OUTLINE FOR SESSION PROPOSAL SUBMITTED TO THE INTERNATIONAL SYMPOSIUM OF THE ANALYTIC HIERARCHY PROCESS

AHP METHODOLOGY AND APPLICAITON

Kè-yù ZHÜ
School of Management,
Hefei University of Technology,
Hefei, China
cauchyandy@163.com

SESSION ABSTRACT

In this session, four scholars were invated to present their studies in AHP methodology and its applicaiton. Dr. Wang proposes an AHP methology with gray theory to establish judgment matrix for the AHP, PhD. Peng constructs an AHP model with maximizin deviation method to evaluation regional innovation, Dr. Dong proposes a group AHP consensus reaching model for supplier selection in collabortive product development, Dr. Zhu proposes a weighted geometric aggregation AHP model to preserve rank in AHP.
Key Words: AHP, regional innovation, Group AHP,*Grey number*, rank reversal

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OUTLINE FOR PAPER PROPOSAL 1 SUBMITTED TO THE INTERNATIONAL SYMPOSIUM OF THE ANALYTIC HIERARCHY PROCESS

A GREY NUMBER APPROACH TO ESTABLISH JUDGMENT MATRIX IN AHP

Xiaojia Wang
School of Management
Hefei University of Technology
Hefei, Anhui, China

Jennifer Shang
Katz Graduate School of Business
University of Pittsburgh
Pittsburgh, PA USA

ABSTRACT

Multiple attribute decision making (MADM) is becoming an important part of modern decision science. It has been extensively applied to various areas such as society, economics, management, etc., and has been receiving more and more attention over the last decades. However, owing to the increasing complexity of decision, the uncertainty of decision information growing sharply and the multi-period multi-attribute decision making has become the focus of people's attention. Therefore, this study proposes a multiple attribute decision making model (MADM) which takes the AHP technique as main structure, integrating the concepts of grey number into it to cope with uncertain information. An emerging market stock selection example is employed to demonstrate the feasibility and practicability of the proposed model. Results show that the proposed model is efficient and robust, and is practical for real world applications.

Keywords: *Comparison matrix; AHP; Multi-attribute decision making; Grey number*

1. Introduction

Multi-attribute decision making (MADA) problem is to select an appropriate alternative (choice) from a finite number of feasible alternatives based on the features of each attribute (objective or criteria) with respect to every alternative. AHP（technique for organizing and analyzing complex decisions）, proposed by Saaty (1977, 1980, 1986) is one of the most widely used techniques to solve MADM problems. AHP provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions. The basic concept of AHP technique is that AHP is a decision analysis technique used to evaluate complex multi-attributed alternatives with conflicting objectives among one or more actors. The process involves hierarchical decomposition of the overall evaluation problem into sub-problems that can be easily understood and evaluated. In recent years, AHP has been successfully adopted to solve various MADM problems, such as accounting (Webber et al. 1996), assessment (Sloane et al. 2003), programming (Ngai, 2003;Yang and Kuo, 2003), and information management (Liu & Shih, 2006). Hence, we use the AHP to resolve the MADM problem and extend it to the uncertain environment on account of its effectiveness and practicability.

A problem in applying the AHP is that the judgments are not transitive, i.e. the judgment matrix is inconsistent. For this reason, this paper starting from the objective property of grey number division operation, modify the definition of judgment matrix and make it only has reciprocity. Then, we can give out the determine condition of consistency for judgment matrix to overcome this problem and also to enhance the reliability of decision.

Besides, along with the increasing complexity of decision, the uncertainty of evaluation will be growing. Under this situation, the decision making scheme is not enough to reflect the decision environment, or the understanding of the expert is far from being comprehensive and accurate, at this time, the judgment has a variety of possible and decision maker(s) are unable to point out a precise numbers to express the important degree in paired comparisons. But, if they can give approximate ranges, i.e., grey numbers or interval numbers, by their knowledge and cognition. This way will represent their judgment and feelings more truly. Therefore, applying the concepts and operations of grey number will be helpful to deal with the uncertain information.

Hence, this study attempts to propose an effective MADM model, which adopts AHP as the main structure as well as integrating with the concepts of grey number, to effectively deal with the uncertain information and comprehensively aggregate the different decisions among all periods.

This paper is organized as follows: Section 2 describes the concepts of AHP technique and grey number, respectively. Based on the concepts in Section 2, Section 3 discusses the properties of consistent judgment matrix and presents the criteria of consistency in grey numbers. Finally, Section 4 concludes the paper.

2. Literature survey

To arrange the survey in various aspects, we divide it into two parts: the AHP technique and the concept of grey number.

2.1 AHP technique

Analytic hierarchy process (AHP, Saaty, 1977, 1980, 1990) is a multi-criteria decision making method based on pairwise comparisons for elements in a hierarchy. It decomposes problems in a hierarchical structure, and explicitly incorporates decision makers' expertise/experience in AHP evaluation. Decision makers make use of the subjective judgments, but can also integrate objectively measured information when necessary. The three principles of AHP are decomposition, comparative judgment, and synthesis.

In the above-mentioned procedure of AHP technique, a problem can be obviously identified. This problem can be further observed from Eq. (1) and (2). After obtaining the judgment matrix, the diagonal elements is defined as a fixed value 1, this man-made regulation led to a contradiction of judgment matrix itself. The reason is, in social and economic systems, decision makers' subjective judgment, choices and preference has a great influence on decision results. It is difficult to obtain the priorities of the alternatives or the weight of every criterion directly, even is impossible. However, this problem can be overcome by using the grey numbers (Deng, 1982).

2.2 Grey number

Grey system theory proposed by Deng (1982), is a mathematical theory originating from the concept of grey set. It can effectively solve uncertainty problems under discrete data and incomplete information. In grey system, if the system information is fully known, the system is called a white system. When the system information is unknown, it is called a black system. A system with partial information known and partial information unknown is grey system. Thus, a grey system contains uncertain information presented by grey number and grey variables, as shown in Figure 2.

