MUDDLED MAGNITUDES

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Abstract

1. Introduction

In the additive AHP model with relative measurement, the overall preferences $V_1,V_2,...,V_n$ of $A_1,A_2,...,A_n$ are estimated by the weighted arithmetic means $f_1,f_2,...,f_n$

$$f_j = w_1ß_1y_{j1} + w_2ß_2y_{j2} + ... + w_mß_my_{jm}, \ j = 1,2,..., \ n \quad (1)$$

where

w_i	is the importance weight of criterion i
y_{ji}	is a ratio derived scale that measures and estimates alternative A_j on criterion i
$ß_i$	is a positive constant that represents different scaling or normalizations of the ratio y_{ji}

In (1), f_j, the overall preference, can be looked upon as the sum of partial $w_iß_iy_{ji}$ values. In order for summation of those partial values to yield a ratio answer, their individual parts, before addition, must be in commensurate units. The values of y_{ji} are usually not known explicitly and a pairwise comparison matrix of preferences is used to estimate the values of $y_{1i}, y_{2i},...,y_{ni}$ for each criterion i. The scaling constants $ß_1$, $ß_2$,..., $ß_m$ in (1) have been included explicitly to show that a positive multiple does not destroy the ratio relationship amongst a criterion's y_{ji}. Usually, $ß_i$ is chosen so that the $ß_iy_{ji}$ of all criteria conform to the same standard format (sum to one for the distributive mode or the best alternative equals 1 for the ideal mode). Other normalizations are possible. A muddled mode uses different formats for different criteria. A muddled mode helps to illustrate the interrelationship between w_i and different normalizations of $ß_i y_{ji}$. Since $ß_i y_{ji}$ values are ratio but not commensurate, the function of the w_i values is to assure commensurability before the partial values in (1) are summed. Not just any set of w_i achieves the commensurability.

2. The process

Vargas' (1997) example of 3 boxes that have different components of 4 objects has been used to analyze different normalization procedures. Each box is considered to be a criterion and the objective is to determine the assembled weight of each object. The true relative overall priorities that must be generated by the various methods are .243, .286, .243 and .229 for Objects 1 to 4 respectively.

Tables of $ß_i$ y_{ji} are developed for the distributive, ideal and muddled modes. In order to generate the correct overall relative preferences for the distributive mode, it is shown that w_i comparisons are between

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the total of each criterion possessed by all relevant alternatives. For the ideal mode, w_i comparisons are between the criteria possessed by the ideal alternatives

Table 5 below presents muddled magnitudes in each column: Box 1,according to the distributive mode (i.e. $ß_1 = 1/\Sigma y_{j1}$); for Box 2, according to the ideal mode (i.e. $ß_2 = 1/\text{best } y_{j2}$); and for Box 3, according to the least desirable alternative (i.e. $ß_3 = 1/\text{worst } y_{j3}$). It is apparent that the unit for column 1 is the sum of all components and for columns 2 and 3, the components for Object 3

Table 5 – Muddled mode $ß_i y_{ji}$ and Composite Priorities

Criteria priorities=	1.667	1.333	1		
	Box 1	Box 2	Box 3	Simple Total	Weighted Total
Object 1	0.1	0.75	1.667	**2.5167**	2.833
Object 2	0.2	0.5	2.333	**3.0333**	3.333
Object 3	0.3	1	1	**2.3**	2.833
Object 4	0.4	0.25	1.667	**2.3167**	2.667
Total	**1**	**2.5**	**6.667**	**10.167**	

For the muddled mode where the $ß_i y_{ji}$ unit of each criterion is established in a different manner, the appropriate w_i values are determined by comparing the referent alternative(s) that form each unit. Thus the weight of Box 1 (10 lbs.) is compared to the weight of item 3 of Box 2 (8 lbs) and item 3 in Box 3 (6 lbs). In Table 5, we have chosen to show the resulting criteria weights in terms of the Object 3 component in Box 3. Using those ratios gives the correct relative overall preference (i.e. 3.333/2.833 =.286/.243 = 1.176). The same relative results occur from normalizing the w_i to sum to one.

3.0 Discussions and Conclusion

Appropriate w_i values depend upon how $ß_i$ values have been established. Very often, the $ß_i y_j$ of the distributive mode ($ß_1 = 1/\Sigma y_{j1}$) are converted to the ideal mode by applying $ß_i = 1/\text{best } y_{ji}$ to the distributive mode priorities. These changes in unit of measure are legitimate transformations that maintain ratio relationships within criteria. However, such a change without a corresponding change in w_i will lead to incorrect ratios. To get the correct values after changing $ß_i$ values, it is necessary to adjust w_i values.

As well, addition or deletion of an alternative can change the unit of $ß_i y_j$ values if renormalization takes place after the change. This is a particular problem for the distributive mode, since its local priorities and unit depend upon the set of alternatives being used. Additions or deletions change the set, the unit of measure of local priorities, and the criteria weights that will generate correct overall preferences.

Since AHP is based upon ratio measurement, natural zero and a unit of measure identify its derived scales. The units of measure in Table 5 for Box 2 and 3 are fairly explicit – they are the best item in Box 2 and the worst item in Box 3. The unit of measure for the items in Box 1 is more difficult to ascertain. In the distributive mode, the total rather than the local preference of any one alternative takes the value of unity and the total criterion possessed by all relevant alternatives should be the unit to compare when deriving criteria weights. Alternatively, a specific alternative from each criterion could be compared Then, these linking priorities for criteria can be scaled upward to reflect the unit sum totality of all local priorities. We suggest that this process is better able to handle non-linear relationships between derived scales and their individual components.

Our use of muddled magnitudes is for illustrative purposes only. In practice, we would not recommend such muddling, because it increases the cognitive load when making criteria comparisons. Nevertheless, the illustration is useful, because the muddled magnitudes, placed side by side, illustrate that different normalizations of derived scales lead to different units of measure. Recognition of the unit of the derived scales assists in generating valid criteria weights.