

COMPLEX NUMBER PAIRWISE COMPARISON AND COMPLEX NUMBER AHP

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Summary: *New mode of ambiguity expression, "complex number", is introduced into pairwise comparison and AHP. The imaginary part of judgment reflects some type of ambiguity. Their applicability is examined through some examples.*

Abstract

There can be many kinds of ambiguity modes. Some of them are probability[KITO02], fuzziness[BUCK85], interval expression[ARBE92], and so on. As one of ambiguity expressions, we propose to use "complex number" and to incorporate it into pairwise comparison and AHP, which are called complex number pairwise comparison (C-comparison) and complex number AHP(C-AHP), respectively.

Complex number pairwise comparison (C-comparison)

The (j, k)th element of pairwise comparison matrix A, a_{jk} , indicates the dominance of item j over item k, or how many times more important item j is than item k. These a_{jk} data are usually measured in the real scale. Even in case of ambiguity existence, they have been expressed in the forms of probability distribution[KITO02], membership function[BUCK85] and interval[ARBE92]. These ambiguity expressions have their merits and demerits, which are summarized in Table 1. As a new mode of ambiguity expression for the pairwise comparison measurement we will propose to use "complex number", instead of "real number". Although its measurement may not be so easy compared to the probabilistic comparison or the interval comparison, the C-comparison has its advantages in transitivity satisfaction in consistency case and understanding its process and result, whose explanation will be tried.

Measuring pairwise comparison judgement in complex number

With the conventional fundamental scale, the intensity of importance ranges from 1 to 9. Say the importance intensity "5" (strong importance) means that experience and judgment strongly favor one activity over another. Mathematically speaking it means that experience and judgment 5 times more favor one activity over another. Here all the pairwise comparison judgments are measured in the real. If we have some kind of ambiguity with this intensity "5" measurement, how do we distinguish among them. Then, complex number a_{jk} will be proposed to distinguish among the measurements with the same intensity but different ambiguity degrees.

$$a_{jk} = r_{jk} \exp(iq_{jk}) \quad (1)$$

$$= r_{jk} (\cos q_{jk} + i \sin q_{jk}) \quad (2)$$

r_{jk} : intensity or amplitude of (j, k) pairwise comparison judgment

q_{jk} :degree of inclination angle from the real toward the imaginary

i = imaginary unit

Table1 Merits and demerits of four ambiguity expressions

	Probabilistic comparison	Fuzzy comparison	Interval Comparison	Complex Comparison
Measurement	Easy	Moderate	Easy	Moderate
Weight estimation	Moderate	Difficult	Difficult	Moderate
Process accountability	Moderate	Questionable	Questionable	Clear
Validity of result	Moderate	Questionable	Questionable	Clear
Transitivity satisfaction in consistency case	Moderate	Difficult	Difficult	Clear

References

Masahiro KITO and Masaaki SHINOHARA : Proposal of Probabilistic AHP , Proceeding of 35th Academic Conference of Nihon University , Mathematical Information Engineering Department , CIT , 7-9 , pp.27-28(2002.12).

J.J.Buckle :Fuzzy Hierarchical Analysis , Fuzzy Sets and Systems ,Vol.17,pp.233-247 (1985).

A.Arbel and L.G.Vargas : The Analytic Hierarchy Process with Interval Judgments , Multiple Criteria Decision Making , Springer – Verlag , pp.61-70(1992).