Traditional, DEA-AHP ranking and possibilistic fuzzy DEA approach for efficiency analysis of city hospitals

ABSTRACT

In this study, we seek to explore six city hospitals efficiency by using public hospitals statistical records for the year 2017. Conventional DEA, DEA-AHP ranking and possibilistic fuzzy DEA models are generated and compared. Study findings provides many lights to compare conventional and fuzzy DEA efficiency results and findings pose that conventional DEA estimates overscores efficiency estimates. Traditional and possibilistic fuzzy DEA estimates with different α-cut levels are consistent and a decreasing possibilistic efficiency trend is observed while increasing α-cut parameter from 0 to 1 in possibilistic fuzzy DEA model.

Keywords: efficiency, DEA, DEA-AHP ranking, possibilistic efficiency, fuzzy DEA

1. Introduction

In every sector, efficiency is a key element of decision-making because it directly affects business and competition results (Rouyendegh et al., 2019). The ability to make accurate decisions is critical in health care management, especially when health care costs are spiking and the number of individuals insured is increasing. To ensure actionable productivity measurements, the sector needs to measure efficiency frequently and accurately (Barnum et al., 2016). Hospitals play a significant role as providers in health sector (O'Hanlon et al., 2019). It is essential for hospitals to evaluate their performance and identify the shortages among competing companies based on their inputs and outputs (Otay et al., 2017). Better location and operationalization of hospital services provides insight into the health systems performance assessment (Plott et al., 2022). Policy makers and stakeholders in health system need to pay special attention to efficiencies in the wake of recent health sector investments, such as health campuses (city hospitals).

Health campuses (city hospitals) which are built with public-private partnership models are current health policy actions of Turkish government (MoH, 2022). Existing literature provide an evidence about opinions of stakeholders about city hospitals in Turkey. Moreover, most participants stressed the importance of standardization and feasibility in public-private partnership models (Top & Sungur, 2019). However, integrated health campuses located in metropolitan areas of the country, such as İstanbul and Ankara negatively impact people's access to health services by taking into account the cities' geographic integrity in terms of urban traffic and distance between hospitals and the city center (Top & Sungur, 2019). A unique evidence provides clues about efficiency of city hospitals build with public-private partnership models in Madrid Health Service. It has been stated that greater efficiency is obtained from public-private collaboration models than in traditionally managed hospitals (Franco Miguel et al., 2019). As far as we know, there are no existing studies about the efficiency of large city hospital investments in Turkey. This study is designed to fill this void, by comparing conventional, AHP-DEA ranking and possibilistic fuzzy DEA models performances to explore efficiencies of city hospitals.

2. Literature Review

There are various optimization techniques for quantitative problems. Additionally, there exists a huge literature about quantitative techniques to better measure hospital performance and data envelopment analysis (DEA) is the pioneering one among other techniques (Kohl et al., 2019). In contrast to multivariate statistics, DEA offers a wide spectrum of perspectives (Rouyendegh et al., 2019). There are two types of units in DEA: efficient and inefficient. Based on a dual arrangement of outputs, several outputs have a positive influence on this classification. The initial DEA does not facilitate fully ranked studies. A dual grouping of efficient and inefficient units is simply presented, rather than a weighted positioning of the units. A positive equivalence is assumed for all units classified as efficient in this model. In DEA, units are categorized into two categories: efficient and less efficient, based on a binary output that subsidizes the overall quantification process. Through DEA, units can be classified into efficient and less efficient dichotomous categories without ranking them. According to the Pareto principle, all units classified as efficient are technically good (Ganley & Cubbin, 1992; Rouyendegh et al., 2019).

Conventional DEA requires crisp numerical values for performance evaluation. In real-world problems, inputs and outputs are often inaccurate or vague. Fuzzy numbers are used in DEA to reflect decision makers' intuition and subjective judgments. An additive fuzzy DEA model is proposed for evaluating the efficiency of decision-making units (DMUs) with fuzzy inputs and outputs instead of the traditional DEA model (Hatami-Marbini et al., 2012). Among the most popular methods for handling epistemic uncertainty is an optimization model based on probabilistic programming (León et al., 2003). In order to measure the probabilities of fuzzy events, three measures are considered: possibility, necessity, and credibility (Peykani et al., 2018).

DEA manipulates uncertain data by using possibility distributions. It is challenging to design probability distributions because they either demand a posteriori frequency determination or a priori known regularity. An alternate strategy is to represent the ambiguous values using fuzzy set theory's membership functions. There is a challenge in designing probability distributions because it requires either a priori predictable regularity or a posteriori frequency estimation. An alternate strategy is to represent the ambiguous values using fuzzy set theory's membership functions (Hatami-Marbini et al., 2012). Lertworasirikul et al. (2003) is based on the (Zadeh, 1978)'s "possibility" and "credibility" approaches to overcome the ranking problem in the DEA-CRS model on the basis of the foundational principles of possibility theory. After that, (Lertworasirikul et al., 2003) is proposed the possibility approach to fuzzy DEA-VRS model. The fuzzy DEA model is transformed into a linear model that can be solved with linear programming software. When dealing with fuzzy and uncertain data, the "possibility" and "credibility" approaches can be useful (Amini et al., 2019). As part of the standard DEA procedure, a pair of patterns is solved, namely multiplier patterns and envelopment patterns. Due to the dual nature of linear programming, one of the most interesting aspects of DEA is that these two types of models are comparable. Several DEA models have been developed in the literature in multiplier and/or envelopment forms, it is still unclear whether and how the same primal-dual correspondence can be solved (Lim & Zhu, 2019). The computational performances of different DEA models are examined in the literature. These include primary and dual versions of one-step and two-step techniques. The implementation based on the main DEA model is faster than that based on the dual model, which has a number of limitations (Green & Doyle, 1997).

Fuzzy set theory applications in DEA are divided into four categories: the possibility method, the fuzzy ranking approach, and the fuzzy ranking approach. These applications are well known for using the α-level strategy (Lertworasirikul et al., 2003). The drawbacks of the possibility approach, the tolerance approach, and the fuzzy ranking technique are that when real information is translated from a fuzzy model to a very exact model, it is sometimes missed. Therefore, α-level approach is applied to account for any vague information in performance evaluation (Pourmahmoud & Bafekr Sharak, 2018). The range of DMU's efficiency score at various probability levels can be determined by using the α-cut technique. The idea behind the α-cut approach is Zadeh's extension principle to transform a fuzzy DEA model into a family of clear DEA models that can be described by a pair of parametric programs to find the α-cuts of functions of the efficiency measures. Numerical solutions for various α-cut levels are computed to approximate the membership function. Since efficiency measures are expressed by membership functions rather than crisp values, a fuzzy number classification method can be used to determine which DMU performs best (Kao & Liu, 2000). Most α-cut approaches require classification of the membership function of fuzzy numbers. The lowest number of α-cuts necessary to maintain the effectiveness and applicability of the organization of the fuzzy numbers aggregated according to the proposed methods (Kao & Liu, 2000).