Figure 2. The concept of grey system.

Let $\otimes a = [\underline{a}, \overline{a}] = \{a \mid \underline{a} \le a \le \overline{a}; \underline{a}, \overline{a} \in R\}$. Then, $\otimes a$ have two real numbers $\underline{a}$ (the lower limit of $\otimes a$) and $\overline{a}$ (the upper limit of $\otimes a$) is defined as follows (Liu & Lin, 2006)

- If $\underline{a} \to -\infty$ and $\overline{a} \to +\infty$, then $\otimes a$ is called the black number which means without any meaningful information or information is totally unknown.
- Else if $\underline{a} = \overline{a}$, then $\otimes a$ is called the white number which means with complete information or information is totally known.
- Otherwise, $\otimes a = [\underline{a}, \overline{a}]$ is called the grey number which means insufficient and uncertain information.

Grey number is a concept of grey theory to deal with the insufficient and incomplete information. Although grey theory has been applied in various fields (Liu & Shih, 2006), the applications are mostly based on the white numbers. Nevertheless, the obtained information from real world is always uncertain or incomplete. Hence, extending the applications from white number to grey number is necessary for real world applications.

3. The proposed multi-attribute decision making model

3.1 Grey number approach to establish the judgment matrix in AHP

We first discuss the grey number judgment matrix.

Definition 3.1. Let $D(\otimes)=[\otimes_{ij}]_{n\times n}$ be a grey number matrix and $i,j=1,2,\cdots,n$ such that

- $\otimes_{ij}=[\underline{d}_{ij},\overline{d}_{ij}]$,and $\frac{1}{9}\le\underline{d}_{ij}\le\overline{d}_{ij}\le 9$
- $\otimes_{ij}=\frac{1}{\otimes_{ji}}$

Then, $D(\otimes)$ is called grey number judgment matrix.

Suppose $D(\otimes)=[\otimes_{ij}]_{n\times n}$ is a grey number judgment matrix, $w(\otimes)=(w_1(\otimes),w_2(\otimes),\cdots,w_n(\otimes))^T$ is the grey number weight vector associated $D(\otimes)$, then, we have $\otimes_{ij}=\frac{w_i(\otimes)}{w_j(\otimes)}$ available for all $i,j=1,2,\cdots,n$.

Definition 3.2. Suppose $D(\otimes)=[\otimes_{ij}]_{n\times n}$ is a grey number judgment matrix and $i,j,k=1,2,\cdots,n$ be such that

$$\otimes_{ij}=\frac{1}{\otimes_{ji}},\quad \otimes_{ij}\times\otimes_{jk}=\otimes_{ji}\times\otimes_{ik} \tag{1}$$

Then, $D(\otimes)$ is called consistent grey number judgment matrix, and Eq. (1) is the consistency condition of $D(\otimes)$.

Next, we discuss the properties of consistent grey number judgment matrix. We first introduce the quasi-uniformity concept of real number matrix.

Theorem 3.1. A necessary and sufficient condition for $D(\otimes)=[\otimes_{ij}]_{n\times n}$ is consistent grey number matrix is there exist grey number $\otimes_i(i=i,2,\cdots,n)$, such that

$$\otimes_{ij}=\frac{\otimes_i}{\otimes_j},(i,j=1,2,\cdots,n) \tag{2}$$

3.2. The modeling mechanism of the grey number multi-attribute decision making model

Base on the operations of grey number, this paper proposes an effective multi-attribute decision making model under the condition of uncertain information. Before we describe the detailed model, we assume a positive grey number judgment matrix, $D(\otimes)=[\otimes_{ij}]_{n\times n}$.

$$D(\otimes)=[\otimes_{ij}]_{n\times n}=\begin{bmatrix}\otimes_{11} & \otimes_{12} & \cdots & \otimes_{1n}\\ \otimes_{21} & \otimes_{22} & \cdots & \otimes_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \otimes_{n1} & \otimes_{n2} & \cdots & \otimes_{nn}\end{bmatrix} \tag{3}$$

where $\otimes_{ij}$ denotes the grey number evaluations of the ith alternative with respect to the jth attribute. $\otimes_i = (\otimes_{i1}, \otimes_{i2}, \cdots, \otimes_{in})$ is the grey number evaluation series of the ith alternative. Assume $w(\otimes) = (w_1(\otimes), w_2(\otimes), \cdots, w_n(\otimes))^T$ is the grey number eigenvector of $D(\otimes)$, and the entries of grey number judgment matrix $D(\otimes)$ is changed at different decision period.

In order to ensure the scientificalness and correction of the proposed MADM model, we first introduce the modeling mechanism.

Theorem 3.2. Suppose $D(\otimes) = [\otimes_{ij}]_{n\times n}$ is consistent grey number matrix, $\underline{w}=(\underline{w}_1, \underline{w}_2, \cdots, \underline{w}_n)^T$ and $\bar{w}=(\bar{w}_1, \bar{w}_2, \cdots, \bar{w}_n)^T$ are nonnegative normalized eigenvector of $(\underline{d}_{ij})_{n\times n}$ and $(\bar{d}_{ij})_{n\times n}$ associated the maximal eigenvalue. Then

$$w(\otimes) = [p\underline{w}, q\bar{w}] = (w_1(\otimes), w_2(\otimes), \cdots, w_n(\otimes))^T \quad (4)$$

and a necessary and sufficient condition for $\otimes_{ij} = \dfrac{w_i(\otimes)}{w_j(\otimes)}, (i, j = 1, 2, \cdots, n)$ is

$$\Omega = \sum_{j=1}^{n} \quad = \frac{1}{\sum_{i=1}^{n} \bar{d}_{ij}} = \frac{1}{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} \underline{d}_{ij}}} \quad (5)$$

Next, the procedure of the proposed model can be shown as the following five steps.

