costs
SB	0	0	1	0	0	0.135135	0	0.135135	0
S	0	1	0	0	0	0	0.535714	0	0.178571
JA	0	1	0	0	0	0	0.071429	0	0.071429
CC	1	0	0	0	0	0.081081	0	0.081081	0
SD	0	0	1	0	0	0.054054	0	0.135135	0
VD	0	1	0	0	0	0.067568	0	0.081081	0
MER	0	0	1	0	0	0.027027	0	0.108108	0
IC	0	0	1	0	0	0.108108	0	0.027027	0
					B/C Ratio	0.7790		2.2703	

Table 6. The Optimal Solution

Optimal Solution		Intensities				Recruiter		Candidate	
	1	2	3	4	5	Benefits	Costs	Benefits	Costs
SB	0	0	1	0	0	0.135135	0	0.135135	0
S	0		1	0	0	0	0.357143	0	0.357143
JA	1	0	0	0	0	0	0	0	0
CC	1	0	0	0	0	0.081081	0	0.081081	0
SD	0	0	0	0	1	0	0	0.27027	0
VD	0	0	0	0	1	0.27027	0	0	0
MER	1	0	0	0	0	0	0	0.216216	0
IC	0	0	0	0	1	0.216216	0	0	0
					B/C Ratio	1.9676		1.9676	

Translated into the original scale values of the dimensions we have Table 7. Note that now both the recruiter and the candidate gain the same.

Table 7. The Terms of the Contract

	SB	S	JA	CC	SD	VD	MER	IC	Total
	6%	$ 56,000.00	Division A	LUX EX2	1-Aug	10 days	100%	Insure Alba	Points
Recruiter	2000	-3000	0	1200	0	4000	0	3200	7400
Candidate	2000	-3000	0	1200	4000	0	3200	0	7400

Obviously, the scales within each dimension do not have to be linear. For example, if the recruiter and the candidate have relative intensities as given in Table 8, the solution

(Table 9) would not be the same as the one in Table 7. The solutions in Table 9 are within 3.125% of each other. No other closer solutions exist.

Table 8. Intensities with Non-Linear Relative Scales

			Relative Scales	
Dimensions	RECRUITER	CANDIDATE	RECRUITER	CANDIDATE
SIGNING BONUS				
10%	0.01	4,000	1E-06	1
8%	1,000	3,000	0.1	0.75
6%	2,000	2,000	0.5	0.5
4%	3,000	1000	0.9	0.1
2%	4,000	0.01	1	1E-06
SALARY	**10000.01**	**10000.01**		
$60,000	-6,000	0.01	1	1E-06
$58,000	-4,500	-1,500	0.75	0.1
$56,000	-3,000	-3,000	0.5	0.5
$54,000	-1,500	-4,500	0.1	0.9
$52,000	0.01	-6,000	1E-06	1
JOB ASSIGNMENT	**-14999.99**	**-14999.99**		
Division A	0.01	0.01	1E-06	1E-06
Division B	-600	-600	0.1	0.1
Division C	-1,200	-1,200	0.5	0.5
Division D	-1,800	-1,800	0.9	0.9
Division E	-2,400	-2,400	1	1
COMPANY CAR	**-5999.99**	**-5999.99**		
LUX EX2	1200	1200	1	1
MOD 250	900	900	0.75	0.75
RAND XTR	600	600	0.5	0.5
DE PAS 450	300	300	0.1	0.1
PALO LSR	0.01	0.01	1E-06	1E-06
STARTING DATE	**3000.01**	**3000.01**		
1-Jun	1,600	0.01	1	1E-06
15-Jun	1,200	1,000	0.75	0.1
1-Jul	800	2,000	0.5	0.5
15-Jul	400	3,000	0.1	0.9
1-Aug	0.01	4,000	1E-06	1
VACATION DAYS	**4000.01**	**10000.01**		
30 days	0	1,600	0	1
25 days	1,000	1,200	0.1	0.75
20 days	2,000	800	0.5	0.5
15 days	3,000	400	0.9	0.1
10 days	4,000	0.01	1	1E-06
MOVING EXPENSES	**10000**	**4000.01**		
REIMBURSEMENT				
100%	0.01	3,200	1E-06	1
90%	200	2,400	0.1	0.75
80%	400	1,600	0.5	0.5
70%	600	800	0.9	0.1
60%	800	0.01	1	1E-06
INSURANCE COVERAGE	**2000.01**	**8000.01**		
Allen Insurance	0.01	800	1E-06	1
ABC Insurance	800	600	0.1	0.75
Good Health Insurance	1,600	400	0.5	0.5
Best Insurance Co.	2,400	200	0.9	0.1
Insure Alba	3,200	0.01	1	1E-06
	8000.01	**2000.01**		

