MEASURING IN WEIGHTED ENVIRONMENTS
(Moving From Metric Topology to Order Topology)

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ABSTRACT

This paper addresses the problem of measuring compatibility in a decision making process with priority vectors. Here compatibility is defined as similarity or closeness between vectors inside a decision-making space (a weighted space). The main question that arises here is when two decision makers (DMs) present the same mode of thinking (making similar decisions) is how to measure that similarity or closeness in a weighted environment.

Keywords: Decision making process, Compatibility Index, priority vectors.

1.- Introduction

About Compatibility, Ordinal and Cardinal Data:
There is a common belief that people making the same choices are compatible (people that think the same), while people that make different choices are not. It is easy to prove that this is not always true, since when working with data from an ordinal scale (first, second, third…) the quality of the data when used for decision-making is too low, becoming easy to reach the erroneous conclusion of the common belief. Ultimately, decision making is very different when working with ordinal data rather than with cardinal data. One common example, the information in the following case: Pedro, Maria, Pablo: (first, second, third), then Pedro, Maria, Pablo: (57.5; 25.5; 17.0), is very different. The ordinal data is the same in both cases, but the quality of the information enclosed is very different. In the first set of numbers the extent to which Pedro is better than Maria is not known, while in the second set the information is precise.

As an example of an extreme situation, a person with an ordinal array of preference, such as: (first, second, third), may be closer (thinking alike) to a second person with an array of preferences: (third, second first), than to a third person with the same ordinal array of preferences as person one (first, second, third). Although having an inverse order of preferences, but at the same time incompatible with person3 with exactly the same order of preferences.

Moreover, many common errors and mistakes arise from using ordinal scales, even the Arrow's Impossibility Theorem is based on ordinal scale data, but if cardinal scaled data is used then the theorem would be no longer valid. There is a tendency to use ordinal behavior because the human mind is a lot more comfortable with linear processes, and ordinal scales are a kind of linear scale, although everyone is aware that the many phenomena in the real world are far from being linear[1].

[1] The Singularity is Near, Ray Kurzweil. Pinguin Group Publications, 2005.

The History of the Compatibility Index:

The application of the Compatibility Index concept to multi-criteria decision making (MCDM), was first introduced by Dr. Thomas Saaty in the 90`s along with the Analytical Network Process (ANP) applied to examples of market shares, especially in the quality of the ANP model predictions compared with the actual market share. The strong predictive capacity of these models in different and complex applications produced a deep impression on this author. The concept of compatibility was also suggested by Dr. Saaty as a tool for fine-tuning in conflict resolutions in his book "Decision Making with Dependence and Feedback, The Analytic Network Process"[2]. This author wants to thanks Dr. Saaty for his help in using this concept and for all our interesting discussions about compatibility and weighted environments (in Pittsburgh and in Santiago), and for including the General Compatibility Index (*G*) in his 2008 book: "*Group Decision Making*"[3]. Without the development of the compatibility concept, it would not be possible to ask oneself about how to measure the closeness between two different value systems (within the order topology domain). In this sense the logical next step is to have a cardinal measurement to determine the closeness (compatibility) of two value systems for a general situation, that is, an index of measurement to measure the closeness between two or more priority vectors that represent two or more decision makers in a weighted environment, an index that present no singularities with an adequate threshold value able to assess (in almost any situation), if two individuals really have similar ways of thinking.

The way to measure similarity or proximity in weighted environments is closely related to the intensity of preferences of decision makers, that is, the strength that one alternative or criteria dominate others. Those preferences belong to a set of decision makers and their decision profiles or behavior patterns. All possible criteria tangible or intangible that may affect the measurement should be considered, and this consideration affects the kind of topology required to produce a measurement of proximity (closeness).

There are two kinds of topology for measurement: metric topology and order topology. One of the main differences between the two is the measure scale on which they are based as well as how the results should be interpreted. In fact, it can be said that intensity of preferences is to order topology as closeness is to metric topology. Similarly, the potential of analysis measuring the intensity of the preferences in order topology will be analog to measuring closeness in metric topology. To illustrate this, a parallel is presented between the Norm of distance of metric topology represented on Cartesian axes and its possible equivalent in order topology, which is intended as closeness of DMs or more specific, compatibility among profiles of decision or compatibility of DM's pattern preferences.

This last idea can be seen in graphical terms with the help of Figure 1:
Consider two persons, P1 and P2 with the following decision profile of preferences over a set of alternatives, the horizontal line in front of each criterion represents the importance or weight of that alternative compared to the others and the vertical line represent the pattern of preferences of P1 and P2 over the alternative set (the behavior of the alternatives related to the criteria).