3. Objectives

The goal of this study is to explore DEA based fuzzy multi strategy decision making model for city hospitals in order to improve their performance. DEA improved with fuzzy AHP is used to evaluate the information and construction the model in decision making. This hybrid model provides several benefits to give most appropriate decision by including the value of the weights determined by the data from the hybrid model (Rouyendegh et al., 2019). Greater efficiency is noticed from the hospitals managed by public private collaboration in Madrid by using conventional efficiency estimates (Franco Miguel et al., 2019). However, there is only scarce evidence on comparing traditional and fuzzy efficiency estimates to compare efficiencies of health campuses (city hospitals). The proposed study provides a unique contribution to the existing knowledge to compare multicriteria decision making alternatives, which are crisp and fuzzy efficiency estimates by considering uncertainties in decision units.

4. Research Design/Methodology

4.1. Basic concept of conventional DEA

Farrell (1957) first developed the basic concept of DEA (Farrell, 1957). Then, Charnes et al. (1979) developed a DEA model is based on the assumption of constant return to scale (CRS) (Charnes et al., 1979). Banker et al. (1984) adopted the CRS model into variable return to scale (VRS) and developed the popular DEA-BCC model (Banker et al., 1984). Different DEA models applied, each with different technical details, to solve real-world efficiency problems better (Berger & Humphrey, 1997; Tone et al., 2019; Wanke & Barros, 2016).

Suppose that there are *n* DMUs, and each DMU_j $(j = 1, \dots, n)$ has *m* inputs denoted by X_j $(j = x_{1j}, \dots, x_{mj})$ and *s* outputs denoted by Y_j $(j = y_{1j}, \dots, y_{sj})$. Due to the study of Banker et al. (1984), the PPS^V under VRS can be formulated as follows (Chen & Wang, 2021).

$$PPS^V\left\{(X,Y)\middle|\sum_{J=1}^{n}\lambda_j X_J \leq \text{X};\ \sum_{J=1}^{n}\lambda_j Y_J \geq Y;\ \sum_{J=1}^{n}\lambda_j = 1;\ \lambda_j \geq 0; j = 1, \dots, n\right\}$$

Where λ_j presents the intensity vector. If the constraint $\sum_{j=1}^{n}\lambda_j = 1$ is removed from PPS^v, then it would be converted to the PPS^C under CRS, which is shown as follows.

$$PPS^C=\left\{(X,Y)\middle|\sum_{J=1}^{n}\lambda_j X_J \leq \text{X};\ \sum_{J=1}^{n}\lambda_j Y_J \geq Y;\ \lambda_J \geq 0; j = 1, \dots, n\right\}$$

According to the definition of PPS^V, the traditional DEA model under VRS, namely BCC model, A certain DMU's relative efficiency can be evaluated using this method DMU_d as follows.

$$\theta_d^I = \min \quad \{\theta_d \ : \ (\theta_d X_d, \quad Y_d) \in PPS^V\} \qquad \textbf{(1)}$$

$$\phi_d^O = \max \quad \{\phi_d : \quad (\theta_d X_d, \quad Y_d) \in PPS^V\} \qquad \textbf{(2)}$$

Models (2) and (3) correspond to input-oriented and output-oriented models, respectively and θ_d^I and ϕ_d^O represent the optimal efficiencies of DMU_d generated by these models. For model

(2), DMU_d is efficient if $\theta_d^I = 1$, or in efficient if $\theta_d^I < 1$; while for model (3), DMU_d is efficient if $\phi_d^O = 1$, or inefficient if $\phi_d^O > 1$. (Chen & Wang, 2021)

4.2. The AHP/DEA ranking model

This study includes two well-known methods DEA and AHP. In the following sections, each model is described, after which the multilevel model is developed. In the first stage of the procedure DEA-AHP method can be summarized as in the following steps:

Step 1: Estimate the decision matrix of the DEA method ($e_{k,k'}$). With *m* alternatives and *n* criteria, $e_{k,k'}$ is as follows (Rouyendegh & Erol, 2010; Rouyendegh et al., 2019; Sinuany-Stern et al., 2000):

$e_{k,k'} = max \sum_{r=1}^{t} u_r y_{rk}$ **(3)**

Subject to

$\sum_{i=1}^{m} v_i \, x_{rk} = 1$ **(4)**

$\sum_{r=1}^{s} u_r y_{rk} - \sum_{i=1}^{m} v_i \, x_{rk} \leq 0$ **(5)**

$\sum_{r=1}^{s} u_r y_{rk'} - \sum_{i=1}^{m} v_r x_{rk'} \leq 0$ **(6)**

$u_r \geq 0, \mathrm{r} = 1, \dots, \mathrm{t}$

$v_i \geq 0, \mathrm{i} = 1, \dots, \mathrm{m}$

The solution of this problem yields the values for $e_{k,k'}$ elements as well as the binary E comparison matrix ($k' = 1, \dots, n, k = 1, \dots, n$ and $k \neq k'$).

$a_{k,k'}$ *Step 2:* Compute the components of the pairwise comparison matrix from which are derived Eq. (**7**):

$$a_{k,k'} = \frac{e_{k,k'}}{e_{k',k}} \quad \textbf{(7)}$$

Step 3: Each component derived from the second step is divided by the total value of the column.

