LOCAL PROPERTIES OF SYNTHESES FOR CATEGORIZED AHP

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ABSTRACT

If alternatives are divided into some categories, decision makers often enforce AHP on every category and synthesize their results. We provide principles that the synthesis must satisfy. These principles have axiomatic descriptions and represent a relation between local properties and results of the synthesis.

Keywords: categorized alternatives, axiomatic descriptions

1. Introduction

If alternatives are divided into some categories, difference of the categories complicates enforcement phases in AHP. So decision makers often enforce the phases for each category independently, and then decision makers synthesize results on categories by their convenient way. We provide principles for the syntheses.

2. Literature Review

In actual applications of AHP, decision makers often divide a set of alternatives into some categories, and they synthesize results of AHP of the categories. There are many reports and papers treat the divisions and syntheses implicitly. Mizuno and Kinoshita (2014) describe a concrete procedure for categorized alternatives by using Dominant AHP technique.

3. Hypotheses/Objectives

This research deals with AHP whose alternatives are categorized. We regard syntheses of results on these categories. Our purpose is to provide principles that the syntheses must satisfies.

4. Research Design/Methodology

We write alternatives in $a_1, a_2, \cdots$ or $b_1, b_2, \cdots$. Categories of alternatives is $Alt_1 = \{a_1, a_2, \cdots\}$, Alt_2, ⋯, Alt_k and a set of all alternatives is $Alt = Alt_1 \cup Alt_2 \cup \cdots \cup Alt_k$. A result of AHP, that is set of scores of alternatives in Alt, is $\mathcal{F}(Alt)$ (Fig.1). We use same notation $\mathcal{F}$ without confusion, and we write a result of Alt_i in $\mathcal{F}(Alt_i)$.

We describe relations between $\mathcal{F}(Alt)$ and each $\mathcal{F}(Alt_i)$; What are conditions for syntheses of $\mathcal{F}(Alt_1), \cdots, \mathcal{F}(Alt_k)$ into $\mathcal{F}(Alt)$?

5. Data/Model Analysis

There are two problems of synthesizing results of categories. First, for examples, when a alternative ξ belongs to Alt_1 and Alt_2, a score of ξ as an element of Alt_1 must equal a score of ξ as an element of Alt_2 (Fig.2). Second, relative scores of alternatives in a category against alternatives in another category must not depend on the number of elements in the category.

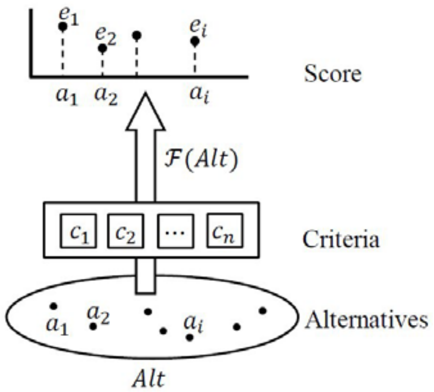

Fig.1

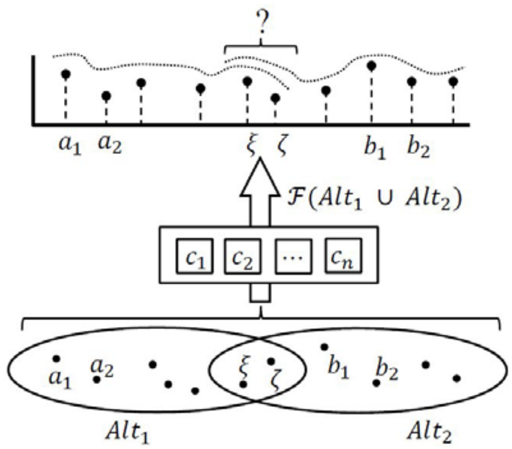

Fig.2.

We write a score of an alternative a_i in (a_i, e_i), and a possible sets of scores of Alt in $\mathcal{F}(Alt) = \{\{(a_1, e_1), (a_2, e_2), \cdots\}, \{(a_1, e_1'), (a_2, e_2'), \cdots\}\}$. And we define a restriction r. If $U \subseteq Alt$, there are $\zeta' \in \mathcal{F}(U)$ and $\zeta \in \mathcal{F}(Alt)$ that $\zeta' = r_U^{Alt}(\zeta)$, $\zeta' \subseteq \zeta$, and there is constant c which $(a, e) \in \zeta'$ and $(a, ce) \in \zeta$. When alternatives Alt is divided into categories U and V, our requirements are [R1] and [R2].

[R1] For all $\zeta, \xi \in \mathcal{F}(Alt)$, and for all $W \in \{U, V\}$, if $r_W^{Alt}(\zeta) = r_W^{Alt}(\xi)$ then $\zeta = \xi$.

[R2] If $\zeta \in \mathcal{F}(U), \xi \in \mathcal{F}(V)$, and $r_{U\cap V}^{U}(\zeta) = r_{U\cap V}^{V}(\xi)$ then there is a scores $\alpha \in \mathcal{F}(Alt)$ that $\zeta = r_U^{Alt}(\alpha)$ and $\xi = r_V^{Alt}(\alpha)$.

Conditions [R1] and [R2] are local properties. A term "local" means a subset of relations between alternatives and its scores. The conditions is an axiomatic description of relations of local properties of the syntheses of categorized AHP.

6. Limitations

We describe the required conditions in [R1] and [R2] abstractly, and there is no concrete procedure for synthesis scores. Mizuno and Kinoshita (2014) describe a procedure for the synthesis that satisfies the conditions.

7. Conclusions

Saaty (1996) described an axiomatic foundation of AHP. It represents micro properties of pairwise comparisons and their syntheses. Conditions [R1] and [R2], which we provide, are interpreted as axiomatic representation of local properties of synthesis in AHP.

8. Key References

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