Step 1: Construct the grey number judgment matrices.

Step 2: Calculate the maximal eigenvalue of grey number matrix $(\underline{d}_{ij})_{n\times n}$ and $(\bar{d}_{ij})_{n\times n}$ and the associated nonnegative normalized eigenvector $\underline{w}=(\underline{w}_1, \underline{w}_2, \cdots, \underline{w}_n)^T$ and $\bar{w}=(\bar{w}_1, \bar{w}_2, \cdots, \bar{w}_n)^T$.

Step 3: Determine parameter p and q.

$$p = \sqrt{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} \bar{d}_{ij}}}, \quad q = \sqrt{\sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{n} \underline{d}_{ij}}}$$

Step 4: Normalize the weight vector.

$$w(\otimes) = [p\underline{w}, q\bar{w}] = (w_1(\otimes), w_2(\otimes), \cdots, w_n(\otimes))^T$$

Step 5: Rank the preference order.

A set of alternatives now can be preference ranked by the descending order of the weight vector.

4. Conclusions

Due to the increasing complexity of decision, the uncertainty in evaluation will be growing. It is difficult for decision makers to make evaluations with precise number. However, they can still use an approximate range of evaluation by their knowledge and cognition. Under this circumstance, dealing with the uncertain information is necessary on developing the decision making model.

In this study, we proposed a new grey number approach to modify the definition of judgment matrix in the original AHP technique, overcome the problem of inconsistency when the judgments are not transitive. Finally, we have integrated the above mentioned concepts, AHP technique to establish an effective multi-attribute decision making model.

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OUTLINE FOR PAPER PROPOSAL 2 SUBMITTED TO THE INTERNATIONAL SYMPOSIUM OF THE ANALYTIC HIERARCHY PROCESS

A INTEGRATED MODEL BASED ON AHP AND MAXIMIZING DEVIATION METHOD AND ITS APPLICATION

Zhanglin Peng
School of Management
Hefei University of Technology
Hefei, Anhui, China
E-mail: pengzhanglin@163.com

Shanlin Yang
School of Management
Hefei University of Technology
Hefei, Anhui, China
E-mail: hgdysl@gmail.com

Xiaojia Wang
School of Management
Hefei University of Technology
Hefei, Anhui, China
E-mail: tonysun800@sina.com

ABSTRACT

This study explores the potential of applying analytic hierarchy process (AHP) and maximizing deviation (MD) to determine the regional innovation capability of Chinese districts that need evaluation. Compared with some conventional single methods, the proposed combined model incorporates a much wider range of quantitative and qualitative criteria, and deals with a much more certain and uncertain information, and provides a more detailed and thorough research. Firstly, we use the Analytic hierarchy process to deal with uncertain information, then get the first weight vector, which is determined by the importance or priorities of the attributes or criterions; secondly, we use maximizing deviation method to handle some certain information, then we get the second weight vector, which is determined by the discrepancy of the attribute values; finally, we integrate these two weight vector and apply them in evaluating and ranking the regional innovation capability n 31 districts (provinces, municipalities & Autonomous Regions) in China.
Keywords: analytic hierarchy process, maximizing deviation, regional innovation capability

1. Introduction

As we known, Chinese economy's progress is very uneven, and there is a big gap among different districts, especially, between the eastern and western area. In some extent, the uneven economy development is resulted from the uneven science & technology development. In Chinese Mainland, there are 31 provinces, municipalities & Autonomous Regions (not include Hong Kong, Macao, and Taiwan). Due to different polices, more or less R&D investment, different human resources, etc., the imbalance of the regional innovation capability has been produced among these districts. But how much the imbalance is? So we need to evaluate these regional innovation capabilities and compare them.

How should we evaluate them, maybe there are so much methods, for example Data envelopment analysis (DEA), the Evidential Reasoning (ER), Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS), Fuzzy Theory, AHP, etc. when most of these methods deal with the MADM Problems, some problems such as that the evaluation results are close to each other, the evaluation values differ to each other scarcely, the difference between these evaluations is not obvious and so on can easily occur in practice. Although the multiple attribute decision model based on maximizing deviation can enlarge the evaluation values discrepancy for the optimizing and ranking of the decision making schemes, it only considers the variable weight caused by the discrepancy of the attribute values and neglects the weight of the evaluation indicator itself in practice, causing that the evaluation results easily deviate the actual results.

In this study, we combine both the Analytic Hierarchy Process (AHP: Saaty, 1977, 1980) and Maximizing Deviation Method (a mathematical theory) to address the regional innovation capability. Both are useful evaluation tools and are widely accepted methods for improving efficiency and productivity. The synergistic model provides an unequivocal and replicable tool and method in evaluating some units including both certain and uncertain information.

This study organizes the remaining structure as follows. In section 2, we will review some related literatures. Section 3 describes the specific objectives in this paper. Section 4 proposes the in integrated evaluation model based on AHP and MD, and gives detailed computational processes and steps. Section 5 gives a numerical experiment and result analysis.

2. Literature Review

2.1 Analytic Hierarchy Process (AHP)

Analytic Hierarchy Process (Saaty, 1977), (Saaty, 1980), (Saaty, 1986), (Saaty, 1990) is a multi-objective multi-criteria decision making method or approach based on pairwise comparisons for elements in a hierarchical decision model, and is presented to solve some decision making problems, such as choosing a best one in a set of competing alternatives, or ranking all potential alternatives. In this method or approach, the problem can be decomposed into a hierarchical structure including levels of factors or elements. Meanwhile, the strength of the influence of these factors or elements at a particular level can be measured by pairwise comparisons. It has strong axiomatic foundation which highlights: (1) the reciprocal property; (2) homogeneity; (3) dependence; and (4) expectations. The three typical principles of AHP are decomposition, comparative judgments, and synthesis of priorities. AHP is also a problem-solving framework, which can enable us to cope with the intuitive, the rational, the irrational, deal with certain and uncertain information, and combine objective and subjective judgments at the same time, when we make multi-criteria and multi-actor decisions or evaluations.