The matrix obtained here is a normalized matrix as displayed in Eq. **(8)**

$$\acute{a}_{k,k'} = \frac{a_{k,k'}}{\sum_{k=1}^{n} a_{k,k'}} \quad \textbf{(8)}$$

Step 4: The column vector elements are estimated via the collection over the rows as in Eq. **(9)**

$$a_{k,k'}^{n} = \sum_{k=1}^{n} \acute{a}_{k,k'} \quad \textbf{(9)}$$

Step 5: Normalize the column vector via Eq (**10**):

$$a_{k,k'}^{m} = \frac{a_{k}^{n}}{\sum_{k=1}^{n} a_{k}^{n}} \quad \textbf{(10)}$$

In the second phase of the procedure the AHP ranking is summarized. Based on the pairwise comparison matrix *A*, generated in the first step, a single hierarchical level AHP is run to calculate the maximal eigenvalue λ_{max} and its corresponding eigenvector $\vec{w}$. It is not necessary to impose rank 1 here on the sum of the elements of the eigenvector, because we have only one level of AHP. The *j*th component of $\vec{w}$ reflects the relative importance given to unit j. We assign the rank 1 to the unit with maximal value of w_j etc., in a decreasing order of w_j. (Sinuany-Stern et al., 2000)

During the pairwise comparison matrix generation process, we run the DEA for two units at a time; this often gives many effective values (Eq. (4)), especially as the number of inputs and outputs increases. Due to the multiple outputs and inputs embedded in this multicriteria analysis, this phenomenon is reflected in the pareto optimum results. If there is a pair of inputs and outputs for which one unit outperforms the other, it will get a DEA score of 1 and vice versa; i.e., a unit receives a comparison value less than 1 with respect to another unit if it is worse in all possible combinations of inputs and outputs (Sinuany-Stern et al., 2000).

4.3. Lertworasirikul-Fang-Joines-Nuttle fuzzy DEA Model

Lertworasirikul et al. (2003) developed a possibility perspective in which constraints are treated as fuzzy events. This approach transforms fuzzy DEA models into possibility DEA models using fuzzy constraint probability measures (Lertworasirikul et al., 2003). The well-known DEA model used is the CCR model, named after Charnes, Cooper and Rhodes (Charnes et al., 1978). Suppose that there are *n* DMUs, each of which consumes the same type of inputs and produces the same type of outputs. Let *m* be the number of inputs and let *r* be the number of outputs. All inputs and outputs are assumed to be non-negative, but at least one input and one output are positive. The following notation used in the rest of the paper.

Symbols used

DMU_i is the *i*th DMU,

DMU_0 is the target DMU,

$\boldsymbol{x_i} \in R^{m\,x\,1}$ is the column vector of inputs consumed by DMU_i,

$\boldsymbol{x_0} \in R^{m\,x\,1}$ is the column vector of inputs consumed by the target *DMU*,

$\boldsymbol{X} \in R^{m\,x\,n}$ is the matrix of inputs of all DMUs,

$\boldsymbol{y_i} \in R^{r\,x\,1}$ is the column vector of outputs produced by DMU_i,

$\boldsymbol{y_0} \in R^{r\,x\,1}$ is the column vector of outputs produced by the target DMU,

$\boldsymbol{Y} \in R^{r\,x\,n}$ is the matrix of outputs of all DMUs,

$\boldsymbol{\lambda} = (\lambda_i)_{nx1}$, $\lambda \in R^n$ is the column vector of a linear combination of *n* DMUs,

0 is the objective value (efficiency) of the CCR model,

$\boldsymbol{u} \in R^{mx1}$ is the column vector of input weights and,

$\boldsymbol{v} \in R^{rx1}$ is the column vector of output weights.

4.3.1. DEA and Dual DEA

In the CCR model, the multiple inputs and multiple outputs of each DMU are merged into a single virtual input and a single virtual output, respectively. The CCR model and its dual are figured out as the following linear programming models (Lertworasirikul et al., 2003):

$$\text{(CCR)} \quad \max_{\boldsymbol{u},\boldsymbol{v}} \quad v^T y_0$$
$$\text{s.t.} \quad u^T x_0 = 1,$$
$$-u^T X + v^T Y \leq 0, \qquad \textbf{(11)}$$
$$u \geq 0,$$
$$v \geq 0.$$

$$\text{(DCCR)} \min \quad \theta$$
$$\text{s.t.} \quad \theta x_0 - X\lambda \geq 0, \qquad \textbf{(12)}$$
$$Y\lambda \geq y_0,$$
$$\lambda \geq 0.$$

From the duality of the linear programming technique, the ideal objective upsides of the CCR and DCCR models are similar. Let θ^* be the optimal objective value (efficiency value). Using the constraints $u^T x_0 = 1$ and $-u^T X + v^T Y \leq 0$ in (1), an efficiency value θ^* of the target DMU falls in the range of $(0,1]$ (Lertworasirikul et al., 2003)

4.3.2. Fuzzy DEA model

Fuzzy set theory has been proposed as a way to quantify imprecise data in DEA models. Fuzzy DEA models take the form of fuzzy linear programming models. The CCR model with fuzzy coefficients and its dual is presented in Eq. (**13**) and (**14**):

$$\text{(FCCR)} \quad \max_{\boldsymbol{u},\boldsymbol{v}} \quad v^T \tilde{y}_0$$
$$u^T \tilde{x}_0 = 1,$$

$$-u^T\tilde{X} + v^T\tilde{Y} \le 0, \quad \textbf{(13)}$$
$$u \ge 0,$$
$$v \ge 0.$$

(DFCCR) min $\tilde{\theta}$

s.t. $\theta\tilde{x}_0 - \tilde{X}\lambda \ \ge 0,$ **(14)**

$$\tilde{Y}\lambda \ \ge \ \tilde{y}_0,$$

$$\lambda \ge 0,$$

Where $\tilde{x}_0$ is the column vector of fuzzy inputs consumed by the target DMU (DMU_0), $\tilde{X}$ is the matrix of fuzzy inputs of all DMUs, $\tilde{y}_0$ is the column vector of fuzzy outputs generated by the target DMU (DMU_0), and $\tilde{Y}$ is the matrix of fuzzy outputs of all DMUs (Lertworasirikul et al., 2003).

Like the CCR model, the constraints $u^T\tilde{x}_0 = 1$ and $-u^T\tilde{X} + v^T\tilde{Y} \le 0$ in the FCCR model are used to normalize the value $v^T\tilde{y}_0$. However, the objective value $v^T\tilde{y}_0$ can now exceed once since the second and third constraints of Eq. (3) are satisfied "possibilistically". That is, since their parameters are fuzzy sets, $u^T\tilde{x}_0$ is "approximately equal to one", which implies that $v^T\tilde{Y}/u^T\tilde{X}$ is "roughly less than or equal to one" (Lertworasirikul et al., 2003).