Table 9. Terms of the Contract for the Non-Linear Intensity Case

	SB	S	JA	CC	SD	VD	MER	IC	Total
	10%	$ 56,000.00	Division A	LUX EX2	1-Jul	10 days	90%	Insure Alba	Points
Recruiter	0	-3000	0	1200	800	4000	200	3200	6400
Candidate	4000	-3000	0	1200	2000	0	2400	0	6600

4. The General Contract Model

In many contract negotiations, the parties not always act in good faith or share information with the other party. In this case, one should also consider the perceptions of the parties about the benefits and costs of the tradeoffs. For example, in a merger transaction, the buyer (A) and the seller (B) may not always agree as to the terms of the merger, and hence the transaction fails. The steps to make tradeoffs in this more general situation are as follows:

1. Identify the dimensions of the problem
2. Identify the tradeoffs of each party within the dimensions
3. Identify the benefits accrued by a party from the other party's tradeoffs
4. Identify the costs incurred by a party from its own tradeoffs
5. Identify the perceived benefits that the other party received from your tradeoffs
6. Identify the perceived costs incurred by the other party from their tradeoffs
7. Find out what tradeoff each party must make to maximize the total minimum gain they obtain, ensuring that what each party gains is as close as possible to the other party gains. This is what makes the final contract equitable and balanced.

The mathematical model that helps identify the proper contract is given below.

Let $X_k(x)$ the scale of the *k*th dimension. The parties will negotiate on the value of that scale according to their preferences. The realized value of the scale is determined by the benefit, the cost, the perceived benefit and the perceived cost that that value has for each party.

Let $B_i(x_k)$ be the benefits accrued by party *i* from the other party tradeoffs in dimension *k*. Let $C_i(x_k)$ be the costs incurred by party *i* from its own tradeoffs in dimension *k*. Let $PB_i(x_k)$ be the benefits party *i* perceives the other party receives from its tradeoffs in dimension *k*, and let $PC_i(x_k)$ be the costs the other party perceives that party *i* incurs from its tradeoffs in dimension *k*. Thus, for a given dimension *k*, the gain of party *i* is given by the benefits it accrues from the tradeoffs of the other party in that dimension times the costs it perceives the other party incurs in that dimension, i.e., $B_i(x_k)PC_i(x_k)$. Similarly, the loss in a given dimension *k* is given by $C_i(x_k)PB_i(x_k)$. Thus, the gain to loss ratio for a party for a given dimension *k* is given by:

$$\frac{B_i(x_k)PC_i(x_k)}{C_i(x_k)PB_i(x_k)}$$

and the total gain-to-loss ratio for a party is given by

$$r_i \equiv \sum_{all\ k} \frac{B_i(x_k)PC_i(x_k)}{C_i(x_k)PB_i(x_k)}.$$

Let $x_k(s)$ be a binary variable, where $x_k(s)=1$ if the parties agree on selecting the intensity *s* of the *k*th dimension as the best decision for both of them. The problem now consists in finding values of *s* for each dimension that maximizes the smallest gain-to-loss ratio of both parties, i.e.,

$$\underset{s}{Max}\left\{Min\left\{r_i(s) \equiv \sum_{all\ k} \frac{B_i[x_k(s)]PC_i[x_k(s)]}{C_i[x_k(s)]PB_i[x_k(s)]}, r_j(s) \equiv \frac{B_j[x_k(s)]PC_j[x_k(s)]}{C_j[x_k(s)]PB_j[x_k(s)]}\right\}\right\}$$

Subject to $\sum_s x_k(s)=1$ and $\left|1-\frac{r_i(s)}{r_j(s)}\right| \le \varepsilon$, where ε is the tolerance that measures how far the two parties are in terms of their total gain-to-loss ratio.

5. Conclusions

The main difference between this approach and that used in the analysis of the Palestinian-Israeli conflict (Saaty and Zoffer 2011; 2013) is that the tradeoffs were analyzed in pairs. In the case of a contract, the scales in which the dimensions are measured makes it impossible to analyze all possible pairs of tradeoffs. For example, in the simple case given above, the number of tradeoffs is $2^{8\times 4} = 4,294,967,296$. Thus, we use a non-linear optimization model, albeit it is also a MaxMin model.

6. References

Fisher, R. and W. Ury (1981). *Getting to YES: Negotiating Agreement Without Giving In*. New York, Penguin Books.

Saaty, T. L. and H. J. Zoffer (2011). "Negotiating the Israeli Palestinian Controversy from a New Perspective." *International Journal of Information Technology and Decision Making* **10**(1): 5-64

Saaty, T. L. and H. J. Zoffer (2013). "Principles for Implementing a Potential Solution to the Middle East Conflict." *Notices of the American Mathematical Society* **60**(10): 1300-1322