2.2 Maximizing Deviation

But in some extent, we often face a problem that the evaluation results are so close to each other, and the difference among all the results isn't significant obviously. In order to enlarge the evaluation results, (Yingming, 1997) presented a fully objective decision method-maximizing deviation method, and use them to evaluate about industrial economic benefits under multi-criteria, the evaluation results is very significant, exact, reliable without subjectivity. The method has been applied by many researchers, such as, (Wu & Chen, 2007), (Wei, 2008) etc.

But, in practice, we often face a decision problem include certain information and uncertain information, so we not only need objective method to deal with certain information, and need other methods to handle uncertain information. Only by this way, we can get more exact, more precise, more reliable results. So, in this paper, we connect the AHP and maximizing deviation method, and use them to evaluate the Chinese regional innovation capability.

3. Hypotheses/Objectives

This study is mainly focused on the evaluation about the regional innovation capability in different Chinese districts. For simplicity, we just measure the innovation capability by science & technology achievements, including Basic-Theory Research Achievements (BTRAs), Technical Achievements (TAs), and Soft S&T Achievements (SSATs). The alternatives are 31 districts from Chinese Mainland, with 22 provinces, 4 municipalities, and 5 Autonomous Regions.

4. Research Design/Methodology

In this part, we construct a decision or evaluation model connected AHP with Maximizing Deviation Method. Firstly, we built the AHP model to get the priorities of criteria, that is weight vector of BTRAs, TAs and SSATs, then we add the weight vector to the original maximizing deviation decision model, in order to get the total weight vector including the attribute weight and variable weight. So the model built in this paper has two parts: (1) the AHP model; and (2) the integrated model based on AHP and Maximizing Deviation Model.

4.1 The AHP Model

Based on the AHP theory, and with the main kinds of S&T Achievements and the current situation, three criteria are used to evaluate the regional innovation capability of different Chinese districts. These criteria are Basic Theory Achievements (BTAs), Technological Achievements (TAs) and Soft Science Achievements (SSAs).

The 31 districts in China Mainland assigned as the alternatives and their values are given, according to appendix 1, there are 3 years' statistic data which is from 2006 to 2008. 5 experts were invited to rank the 3 criteria according to Chinese S&T development situation, then we can get the priorities of the criteria, that is, we also get the attribute weight vector. Next, we should add this weight vector to the original Maximizing Deviation Model, then work out the final weights of these 3 criteria.

4.2 The integrated model based on AHP and Maximizing Deviation

In conclusion, the main parts of the integrated model are as follows:

Step1: Determine all alternatives (Chinese cities or areas) and attributes (representing the regional innovation capability), and built a set of $A=\{A_1, A_2, \cdots, A_m\}$ and a set of $G=\{G_1, G_2, \cdots, G_n\}$.

Step2: Use AHP to work out the attribute weight or priority, and get the first weight vector $W=(w_1, w_2, \cdots, w_n)^T$.

Step3: Construct the normalized and weighted decision matrix C^* , which be added w_j .

Step4: Construct the maximizing deviation model, and use the Lagrange Function to get the second weight vector $W^*=\left(w^*{}_1, w^*{}_2, \cdots, w^*{}_n\right)^T$, then we get its normalization weight is $w_j^{*'}$, that is , get the finally weight vector corresponding to each decision criteria.

Step5: Calculate the regional innovation capability of each city or area in China using the formula $E_i^{'} = w_1^{*'} w_1 u_{i1} + w_2^{*'} w_2 u_{i2} + \cdots + w_n^{*'} w_n u_{in}$, then we can compare and rank them, and we can give some further analysis.

5. Data/Model Analysis

According to the AHP and the constructed criteria, we invited some related experts to make pairwise comparison, and get the priorities by Software *SuperDecisions*. Then we get the first weight vector $W=(w_1, w_2, \cdots, w_n)^T$ is $W=(0.3125, 0.6250, 0.0625)^T$.

According to the first weight vector and the statistical data of S&T Achievements from 31 districts (Provinces municipalities &Autonomous Regions): 2006-2008, we use the integrated model to calculate the second weight vector, and use software Matlab by programming to solve the optimization model. Then we get e can get the 3 final weight vectors corresponding to 3 years respectively. They are $W^* = (0.3048, 0.6228, 0.0724)$, $W^* = (0.3062, 0.6308, 0.0630)$ and $W^* = (0.2525, 0.6848, 0.0627)$.

Next, we can get the ranking results.

6. Limitations

The paper has the following limitations: (1) due to some difficulties in collecting data of each district in recent years, so we have to adopt the data from 2006 to 2008. (2) in this paper, we give a rank about 31 alternatives, but we don't give more further quantitative analysis about why the model connection AHP and maximizing deviation is better than the single method, AHP or maximizing deviation method.

7. Conclusions

A combination evaluation model based on AHP and Maximizing Deviation Method has been presented and constructed in this paper, which not only can deal with certain information related to attribute values to get the variable weight of criteria, but also can handle uncertain information related to the importance of attributes themselves to get the attribute weight of criteria. In a word, the model can cope with kinds of information. Furthermore, the evaluation results will be significant.

Another important work in this paper is the application of this combination evaluation model. We apply it to evaluate the regional innovation capability in 31 Chinese districts, then compare them and analyze them. The evaluation and sorted results have shown that Chinese S&T development had a so much obviously imbalance in 31 districts from 2006 to 2008, especially, the western region had lower development in S&T than eastern region, and lower than central region.