Fuzzy CCR models cannot be settled by a standard linear programming solver, for example, a CCR crisp model in light of the fact that the coefficients in the fuzzy CCR model are fuzzy sets. With fuzzy inputs and fuzzy outputs, the optimality conditions for the clear DEA model should be clarified and generalized. The related fuzzy linear programming issue is generally tackled utilizing some fuzzy set grouping techniques. Four particular methodologies exists in the literature to solve fuzzy DEA problems, for example, tolerance approach, defuzzification approach and α-level based perspective and fuzzy ranking perspective (Lertworasirikul et al., 2003) The tolerance approach for fuzzy DEA incorporates vulnerability to be integrated into the DEA models by characterizing tolerance levels for limitation violations (Sengupta, 1992). Fuzzy data sources and results are defuzzified to net qualities in the defuzzification approach. Using these crisp values, the crisp model can be estimated by a linear programming solver (Yager & Filev, 1993). In the α-level based approach, the fuzzy DEA model is tackled by parametric programming utilizing α-cuts. Fuzzy efficiencies can be constructed by using number of intervals. Solving the model at a given α-cut level generates a corresponding interval efficiency for the target DMU (Kao & Liu, 2000; Wanke et al., 2016). A DMU said to be α-possibilistic nondominated if the maximum value of the fuzzy efficienct at that α level is greater than or equal to 1 (Lertworasirikul et al., 2003). The α-level based approach gives fuzzy productivity yet requires the ranking of fuzzy effectiveness sets. The fuzzy ranking methodology gives fuzzy effectiveness to an assessed DMU at a predetermined α-level. (Guo & Tanaka, 2001)

4.3.3. Possibility DEA model

Different from the crisp CCR model, in the case of fuzzy inputs and fuzzy outputs, the relationship between the primal and the dual of the CCR model is not clear (Lertworasirikul et al., 2003). The concept of chance-constrained programming (CCP) developed by Charnes and is used in this study, which is a method to solve fuzzy DEA models. CCP specifies the level of confidence in constraints when dealing with uncertainty (Charnes & Cooper, 1959). Using the concepts of CCP and the possibility of fuzzy events, the FCCR model becomes the following CCR possibility model (PCCR):

$$\text{(PCCR)} \quad \max_{\boldsymbol{u},\boldsymbol{v},\bar{f}} \quad \bar{f}$$

s.t. $\pi\ (v^T\tilde{y}_0 \geq \overline{f}) \geq \beta$, **(15)**

$$\pi\ (u^T\tilde{x}_0 = 1) \geq \alpha_0\,, \quad \textbf{(16)}$$

$$\pi\ (-u^T\tilde{X} + v^T\ \tilde{Y}\ \leq 0) \geq \alpha\,, \quad \textbf{(17)}$$

$$u \geq 0,$$
$$v \geq 0,$$

Where β and $\alpha_0 \in [0, 1]$ are prespecified acceptable levels of possibility for constraints (1) and (2), respectively, while $\alpha = [x_1, \ldots., x_n]^T \in [0,1]^n$ is a column vector of prespecified acceptable levels for the vector of the possibility constraints (3).

The presentation of the PCCR model is that the objective value $\bar{f}$ should be the maximum value that the return function $v^T\tilde{y}_0$ can achieve with "possibility" level β or higher, subject to the possibility levels of constraints (2) and (3) being at least α_0 and α, respectively. In other words, at the optimal solution, we obtain the value of $v^T\tilde{y}_0$ at least equal to $\bar{f}$ with possibility level β, while at the same time all constraints are satisfied at the prespecified possibility levels (Lertworasirikul et al., 2003).

In the crisp CCR model, a $v^T y_0(\theta)$ the value of one in the optimal solution indicates that the considered DMU is technically efficient. By following this concept, we use $v^T\tilde{y}_0$ to determine if a DMU is technically efficient for the FCCR model. Correspondingly, the $\bar{f}$ in the PCCR model is used to determine if a target DMU is technically efficient (in possibilistic sense) at the predefined possibility level. Let α' be the set of $\beta, \alpha_0, \alpha_1, \alpha_2, \ldots\ldots \alpha_n$. We define an α'-*possibilistic* efficient DMU and α'- *possibilitistic inefficient* DMU as follows: (Lertworasirikul et al., 2003).

Definition of *possibilistic efficient* and *inefficient* DMUs: A DMU is α'-*possibilistic* efficient is its $\bar{f}$ value at the α' possibility level is higher than or equal to 1; otherwise, it is α'-*possibilistic inefficient*. (Lertworasirikul et al., 2003). Following Lemma is proven in this model.

Lemma 1. Let $\tilde{a}_1,\ \tilde{a}_2, \ldots\ldots, \tilde{a}_n$ be fuzzy variables with normal and convex membership characteristics. Let $(.)_{\alpha_i}^L$ and $(.)_{\alpha_i}^U$ denote the lower and upper bounds of the α-level set of $\tilde{a}_i$, $i = 1, \ldots, n$. Then for any given possibility levels α_1, α_2, and α_3 with $0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$,

(1) $\pi\ (\tilde{a}_1 + \cdots + \tilde{a}_n \leq b)\ \geq\ \alpha_1$ if and only if

$$(\tilde{a}_1)_{\alpha_1}^L + \cdots + (\tilde{a}_n)_{\alpha_1}^L \leq \mathrm{b},$$

(2) $\pi\ (\tilde{a}_1 + \cdots + \tilde{a}_n \geq b)\ \geq\ \alpha_2$ if and only if

$$(\tilde{a}_1)_{\alpha_2}^U + \cdots + (\tilde{a}_n)_{\alpha_2}^U \geq \mathrm{b},$$

(3) $\pi\ (\tilde{a}_1 + \cdots + \tilde{a}_n = b)\ \geq\ \alpha_3$ if and only if

$$(\tilde{a}_1)_{\alpha_3}^L + \cdots + (\tilde{a}_n)_{\alpha_3}^L \leq \mathrm{b} \text{ and } (\tilde{a}_1)_{\alpha_3}^U + \cdots + (\tilde{a}_n)_{\alpha_3}^U \leq \mathrm{b}.$$

Proof. Only the evidence for the first case is provided. The other cases can be proved with similar arguments.

