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ESTIMATING PROBABILITIES IN DECISION-MAKING PROCESSES USING AHP

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Abstract: Probabilities and transition probabilities need to be estimated in many decision-making processes which involve uncertainty. The purpose of this paper is to explain that AHP can be used as a effective tool to estimate these probabilities and transition probabilities to support decision-makers. In the paper, the transition probabilities of a Markov process model are derived by AHP to make the model available for describing student progression process and predicting teaching load in the College of West Virginia.

Introduction

The analytic hierarchy process (AHP), developed by Thomas L. Saaty, is designed to solve complex problems involving multiple criteria. The process requires the decision maker to provide judgments about the relative importance of each criterion and then specify a preference on each criterion for each decision alternative. The output of AHP is a prioritized ranking indicating the overall preference for each of the decision alternative.

AHP utilizes pairwise comparisons to establish priority measures for both the criteria and the decision alternatives. Pairwise comparisons are fundamental building blocks of AHP. In the following discussion, we will demonstrate how the probabilities or transition probabilities in many decision-making processes can also be determined in a similar fashion.

Estimating Probabilities Using AHP

Using pairwise comparisons to provide judgements about each of state probabilities in decision-making processes, we need an underlying scale to compare the likelihood of occurrences of two states. AHP employs an underlying scale with values from 1 to 9 to rate the relative preferences for two items (Saaty, 1980). Since research and experience have confirmed the nine-unit scale as a reasonable basis for discriminating between the preferences for two items, we can use this scale as a basis for comparison between the occurrence likelihood of two states. However, the verbal judgment of preferences in AHP scale needs to be modified to allow the verbal judgment of likelihood. We provide a new pairwise comparison scale for judgements about probabilities, the numerical ratings recommended for the verbal judgment of likelihood of state occurrences are given in the following table.

Table 1 Pairwise Comparison Scale for Occurrence Likelihood of States

Verbal Judgment of Occurrence Likelihood of States	Numerical Ratings
Extremely high	9
Very to extremely high	8
Very high	7
High to very high	6
High	5
Moderately to high	4
Moderately high	3
Equally to moderately high	2
Equally high	1

To introduce this new nine-unit scale, let us consider a simple decision-making problem with three states of nature. We need to estimate the probabilities of the states in the problem. Suppose that we have compared the likelihood of occurrence of state A with that of state B and are convinced that that of state A is high, then a value of 5 is utilized. If we believe that the likelihood of state A is extremely high compared to that of state C, a value of 9 is utilized; if we believe that the likelihood of state B is moderately high compared to that of state C, a value of 3 is utilized. A value of 1 is reserved for the case where the two states are judged to have the equal chances to occur.

To estimate the probabilities of the states, we can use the matrix of the pairwise comparison of AHP. Since probabilities of three states are being considered, the matrix can be constructed as below based on the likelihood we have specified.

	State A	State B	State C
State A	1	5	9
State B	1/5	1	3
State C	1/9	1/3	1

To determine the remaining entries in the matrix, first note that when we compare the chance of occurrence of any state against that of itself, the judgment must be "equally high". Thus, using the scale shown in Table 1, the rating of state A compared to state A, state B to state B, and state C to state C must be 1. Hence, we assign a 1 to all elements on the diagonal of the pairwise comparison matrix. The likelihood rating for state B when compared to state A is simply the reciprocal of the likelihood rating for state A when compared to state B: 1/5. Using this method, we obtain the likelihood rating for remaining entries.

Once the matrix of pairwise comparisons has been developed, we can carry out the AHP synthesis procedure to calculate the probability for each of the states being compared. The probabilities of the three states provided by the procedure are written as follows:

State A	State B	State C
0.75	0.18	0.07

The consistency of pairwise comparison judgments is measured by the consistency ratio provided by AHP, and a consistency ratio of 0.10 or less is considered to be a reasonable level of consistency in the pairwise comparisons.

Estimating Probabilities in a Markov Process Model for Student Progression

The College of West Virginia (CWV) is using a planning process to set the course of actions for meeting the needs of the next five years, and is positioning itself for the challenges of the 21st century. The five-year plan is based on the review and assembly of a wide range of documents and statistical materials on the activities at the College of West Virginia in the past, at present, and projected into the future.

Some of the most significant decisions in the long-term planning are made on the basis of judgment forecasts (A Five-Year Action Plan, 1994) (Jiang, 1994). Forecast of teaching load in the next five years is one of key issues, which provides a vision of what may be accomplished for the college looking to the long-term future of the institution. The CWV wants to predict the demand for each of its four levels of teaching load to determine its investment level, work-force size, budgets, and facility capacity, among many other considerations. It is the accuracy of the forecast that will ultimately determine the degree of success of the planning, and the future of the institution.

As is the case with all long-term forecasts, that of CWV is based on a number of factors and broad assumptions believed to be reasonable. The forecast of teaching load involves in various aspects of judging process: the scope of the current academic program, recent enrollment trends, the continuing efforts of the faculty to offer a variety of degree and certificate programs, and above all, the progression of the students in the college.