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OUTLINE FOR PAPER PROPOSAL 3 SUBMITTED TO THE INTERNATIONAL SYMPOSIUM OF THE ANALYTIC HIERARCHY PROCESS

A GROUP AHP CONSENSUS REACHING MODEL FOR SUPPLIER SELECTION IN COLLABORATIVE PRODUCT DEVELOPMENT

Qingxing Dong
School of Information Management
Central China Normal University
Wuhan, China
E-mail: qx_dong@126.com

Keyu Zhu
School of Management
Key Laboratory of Process Optimization and Intelligent Decision-making, Ministry of Education
Hefei University of Technology
Hefei 230009, China
E-mail: cauchyandy@163.com

ABSTRACT

Group consensus is an essential factor of a successful group decision. However, judgments are always diverse in the real world. Thus supporting the process of consensus reaching is of great significance. To improve the group consensus, the moderator of a group can give some recommendations to the incompatible decision makers to revise extreme opinion. Also, in an autocratic group, where the decision makers are the experts or consultants providing their suggestion to the leader or client, the moderator can adjust the weight or importance of the incompatible decision maker to reduce the perturbation from the extreme opinion. In this paper, we propose a consensus reaching model for the autocratic group decision, where the members use the Analytic Hierarchy Process (AHP) to express their judgment. In this dynamic and interactive model, a moderator suggests the incompatible expert to revise his/her judgment. If the expert rejects this suggestion, his/her importance weight will be adjusted downward. This process supports the leader or client to make a successful decision with a dispersed group of expert by improving the consensus level in this group. Finally, a numerical example is given to illustrate the validity of the proposed consensus reaching model.

Keywords: Analytic Hierarchy Process (AHP); Group AHP, Consensus reaching; Weight adjustment; Judgment updating

1. Introduction

To make a successful group decision, a certain level of consensus in a group must be achieved. Consensus is commonly meant as a total agreement of all the decision makers with respect to all judgments. However, the opinions in a group are always diverse. Thus measuring and improving the consensus level in a group is of great significance in group decision making.

2. Literature Review

There is considerable literature on improving group consensus. defined the degree of consensus and presented a consensus model based on linguistic preference. In a group AHP decision making context, proposed a framework to measure the group consensus level and then use such information to support the process of consensus building. proposed a consensus reaching model for group AHP under row geometric mean prioritization method. This model first defines the consensus indices among the PCMs and then the moderator suggests decision maker to adjust his/her PCM. presented a model to improve both the consistency and consensus in group AHP, in which the consensus was measured by the compatibility index of two PCMs and then the decision maker would revise his/her PCM according to the moderators suggestion.

The consensus models described above are focused on revising or updating the decision makers' judgments. The weights of the decision makers, however, which are usually associated with the quality of their judgments, are kept fixed in the negotiation and discussion process. It is well-known that in democratic group decision making (e.g. a presidential election, congressional vote), it is inadvisable and infeasible to change the weight of a decision maker simply because his opinion is incompatible with that of the group. But it is feasible when a group making decision in an autocratic environment, e.g., the decision about the date of D-Day , where the decision makers in this group are the experts or consultants who input their opinions to a powerful decision maker. Thus changing the importance weight vector of individuals in a group is an alternative way of encouraging them to reach a group consensus.

3. Hypotheses/Objectives

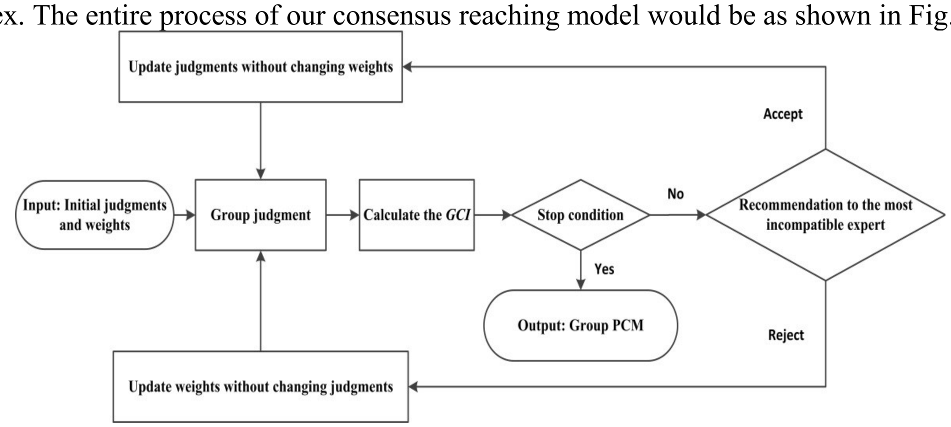
The consensus model presented in this paper attempts to update both the opinion and weights of the decision makers in a negotiation process. The weight adjustment method is simplified and easy to execute in each round of the process. In our model, a decision maker is able to either insist on using his/her own judgment or update his/her judgment based on the moderator's suggestion and his/her weight is adjusted automatically according to which choice he/she makes in the negotiation process.