Given that fuzzy inputs and outputs of the PCCR model are normal and convex, it follows from Lemma 1 that the PCCR model can be solved by considering:

(PCCR1) $\max\limits_{u,v,f}$ $\quad \bar{f}$

s.t. $\quad (v^T \tilde{y}_0)_\beta^U \geq \bar{f},$ **(18)**

$(u^T \tilde{x}_0)_{\alpha_0}^U \geq 1,$ **(19)**

$(u^T \tilde{x}_0)_{\alpha_0}^L \leq 1,$ **(20)**

$$(-u^T\tilde{X} + v^T \tilde{Y})^L_\alpha \leq 0, \quad \textbf{(21)}$$

$$u \geq 0,$$

$$v \geq 0.$$

Depending upon the membership functions of fuzzy parameters in the model, the PCCR1 model take the form of a linear programming model or a nonlinear programming model. (Lertworasirikul et al., 2003)

The original contributions of this study provided as follows: (i) despite huge investments for city hospitals is controversial in the literature, to the best of our existing knowledge, this is the first experiment that explores city hospital efficiencies in Turkey (ii) conventional, DEA-AHP ranking and possibilistic fuzzy DEA efficiency estimates are presented comparatively to explore city hospital efficiencies (iii) there exists number of studies in the literature about the application of traditional DEA techniques to estimate efficiencies of hospitals in Turkey, this is the first experiment that incorporated DEA-AHP ranking and possibilistic fuzzy DEA models to explore efficiencies of city hospitals (iv) changes in efficiency scores is examined by changing α-cut parameters in fuzzy DEA estimates representing uncertain knowledge.

5. Data/Model Analysis

Table 1 presents the baseline statistics of city hospitals in terms of input and output indicators. Minimum, maximum, mean and standard deviation scores of variables are presented in this table. Mean values of number of beds (mean 1003,33; sd 384,62); number of specialist physicians (mean 39,17; sd 10,18); number of operations (mean 18922,67; sd 7424,32); number of outpatient visits (mean 1776204,17; sd 743725,66).

Table 1. Baseline characteristics of input/output indicators of city hospitals

Input/Output variables	N	Basic statistics	
Inputs			
NumBeds	6	Minimum	475
		Maximum	1550
		Mean	1003,33
		Standard deviation	384,62
NumSpecPhysicians	6	Minimum	31
		Maximum	54
		Mean	39,17
		Standard deviation	10,18
Outputs			
NumOperations	6	Minimum	6309
		Maximum	25739
		Mean	18922,67
		Standard deviation	7424,32
NumOutVisits	6	Minimum	747068
		Maximum	2754862
		Mean	1776204,17
		Standard deviation	743725,66

Table 2 presents conventional DEA estimation results. K. Maras Necip Fazıl City Hospital, Isparta City Hospital and Yozgat City Hospital are efficient city hospitals and all of the other hospitals are inefficient. Adana City Hospital is inefficient city hospital. Figure 1 presents geographic location of city hospitals in a Turkey map and conventional efficiency scores are labeled on them. It is understood that, the geographic proximity between city hospitals is not considered when planning their locations. In order to verify that, Figure 1 shows that Mersin City Hospital, Adana City Hospital and K. Maraş Necip Fazıl City Hospital are located very next to each other.

Table 2. Conventional DEA results

No	City Hospitals	Labels	Conventional efficiency scores
1	Adana City Hospital	ACH	0,659
2	Balıkesir Atatürk City Hospital	BACH	0,864
3	Isparta City Hospital	ICH	1
4	K. Maras Necip Fazıl City Hospital	KMNFCH	1
5	Mersin City Hospital	MCH	0,737
6	Yozgat City Hospital	YCH	1

Figure 1. City hospitals located on a Turkey map and conventional DEA scores

Table 2 shows DEA-AHP ranking results comparative with conventional efficiency estimates. The DEA-AHP model ranks DMUs in terms of efficiency scores. Conventional and DEA-AHP ranking results are consistent with each other and shows that Kahraman Maras Necip Fazıl City Hospital is the most efficient city hospital compared with other city hospitals.

Table 3. DEA-AHP ranking results

No	City Hospitals	Labels	DEA-AHP ranking efficiency scores	Ranking	Conventional DEA scores
1	Adana City Hospital	ACH	0,162	5	0,659
2	Balıkesir Atatürk City Hospital	BACH	0,163	4	0,864
3	Isparta City Hospital	ICH	0,170	3	1
4	K. Maras Necip Fazıl City Hospital	KMNFCH	0,174	1	1
5	Mersin City Hospital	MCH	0,173	2	0,737
6	Yozgat City Hospital	YCH	0,155	6	1

Primal fuzzy DEA results by changing α-cut parameters are presented in Table 4. 21 different α-cut parameters are determined by changing from 0 to 1. In this case, α=0 represents the fuzzification parameter that has high degree of fuzziness and upper bound of fuzzy efficiency rankings. Primal fuzzy DEA scores generated by using α=0 presented as follows: Yozgat City Hospital is 5.43; Isparta City Hospital is 2.36; K. Maras Necip Fazıl City Hospital is 1.81; Balıkesir Atatürk City Hospital is 1.62; Mersin City Hospital is 1.20 and Adana City Hospital is 1, respectively. In this case, α=1 presents the fuzzification parameter that has low degree of fuzziness and lower bound of fuzzy efficiency scores. Primal fuzzy DEA scores generated by α=1 presented as follows: Yozgat City Hospital is 0.59; Adana City Hospital is 0.65; Balıkesir Atatürk City Hospital is 0.72; Mersin City Hospital is 0.72; Isparta City Hospital is 0.91 and K. Maras Necip Fazıl City Hospital is 1. Primal efficiency scores obtained from Kahraman Maras Necip Fazıl City Hospital shows possibility levels greater than 1 in all α-cut levels and it is noticed as a possibilistic efficient city hospital.

Table 4. Primal fuzzy DEA results by changing α-cut parameters

No	City Hospitals	**α = 0**	**α = 0.05**	**α = 0.10**	**α = 0.15**	**α = 0.20**	**α = 0.25**	**α = 0.30**
1	ACH	1,000873	0,981734	0,970758	0,9447713	0,926417	0,907517	0,888620
2	BACH	1,626500	1,556869	1,517088	1,428970	1,370060	1,314139	1,259738
3	ICH	2,363950	2,232855	2,159937	2,002357	1,900359	1,805798	1,716179
4	KMNFCH	1,810752	1,754913	1,722498	1,649637	1,599960	1,552111	1,505992
5	MCH	1,200028	1,171449	1,154509	1,115844	1,088824	1,062330	1,035331
6	YCH	5,437998	4,401463	3,939630	3,129240	2,719891	2,392869	2,123271
No	City Hospitals	**α = 0.35**	**α = 0.40**	**α = 0.45**	**α = 0.50**	**α = 0.55**	**α = 0.60**	**α = 0.65**
1	ACH	0,871638	0,853952	0,836421	0,818507	0,801415	0,784526	0,767853
2	BACH	1,210368	1,162113	1,116046	1,072011	1,029872	0,989503	0,950787
3	ICH	1,635852	1,559127	1,487173	1,419529	1,355802	1,295639	1,238728
4	KMNFCH	1,461509	1,418578	1,377120	1,337058	1,298325	1,260854	1,224587
5	MCH	1,010891	0,985940	0,961493	0,937541	0,914079	0,891088	0,868586
6	YCH	1,902107	1,712705	1,549803	1,408044	1,283431	1,172924	1,074146
No	City Hospitals	**α = 0.70**	**α = 0.75**	**α = 0.80**	**α = 0.85**	**α = 0.90**	**α = 0.95**	**α = 1**
1	ACH	0,750830	0,734589	0,718562	0,702750	0,686672	0,671307	0,656161
2	BACH	0,913623	0,877914	0,843573	0,810519	0,778677	0,747982	0,718367
3	ICH	1,184807	1,133631	1,084983	1,038670	0,994520	0,952377	0,912096
4	KMNFCH	1,189463	1,155432	1,122443	1,090447	1,059403	1,029267	1
5	MCH	0,846534	0,824933	0,803774	0,783046	0,762740	0,742845	0,723354
6	YCH	0,985285	0,904839	0,831614	0,764633	0,703092	0,646319	0,593745
See Table 1 and 2 for labels and conventional and DEA-AHP ranking results								

