PERFORMANCE OF COMPATIBILITY INDICES FOR HIGH N VECTORS

ABSTRACT

Compatibility indices measure the closeness level between priority vectors. This paper presents a performance assessment of three indices (Saaty's *S* and Garuti's *G*, and Salomon's *V* indices) for high *n* vectors. It found a better performance of indices *S* and *V* in relation to index *G*. It also reinforces assumptions of earlier studies about sensitivity of index *G* to high *n*; and that due index *V* to use ordinal vectors is less sensitive than *S*.

Keywords: Compatibility index, priority, vectors.

1. Introduction

Although the discussion about compatibility indices have appeared in the 1990s, only a few studies have been developed to assess their performance. That is, as an index is efficient in assessing the closeness between vectors, whereas poor performance means to indicate incompatibility for similar vectors.

This work seeks to contribute to performance study of compatibility indices, in particular related to analyzing high n vectors. We consider high *n*, when *n* is greater than 4.

2. Literature Review

The compatibility indices have particular characteristics related to their calculation procedure, as well as sensitivity related to characteristics of vectors under consideration. Saaty (1994) proposed the compatibility index *S*, using the Hadamard Product. Saaty & Peniwati (2013) verifying sensitivity of index *S*, in comparing small value to other with greater magnitude, suggest as an alternative to use index *G* proposed by Garuti (2007), which one is based on the inner product between vectors. In the same sense, Salomon (2010) has proposed the index *V*, which uses index *S* formula, however with ordinal vectors, and had proposed to use indices *S* and *V* together. On the other hand, it has also been observed a tendency to reduction of value of the index *G* when the vectors have high *n* (Garuti & Salomon, 2012).

3. Hypotheses/Objectives

The aim is to compare the compatibility of indices in assessing closeness between vectors of order 9. The assumptions related to sensitivity discussed in item 2 should be considered.

4. Research Design/Methodology

The indices *S* and *V* are obtained with Equation 1, where *n* is the number of elements of the x and y vectors, **e** is a column-matrix with all $e_i = 1$, $a_{ij} = x_i/x_j$, $b_{ij} = y_i/y_j$ ($i, j = 1, 2,.. n$), and • is the Hadamard Product operator (Saaty, 2001).

$$S = (1/n^2)\mathbf{e}^T\mathbf{A}\bullet\mathbf{B}^T\mathbf{e} \quad (1)$$

Index G, for vectors x and y, is obtained with Equation 2 (Garuti & Salomon, 2012).

$$G = \sum_i[(\min(x_i,y_i)/\max(x_i, y_i))(x_i + y_i)/2] \quad (2)$$

5. Data/Model Analysis

Table 1 shows priority order obtained from real priority vectors of an ongoing research: P1 is the aggregated priority for companies from different branches, and P2 is the priority for a small plant of shoes.

Table 1
Priority vectors

P1	Priority order	1	2	3	4	5	6	7	8	9
	Priority [%]	30.2	20.7	13.2	8.9	8.4	7.8	4.2	3.5	3.1
P2	Priority order	1	2	3	4	6*	8*	7	5*	9
	Priority [%]	25.8	21.43	11.38	10.01	7.14	7.07	6.11	5.55	5.51

Only the priority order of the components 5, 6 and 8 changed to 8, 5 and 6, respectively, did not change the order of the six other components. The compatibility indices are $G = 0.833$ (lower limit is 0.9), $S = 1.128$, and $V = 1.038$ (for S and V, upper limit is 1.129 to $n = 9$). According G vectors are incompatible, while for V and S are compatible.

6. Limitations

Few studies until this moment, and need more evaluations with more different vectors.

7. Conclusions

This work reinforces results of earlier studies. According to data analyzed, there is greater sensitivity of the index G to high n vectors, and better performance of indices S and G in relation to G. The index V has showed lower sensitivity than the S. There is a need for further studies related to the use of S and V together, as well as to assess greater changes in vectors.

8. Key References

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