4. Research Design/Methodology

For simplicity, we use $N = \{1,2,\cdots,n\}$, $M = \{1,2,\cdots,m\}$ to denote elements in sets. For a finite set of alternatives $X = \{x_1, x_2, \cdots, x_n\}$, the judgment information is represented as an $n \times n$ PCM $\boldsymbol{A} = \left(a_{ij} = w_i / w_j\right)_{n\times n}$, where $a_{ij} = 1/a_{ji}$ and a_{ij} belongs to Saaty's 1-9 fundamental scale and represents the relative importance or better, dominance of x_i over x_j .We assume that there are m decision makers $DM_1, DM_2, \cdots, DM_m$ with PCM $\boldsymbol{A}_k = \left(a_{ij(k)}\right)_{n\times n}$, for $k \in M$, and let $\boldsymbol{\rho} = (\rho_1, \rho_2, \cdots, \rho_m)^{\mathrm{T}}$ be the weight or importance vector of the decision makers, where $\rho_k \geq 0, \sum_{k=1}^{m} \rho_k = 1, k \in M$. Then by aggregating with the weighted geometric mean, the group PCM $\boldsymbol{G} = \left(g_{ij} \right)_{n \times n}$ can be calculated as

$$g_{ij} = \prod_{k=1}^{m} (a_{ij(k)})^{\rho_k}.$$

Since there is almost always a diversity of opinions, a consensus reaching process is needed to drive decision makers towards consensus. To measure the consensus level in a group, one first measures the closeness of two Pairwise Comparison Matrices(PCMs). Considering that PCM belongs to an absolute scale and thus also to a ratio scale, Saaty suggested that the closeness of two PCMs can be measured by using the compatibility index. The entire process of our consensus reaching model would be as shown in Fig. 1.



5. Data/Model Analysis

We use the following group decision making problem presented by to demonstrate the validity of our proposed process. Suppose we have four alternatives X_1, X_2, X_3 and X_4 to be ranked and five decision makers DM_1, DM_2, DM_3, DM_4, and DM_5 with PCMs $\boldsymbol{A}_k = \left(a_{ij(k)} \right)_{4 \times 4}$, $k = 1,2,3,4,5$, where

$$\boldsymbol{A}_1 = \begin{pmatrix} 1 & 4 & 6 & 7 \\ 1/4 & 1 & 3 & 4 \\ 1/6 & 1/3 & 1 & 2 \\ 1/7 & 1/4 & 1/2 & 1 \end{pmatrix}, \boldsymbol{A}_2 = \begin{pmatrix} 1 & 5 & 7 & 9 \\ 1/5 & 1 & 4 & 6 \\ 1/7 & 1/4 & 1 & 2 \\ 1/9 & 1/6 & 1/2 & 1 \end{pmatrix}, \boldsymbol{A}_3 = \begin{pmatrix} 1 & 3 & 5 & 8 \\ 1/3 & 1 & 4 & 5 \\ 1/5 & 1/4 & 1 & 2 \\ 1/8 & 1/5 & 1/2 & 1 \end{pmatrix}$$

$$\boldsymbol{A}_4 = \begin{pmatrix} 1 & 4 & 5 & 6 \\ 1/4 & 1 & 3 & 3 \\ 1/5 & 1/3 & 1 & 2 \\ 1/6 & 1/3 & 1/2 & 1 \end{pmatrix}, \boldsymbol{A}_5 = \begin{pmatrix} 1 & 1/2 & 1 & 2 \\ 2 & 1 & 2 & 3 \\ 1 & 1/2 & 1 & 4 \\ 1/2 & 1/3 & 1/4 & 1 \end{pmatrix}.$$

The consistency ratios of $\boldsymbol{A}_k$ are $CR_{A_1} = 0.0383$, $CR_{A_2} = 0.0678$, $CR_{A_3} = 0.0339$, $CR_{A_4} = 0.0471$, $CR_{A_5} = 0.0363$, which indicate that the given PCMs are of acceptable consistency. Let $\boldsymbol{\rho} = (0.1, 0.3, 0.1, 0.2, 0.3)^{\mathrm{T}}$ be the initial weight vector of the decision makers. Set the threshold value of the group consensus index $\varepsilon = 1.01$, $\alpha = 0.7$, $\theta = 0.7$, and the maximum number of iterations $T = 10$. Then we show how to apply the proposed consensus reaching model to adjust the weights and update the judgments. Let $t = 0$, $\boldsymbol{A}_k^0 = \boldsymbol{A}_k$, $k = 1,2,3,4,5$, $\boldsymbol{\rho}^0 = \boldsymbol{\rho}$. We get the group PCM

$$\boldsymbol{G}^0 = \begin{pmatrix} 1 & 1.9786 & 2.6547 & 3.3879 \\ 0.5054 & 1 & 2.2894 & 2.8536 \\ 0.3767 & 0.4368 & 1 & 1.7818 \\ 0.2952 & 0.3504 & 0.5612 & 1 \end{pmatrix}.$$

Then we can calculate $GCI_k^0, k = 1,2,3,4,5$. The consensus reaching process should be continued until it fulfills the stop conditions. The group consensus index, weight vector and the choice of decision makers in each iteration are listed in Table 1.

Table 1. Group consensus index, weight vector and the choice of selected decision maker

Round (t)	Group consensus index ($\boldsymbol{GCI}$) and weight vector ($\boldsymbol{\lambda}$)	Selected decision maker	Choice
0	$\boldsymbol{GCI}^0$ =(1.0502,1.1144,1.0382,1.0040,**1.3366**)$^{\mathrm{T}}$ $\boldsymbol{\rho}^0$ =(0.1,0.3,0.1,0.2,0.3)$^{\mathrm{T}}$	DM_5	Accept
1	$\boldsymbol{GCI}^1$ =(1.0301,1.0817,1.0225,1.0253,**1.2177**)$^{\mathrm{T}}$ $\boldsymbol{\rho}^1$ =(0.1,0.3,0.1,0.2,0.3)$^{\mathrm{T}}$	DM_5	Accept
2	$\boldsymbol{GCI}^2$ =(1.0182,1.0603,1.0139,1.0175,**1.1317**)$^{\mathrm{T}}$ $\boldsymbol{\rho}^2$ =(0.1,0.3,0.1,0.2,0.3)$^{\mathrm{T}}$	DM_5	Accept
3	$\boldsymbol{GCI}^3$ =(1.0112,1.0459,1.0094,1.0137,**1.0806**)$^{\mathrm{T}}$ $\boldsymbol{\rho}^3$ =(0.1,0.3,0.1,0.2,0.3)$^{\mathrm{T}}$	DM_5	Accept
4	$\boldsymbol{GCI}^4$ =(1.0071,1.0361,1.0073,1.0122,**1.0497**)$^{\mathrm{T}}$ $\boldsymbol{\rho}^4$ =(0.1,0.3,0.1,0.2,0.3)$^{\mathrm{T}}$	DM_5	Reject
5	$\boldsymbol{GCI}^5$ =(1.0042,**1.0274**,1.0065,1.0119,*1.0626*)$^{\mathrm{T}}$ $\boldsymbol{\rho}^5$ =(0.1225,0.3225,0.1225,0.2225,0.21)$^{\mathrm{T}}$	DM_2	Accept
6	$\boldsymbol{GCI}^6$ =(1.0052,**1.0173**,1.0075,1.1005,*1.0553*)$^{\mathrm{T}}$ $\boldsymbol{\rho}^6$ =(0.1225,0.3225,0.1225,0.2225,0.21)$^{\mathrm{T}}$	DM_2	Accept
7	$\boldsymbol{GCI}^7$ =(1.0063,**1.0110**,1.0086,1.0098,*1.0500*)$^{\mathrm{T}}$ $\boldsymbol{\rho}^7$ =(0.1225,0.3225,0.1225,0.2225,0.21)$^{\mathrm{T}}$	DM_2	Accept