Figure 2 presents changes in alpha-cut parameters from 0 to 1 on six city hospitals primal efficiency scores. The degree of fuzziness is high in alpha 0 level and highest primal fuzzy efficiency scores are obtained in that level. Primal efficiency scores are decreasing while increasing alpha-cut parameters from 0 to 1. Yozgat City Hospital has the highest primal fuzzy efficiency scores compared with other city hospitals.

Figure 2. Changes in alpha-cut parameters on primal efficiency scores

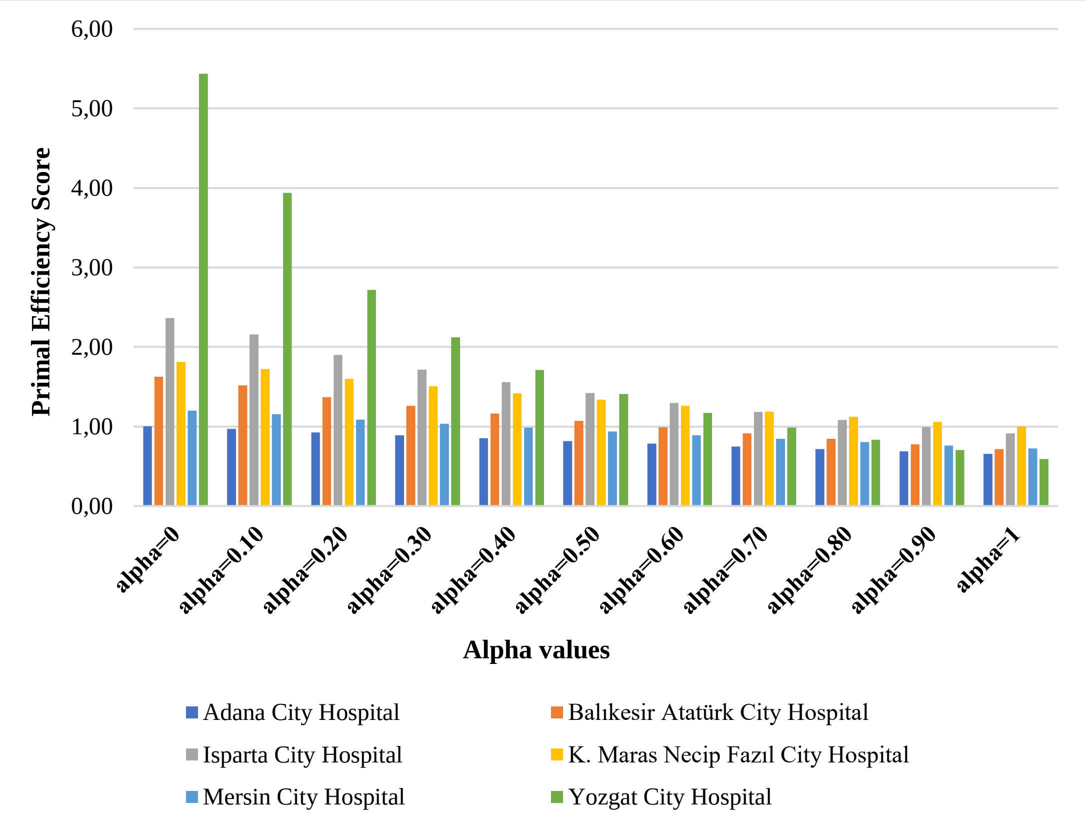


Table 5 presents mean rank differences between primal efficiency values obtained from six city hospitals. It is seen that, there exists statistically significant mean rank differences between primal model efficiency estimates by changing α-cut values from 0 to 1 (X^2=36.16, p<0.001). Therefore, there exists statistically significant mean rank differences between primal efficiency scores gathered from city hospitals while changing α-cut parameters.

Table 5. Differences between primal efficiency scores obtained from six city hospitals

	α-cut values	N	Mean rank	Chi-square	p
Primal model efficiency estimates by changing α-values	**α = 0**	6	51,17	39,161	<0.001
	α = 0.10	6	58,33		
	α = 0.20	6	46		
	α = 0.30	6	42,50		
	α = 0.40	6	38,50		
	α = 0.50	6	34,33		
	α = 0.60	6	29,67		
	α = 0.70	6	24,33		
	α = 0.80	6	19,17		
	α = 0.90	6	14		
	α = 1	6	10,50		
Chi-square: Kruskall Wallis variance analysis					

Figure 3 shows mean primal efficiency scores obtained from six city hospitals with different α-cut levels. A continuously decreasing trend observed in mean primal fuzzy efficiency estimates gathered from six city hospitals while changing α-cut parameters from 0 to 1.