As we consider the student progression process into the future, we cannot say for certain where a student will be during a given year. However, using a Markov process model, we are able to compute the probability that the student stays at each state over successive time periods. We are interested in the probability of the college students being in a particular state at a given time period, and a Markov process model and its transition probabilities are utilized to describe the manner in which the college students make transitions from one period to the next.

In this study we assume that the transition probabilities of the Markov model remain constant over time, and the probability of the students being in a particular state at any one time period depends only on the state of the process in the immediately preceding time period.

Using the terminology of Markov processes, we refer to the yearly periods as the trials of the process. The particular student status in a given year is referred to as the state of the students in that period. Since the college students have six status alternatives in each year, we can identify six states for the Markov process model as follows:

State 1	Freshman
State 2	Sophomore
State 3	Junior
State 4	Senior
State 5	Dropout
State 6	Graduate

We use **P** to represent the matrix of transition probabilities, that is, **P** =

	Freshmen	Sophomore	Junior	Senior	Graduate	Dropout
Freshmen						
Sophomore						
Junior			P_{ij}			
Senior						
Graduate					1.00	
Dropout						1.00

where P_{ij} = probability of making a transition from state i in a given year to state j in the next year.

To determine the probabilities that the students remain with a state or switch to the other states as the process continues from year to year, we collect data and judgments from the college administration office and student service office over a five-year period. These data and judgments show the pattern of student progression. Then the transition probabilities of the Markov process model are estimated using the method of AHP as explained in the last section of this paper.

Based on a study of student progression, the following assessment has been made of the student transitions. For each level of the students, a pairwise comparison matrix is constructed to judge the likelihood of alternative transitions. The matrices showing these judgments are given in the following four tables.

Table 2 Comparisons of Occurrence Likelihood of Three States for the Freshmen Progression

	Freshmen	Sophomore	Dropout
Freshmen	1	1/4	1
Sophomore	4	1	3
Dropout	1	1/3	1

Table 3 Comparisons of Occurrence Likelihood of Three States for the Sophomore Progression

	Sophomore	Junior	Dropout
Sophomore	1	1/8	1/2
Junior	8	1	5
Dropout	2	1/5	1

Table 4 Comparisons of Occurrence Likelihood of Three States for the Junior Progression

	Junior	Senior	Dropout
Junior	1	1/9	1/2
Senior	9	1	9
Dropout	2	1/9	1

Table 5 Comparisons of Occurrence Likelihood of Three States for the Senior Progression

	Senior	Graduate	Dropout
Senior	1	1/9	2
Graduate	9	1	9
Dropout	1/2	1/9	1

In interpreting these values in the pairwise comparison matrices, we see in Table 5 that the likelihood of graduate is extremely high for the Senior students compared to those of Dropout and remaining in Senior (9), and the chance of remaining in Senior is equally to moderately high for the Senior students compared to that of Dropout (2).

Then we carry out the AHP synthesis procedure using Expert Choice to calculate the transition probabilities. The results are given in the Table 6.

Table 6 The Matrix of Transition Probabilities

	Freshmen	Sophomore	Junior	Senior	Graduate	Dropout
Freshmen	0.17	0.63	0.00	0.00	0.00	0.20
Sophomore	0.00	0.09	0.75	0.00	0.00	0.16
Junior	0.00	0.00	0.07	0.82	0.00	0.11
Senior	0.00	0.00	0.00	0.11	0.82	0.07
Graduate	0.00	0.00	0.00	0.00	1.00	0.00
Dropout	0.00	0.00	0.00	0.00	0.00	1.00

The remaining entries in the matrix of transition probabilities have a value of zero. Since the procedure displays a consistency ratio of less than 0.10 for all of the four pairwise comparison matrices, we believe that there is a reasonable level of consistency in the pairwise comparisons.

Using the matrix of transition probabilities of the Markov progress model, we can now compute the probability that a freshmen or a sophomore will graduate or dropout. We can also determine what percentage of the 2000 students attending the college will be in a state at some period in the future.

Conclusion

In this paper, a practical use of AHP in estimation of state probabilities in decision-making processes has been presented. The pairwise comparison matrix and synthesis procedure of AHP have been utilized to calculate the transition probabilities in a Markov progress model. The result of this study has proved that AHP is a simple and effective tool for this purpose. The AHP-derived transition probabilities have helped the CWV to analyze the progression of its students and to predict and manage its teaching load in the long-term plan.

References

Saaty, T. L. (1980) *The Analytic Hierarchy Process*, McGraw-Hill.

(1994) *A Five-Year Action Plan for the College of West Virginia*, College of West Virginia, USA.

Jiang, C. Q. (1994) "AHP in Trend-Changed Forecasting," *The Third International Symposium on the Analytic Hierarchy Process*, Washington, D.C., USA.