8(Stop)	$\boldsymbol{GCI}^8$=(1.0074,1.0070,1.0098,1.0094,*1.0459*)^T $\boldsymbol{\rho}^8$=(0.1225,0.3225,0.1225,0.2225,0.21)^T	DM_2	Accept

As can be seen in Table 2, the algorithm terminates after 8 iterations. The final group PCM is

$$\boldsymbol{G}^* = \begin{pmatrix} 1 & 3.2469 & 4.7410 & 6.3679 \\ 0.3080 & 1 & 3.2353 & 4.0455 \\ 0.2109 & 0.3091 & 1 & 2.1451 \\ 0.1570 & 0.2472 & 0.4662 & 1 \end{pmatrix}.$$

From $\boldsymbol{G}^*$ we can derive the final priorities $\boldsymbol{w} = (0.5691, 0.2559, 0.1094, 0.0656)^T$. Thus the ranking of alternatives should be $X_1 \succ X_2 \succ X_3 \succ X_4$.

6. Limitations

This model is feasible when a group making decision in an autocratic environment, where the decision makers in this group are the experts or consultants who provide their opinion to a powerful decision maker as the reference. It's not a democratic model.

7. Conclusions

In this paper, we have proposed a consensus reaching model for the autocratic group AHP. We presented a consensus reaching process for group AHP based on weight adjustment and judgment updating. The main advantages of this model are: (1) This model allows the involved decision makers to choose to accept or reject the recommendation from moderator; (2) The weight adjustment is used as an incentive for a decision maker to update judgment according to moderator's suggestion; (3) The consistency of PCMs is maintained in the proposed consensus reaching model; (4) The proposed consensus reaching model improves the consensus level of a group.

Additional research is currently underway to improve the accuracy and effectiveness of dealing with incompatibility.

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OUTLINE FOR PAPER PROPOSAL 4 SUBMITTED TO THE INTERNATIONAL SYMPOSIUM OF THE ANALYTIC HIERARCHY PROCESS

RESOLVING RANK REVERSAL IN CONSISTENT AND INDEPENDENT AHP MODEL

Kè-yù ZHŪ
School of Management,
Hefei University of Technology,
Hefei, China
cauchyandy@163.com

Cooper, Orrin
Fogelman College of Business & Economics
The University of Memphis
Memphis, TN
olcooper@memphis.edu

Shàn-lín Yáng
Key Laboratory of Process Optimization and Intelligent Decision
Ministry of Education
Hefei, China
hgdysl@gmail.com

ABSTRACT

An open question that has existed for some time now is how to preserve rank in the AHP when a new alternative is added or when one is deleted. The essential conditions are that all judgments be consistent and all elements are independent; these have not been fully considered by the AHP critics and defenders. When a new alternative is added or when one is deleted, rank should be preserved when the conditions are satisfied. The weighted geometric mean aggregation rule is proposed to achieve the desired outcome. A proof demonstrates that the weighted geometric mean aggregation rule can preserve rank in the normalized priority vector. Finally, the causes of rank reversal are analyzed: the principal eigenvector approach and the relative mode, and derive that they are not the real reasons of rank reversal.
Keywords: decision analysis, AHP, rank reversal, aggregation rule

1. Introduction

We propose a method is through the weighted geometric mean aggregation rule to achieve rank preservation. However, the usage of the modified geometric mean aggregation rule must also address two other concerns: 1) the local priorities should be obtained through the principal eigenvector approach and one should use the modified geometric mean aggregation to synthesize the criteria clusters in the overall model; 2) the shortcomings of synthesizing with the geometric mean should be overcome in the aggregation process (detailed in section 3.3). We not only prove that the modified geometric mean aggregation rule can force rank preservation through the principal eigenvector approach, but also overcome the shortcomings of the geometric mean in the aggregation process. Finally, two traditional numerical examples of rank reversal from the literature are presented to check the validity of the proposed method.

2. Preliminaries

The rank reversal phenomenon can be described as when three alternatives (*A*, *B*, and *C*) are ranked in order $B > A > C$ by the AHP. Then when another alternative *D*, which is an exact copy of *B*, is added, the alternatives are ranked in the order $A > B = D > C$; thus, the introduction of an irrelevant alternative causes *A* and *B* to switch order. The following is the example provided by Belton and Gear .

3. The reason of rank reversal and its resolution

3.1 The reasons of rank reversal

Saaty explained that the major objection raised against the AHP by practitioners of utility theory has been the issue of rank reversal. In reviewing the critiques of rank reversal in the AHP, three reasons can be identified.