Figure 3. Mean primal efficiency scores obtained from six city hospitals

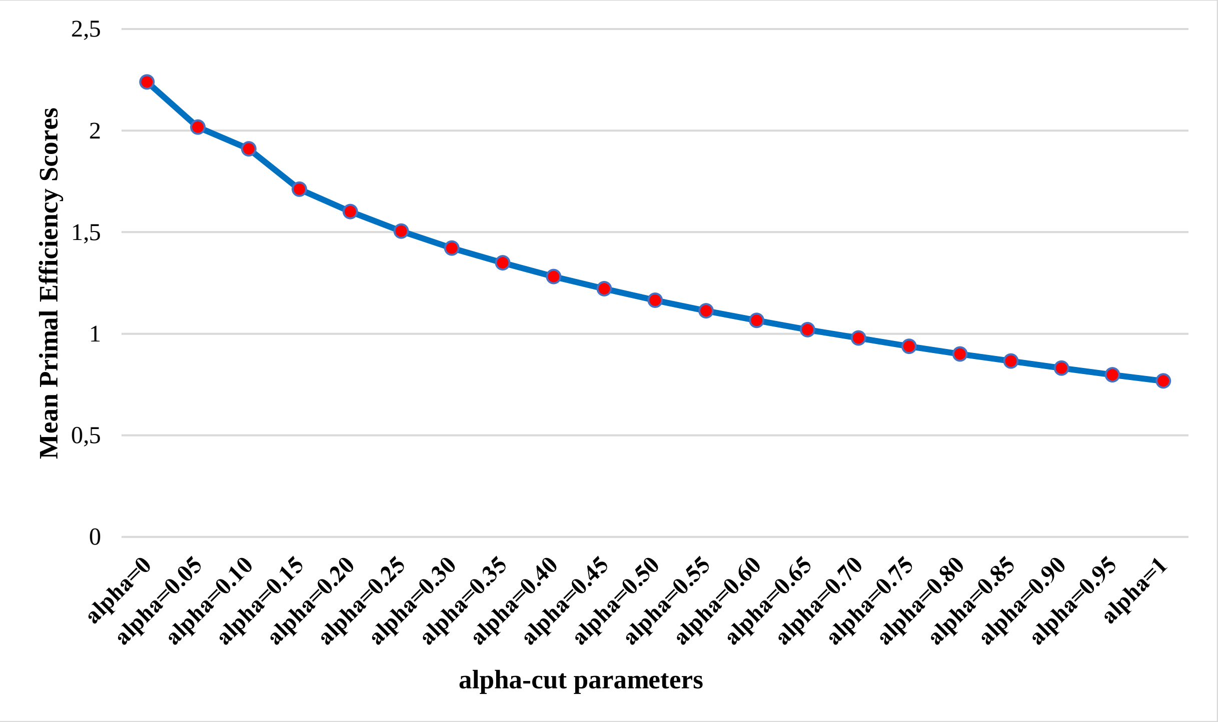
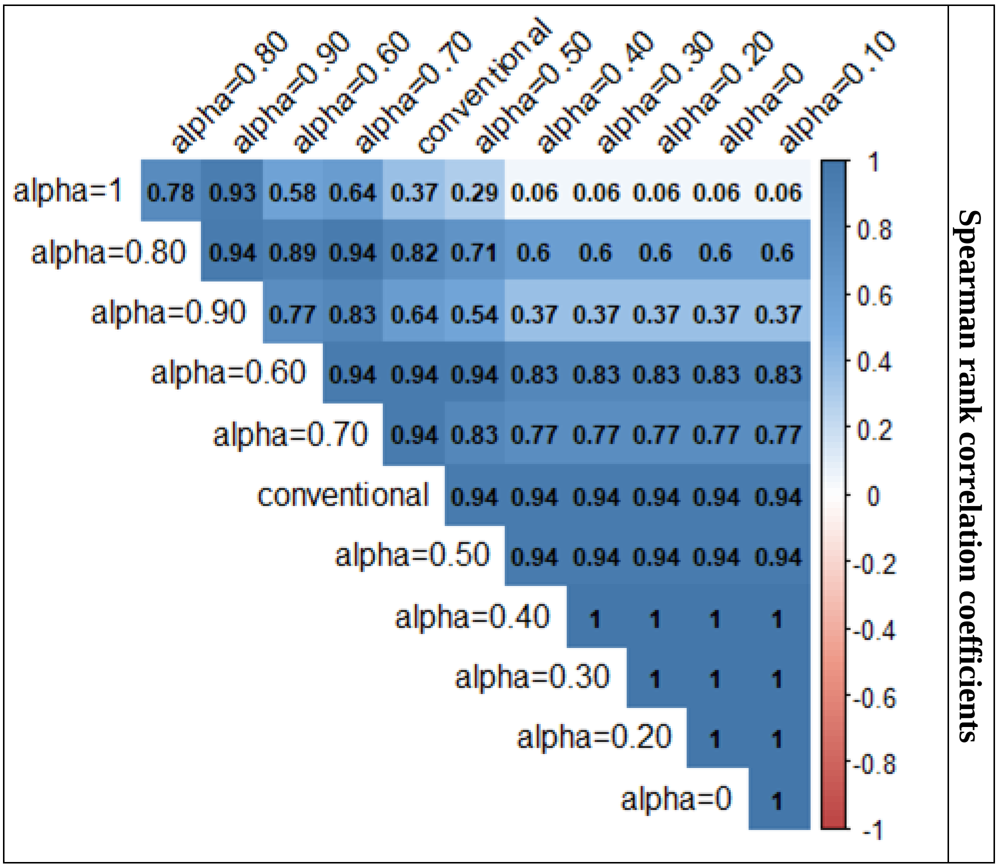


Figure 4 presents correlogram generated by using Spearman rank correlations between conventional and primal possibilistic DEA efficiency estimates, while changing α-cut parameters from 0 to 1. In this figure, blue colors represents for positive correlations and red colors represents for negative correlations. Correlations which are not statistically significant are shown in white color ($p>0.05$). It is seen that, correlogram is mostly colored with dark and light blue colors, therefore there exists statistically significant positive Spearman rank correlations between different crisp and possibilistic fuzzy efficiency estimates. Conventional efficiency scores has high level of similarities with primal possibilistic efficiency estimates generated by using low level of α-cut levels such as α=0 (r_s=0.94; $p<0.01$); α=0.10 (r_s=0.94; $p<0.01$); α=0.20 (r_s=0.94; $p<0.01$); α=0.30 (r_s=0.94; $p<0.01$); α=0.40 (r_s=0.94; $p<0.01$); α=0.50 (r_s=0.94; $p<0.01$); α=0.60 (r_s=0.94; $p<0.01$); α=0.70 (r_s=0.94; $p<0.01$). Moreover, possibilistic fuzzy DEA estimates generated by alpha values with high levels of fuzziness such as α=0, 0.10, 0.20, 0.30, 0.40 levels indicate a perfect positive correlation (r_s=1; $p<0.01$) with each other. However, crisp possibilistic DEA model created with α=1 level has insignificant correlations with possibilistic DEA models created with fuzzy α-cut parameters such as α=0 (r_s=0.06; $p>0.05$); α=0.10 (r_s=0.06; $p>0.05$); α=0.20 (r_s=0.06; $p>0.05$); α=0.30 (r_s=0.06; $p>0.05$) and α=0.40 (r_s=0.06; $p>0.05$).