The first reason rank reversal can occur is the principal eigenvector approach. The second cause of rank reversal is the relative judgment mode.

The third reason is given by AHP defenders, they attribute rank reversal to 1) the dependence and feedback between alternatives and criteria ; and 2) the scarcities and abundance of alternatives .

3.2 A way to preserve rank

Because the current rank preservation methods have theoretical limitations, and also because the conditions of consistency and independence are not fully considered by the AHP researchers and critics alike, another method is needed to preserve rank under the conditions of consistency and independence when a new alternative is added or when one is deleted. With careful consideration of rank reversal in the AHP, it can be shown that with the use of the weighted geometric mean aggregation rule in place of the arithmetic mean aggregation rule, rank can be preserved when a new alternative is added or when one is deleted. However, the usage of the modified geometric mean aggregation rule to preserve rank must tackle the following two problems: one is that the local priorities which will be synthesized by the modified geometric mean aggregation rule should be obtained through the principal eigenvector approach; the other is that the shortcomings of synthesizing with the geometric mean should be overcome in the aggregation process.

In the next section, we will provide a solution to the two problems presented above.

4. Justification on rank preservation

In this section, we prove that the modified geometric mean aggregation rule can preserve rank via the principal eigenvector approach. Then we prove that the shortcomings of the geometric mean will not occur in the aggregation process.

4.1 The modified geometric mean aggregation rule can preserve rank

With pairwise comparison judgments, the priorities of alternatives are relative and depend on each other. It is reasonable to assume that if all the judgments are consistent and all elements are independent when comparing the alternatives with respect to each criterion, adding or deleting an alternative should preserve the final overall priorities of the alternatives with respect to all the criteria. This is the case when using a weighted geometric mean aggregation rule as will be shown in the proof below. The rank preservation idea can be described as the following theorem:

Theorem 1. In the AHP, when a new alternative is added or when one is deleted, the usage of the weighted geometric mean aggregation rule can guarantee that the proportions of the final weights of the old alternatives remain unchanged if all judgments are consistent and all elements are independent.

Theorem 1 is an enhanced version of rank preservation. In theorem 1, not only the rank of original alternatives can be preserved, but the proportions of the final weights of the original alternatives can also be preserved.

4.2 The shortcomings of synthesizing with the geometric mean can be overcome in the aggregation process

In this section, we prove that the shortcomings of geometric mean can be overcome.

5. Validity check

In this section, two familiar examples are presented which have been widely discussed in the complex arguments regarding rank reversal in the AHP to check the validity of our statement.

6. Discussion and conclusion

AHP critics attribute rank reversal to the principal eigenvector approach and the relative judgment . We disagree.

For the eigenvector approach

As was discussed in section 3.1, many researchers attribute rank reversal to the principal eigenvector approach. Barzilai and Golany , in particular, hold that for all judgments there does not exist any synthesis method which avoids rank reversal. However, the principal eigenvector approach has nothing to do with rank reversal. In pairwise comparison judgments, when all judgments are consistent, the results obtained by the principal eigenvector approach are identical with that by arithmetic mean, geometric mean or logarithmic least square method. Therefore, the principal eigenvector approach is not really the root of rank reversal when all judgments are consistent. In fact, rank preservation is irrelevant to the principal eigenvector approach because it has been proven that rank can be preserved with it when the conditions of consistency and independence are satisfied.

For the relative judgment

In the relative measurement the preference for an alternative is determined by all other alternatives . In this sense the alternatives are not independent from each other for the determination of their priorities. This implies that when one meets relative measurement, dependence and feedback should be considered and hence the *Analytic Network Process* (ANP) should be employed. But if the ANP is introduced into relative measurement, the relative mode of the AHP would disappear, so one could argue just use the ANP. This is an interesting phenomenon because the relative mode is a classification of the AHP, but according to its characteristics it should belong to the category of the ANP.

The reason for this phenomenon is because of the misunderstanding of the relationship between the eigenvector approach and the dependence among alternatives. The eigenvector approach is just a data process method. A number of independent elements should not turn into dependent elements after applying the principal eigenvector approach. Thus, the attribution of rank reversal to the principal eigenvector approach in the relative mode is not correct. Regardless of the absolute judgment or the relative judgment, the weighted geometric mean aggregation rule can preserve rank under the conditions of consistency and independence. Section 4.1 is also a proof for relative judgment, where the weighted geometric mean aggregation rule can preserve rank under the conditions of consistency and independence. This is also true for absolute judgment.

The weighted geometric mean aggregation rule is the solution to Dyer's remarks

Probably the most influential critic on the AHP is Dyer's remarks on rank reversal in *Management Science*. He attributes rank reversal as a symptom of a much more global problem with the AHP: the rankings provided by the methodology are arbitrary. Dyer's methodology arbitrarily uses the eigenvector approach on the scores of the alternatives when the principle of hierarchic composition is assumed. He then points out that the AHP theory does not include any "independence conditions" that can be tested empirically. We disagree because it has been proven that rank can be preserved with the principal eigenvector approach when all judgments are consistent and all elements are independent, wherein the "independence conditions" are considered.

In general, this paper does not question the legitimacy of rank reversal, but rather the rank reversal under the conditions of consistency and independence. Theoretically, rank preservation should be guaranteed when one meets the conditions. The weighted geometric mean aggregation rule supersedes any other aggregation rules which can avoid rank reversal. In fact, the AHP employs a ratio scale to measure the intensity of preferences of alternatives and criteria, while the weighted geometric mean aggregation rule is also a ratio scale measurement. They are naturally matched. This body of research can augment and expand the AHP theory.

Future research should consider how to address rank reversal in the ANP supermatrix. For example, one could explore what happens when the criteria depend on the alternatives, as well as with tangible and intangible criteria. Such results could strengthen support for the AHP/ANP and its application.

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