Figure 4. Similarities between conventional and primal possibilistic fuzzy efficiency estimates while changing α-cut parameters from 0 to 1



In this study, dual variables (λ) are generated by using dual model solutions obtained from inefficient city hospitals. Table 6 shows dual variables (λ) generated by using primal efficiency model in α=1 level. In our case, the only possibilistic efficient hospital in α=1 level is Kahraman Maras Necip Fazıl City Hospital, which is a reference city hospital for inefficient hospitals. Therefore, inefficient city hospitals will take Kahramanmaras Necip Fazıl City Hospital as an example fuzzy efficient city hospital in order to become efficient.

Table 6. Dual variables (λ) generated by using dual models in α=1 level

DMU	Possibilistic Inefficient City Hospitals	No	Reference City Hospital (Possibilistic Efficient)	Dual variables (λ)	
1	ACH	4	KMNFCH	λ_4	0.97
2	BACH	4	KMNFCH	λ_4	0.62
3	ICH	4	KMNFCH	λ_4	0.66
5	MCH	4	KMNFCH	λ_4	0.90
6	YCH	4	KMNFCH	λ_4	0.27

All inefficient city hospitals in α=1 level will take the KMNFCH as a reference city hospital to become efficient. The optimal values for inefficient hospitals are calculated by taking the dual variables (λ values) of reference hospital (KMNFCH) and presented in Table 7. Note that, all inefficient hospitals needs to decrease input and output variables to become efficient in α=1 level.

Table 7. Optimal values for inefficient hospitals to become efficient

DMU	Possibilistic Inefficient City Hospitals	NumBeds	NumSpecPhysicians	NumOperations	NumOutVisits
1	ACH	1008	30	24966	2672216
2	BACH	644	19	15958	1708014
3	ICH	686	20	16987	1818208
5	MCH	936	27	23165	2479375
6	YCH	280	8	6949	743812

6. Limitations

The dataset belongs to the year 2017 due to the lack availability of recent data about Public Hospital Statistical Year Books since 2017. It is highly advisable for future studies to use more recent Public Hospital Statistical Yearbook datasets.

7. Conclusions

In the present study, efficiencies of city hospitals investigated by using crisp, DEA-AHP ranking and possibilistic fuzzy DEA methods, comparatively. Conventional, DEA-AHP ranking and possibilistic fuzzy DEA results are mostly in line and emphasize that, small number of city hospitals are efficient and dual possibilistic fuzzy results shows that, possibilistic fuzzy inefficient city hospitals should decrease input and outputs to become efficient by taking efficient city hospital as a reference in $\alpha=1$ level. When some of input and output data are imprecise as consistent with vague nature of health operations, multi-step multicriteria decision making techniques unclosing fuzzy models provides useful insights for health care operation managers to consider uncertainties.

This study offers a broad spectrum of perspective to measure and compare efficiency performances of health campuses (city hospitals) investments. The purpose of this study is to develop a method for finding the membership functions of fuzzy efficiency measures when some observations are fuzzy numbers. This study describes an alternative multi-method analysis perspective for decision-making in health care business practice. Traditional, DEA-AHP ranking model and possibilistic fuzzy DEA methods are used to explore efficiencies of city hospitals. A combined strategy of DEA-AHP approach is used to measure the relative efficiency of slightly non-homogenous DMUs (Saen et al., 2005). DMU efficiency is further explored using a possibilistic DEA procedure. In this method, the decision maker specifies the required possibility levels by using α-level perspective (Lertworasirikul et al., 2003). A fuzzy number with continuous membership functions can be derived by considering data as fuzzy sets and changing the α-cut levels (Omrani et al., 2021).

In spite of the fact that the DEA model is a linear program, one straightforward approach is to use fuzzy linear programming technique (Buckley, 1988; Julien, 1994; Luhandjula, 1987) to the fuzzy DEA problems. The fuzzy DEA technique-based approach and an incorporation of DEA with fuzzy logic improves the diversity in the efficiency score (Zhou & Xu, 2020). Moreover, in order to handle uncertainty in input and output variables, a fuzzy credibility model has been used. DEA models generated with exact perturbations in fuzzy inputs and outputs have a distinguishing power and are more precise than traditional DEA models (Omrani et al., 2021). Existing knowledge about the DEA models are fully fuzzified and weights and input-output data are considered as fuzzy values using the α-cut approach (Singh & Yadav, 2022). In our case, the effect of changes in α-cut parameters on primal efficiency estimates are recorded by presenting the results of an increase in α-cut parameters on fuzzy efficiency results. It is seen that, possibilistic fuzzy efficiency scores have a decreasing trend while improving α-cut levels from 0 to 1. This finding is consistent with the existing knowledge proves that increase in α-cut levels improves possibility DEA results by implementing the stochastic nature in efficiency analysis (Shiraz et al., 2020).

In possibilistic fuzzy DEA approach, the primal fuzzy DEA model identify the upper bounds of fuzzy efficiency estimates and dual fuzzy DEA define the lower bounds of fuzzy efficiency scores. Dual fuzzy possibilistic efficiency results identify ineffective DMUs and shows what value does ineffective DMUs should take in order to become efficient (Kao & Liu, 2000). In our case, mean rank differences observed between primal efficiency scores gathered from city hospitals while changing α-cut parameters from 0 to 1. Moreover, it is seen that, crisp probabilistic fuzzy efficiency estimates generated with α=1 level are uncorrelated with DEA models that utilize fuzzy α-cut levels with a high level of fuzziness. In this study, possibilistic fuzzy DEA scores generated by α-cut values with high levels of fuzziness from 0 to 0.40 shows high similarities.

This study provides valuable insights for practice and future research in fuzzy health care organization planning environment. Study findings also emphasize a remarkable study finding which shows that there exists an imbalance in geographic planning of city hospitals in Turkey. Accessibility and efficiency of health services is important for better planning of health services and to better answer health needs of individuals (O'Hanlon et al., 2019). It is strongly advisable for health policy makers to better answer necessities of health consumers and to consider geographic proximities of health institutions while planning health care investments. Another significant study finding generated by using dual models generated in α=1 level shows that, possibilistic inefficient city hospitals should decrease input and output values to become efficient by taking into account efficient city hospitals as a reference hospital. In other words, city hospitals that are possibilistic inefficient should reduce number of beds, number of specialist physicians, number of operations and number of outpatient visits to become efficient. It is critical to note that, health policy and decision makers should consider equality and equity in the distribution of scarce public health resources.

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