[2] The Analytic Network Process, Thomas L. Saaty. RWS Publications, 2001.
[3] Group Decision Making: Drawing out and Reconciling Differences. Thomas L. Saaty & Kirti Peniwati, 2008, RWS Publication. ISBN 1-888603-08-9.

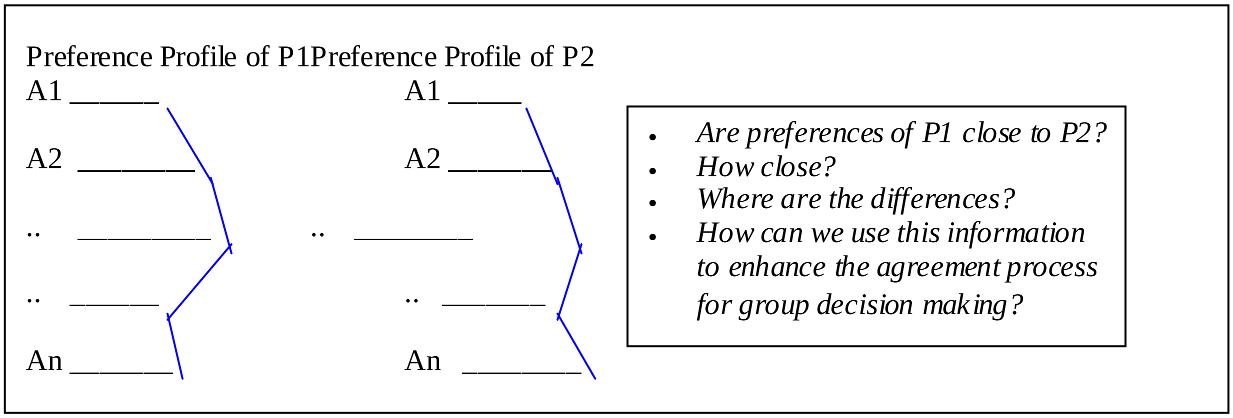

Figure 1: DM's Preference Profiles

The preference profile of P1 can be interpreted as the way that P1 makes decisions, in some way the preference profile represents his/her "rule of measurement" when selecting the best alternative from a set of different possible alternatives. It may become important to evaluate if the decision profile of P1 is close or not to that of P2, to answer a questions such as: is P1 making decisions similarly to P2? Are their patterns of preference close (compatible) to each other?

The same questions can arise for the set of criteria instead the set of alternatives, in that case we get the form that P1 and P2 make decisions, before knowing the alternatives, which will be closest to their general system values, then we might reproduce each rule of measurement produced by each system value (for P1, P2…,Pn) and establish if there are close (compatible) or not. This kind of analysis provides a quantitative form to measure which are their common points and what are their differences, yet more important enables assessing the quantitative values of those differences. This last piece of information helps to establish how to increase their compatibility and overcome potential conflicts due to the differences in their system values.

2.- The General Context:

In this document, it has been argued that any decision process, if well understood and modeled, can always be represented as a vector of priorities, (in a cardinal way) and if desired priorities can be represented by coordinates in a (0, 1) Cartesian graph, with each axis representing the behavior of the alternative in the corresponding dimension or criterion of evaluation. The set of evaluated criteria will create the weighting decision making space, that is, a weighted environment in which the decision makers can recreate their priorities and make their decisions. A weighted environment (weighted space) is necessary, since the preferences of the DMs depend on the criterion used, presenting different preferences for different criteria, this is analog to having a Cartesian Graph with axes of preferences or different weights, representing preferential directions in the space. This condition will become very important when trying to calculate distance in a not "isotropic" preference space, as it is shown in the next two figures (figure 2a and 2b):

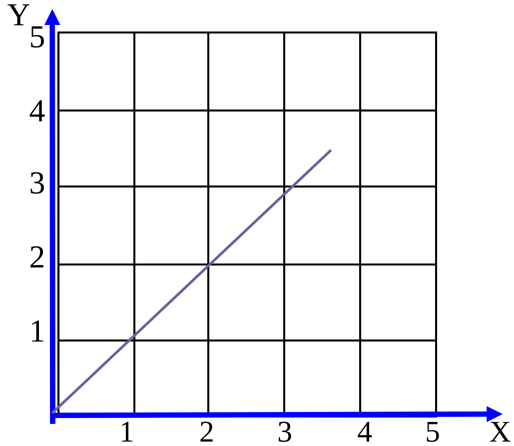

Figure 2a: Equally important (y=x)
Coord. *x* is as preferable as *y*

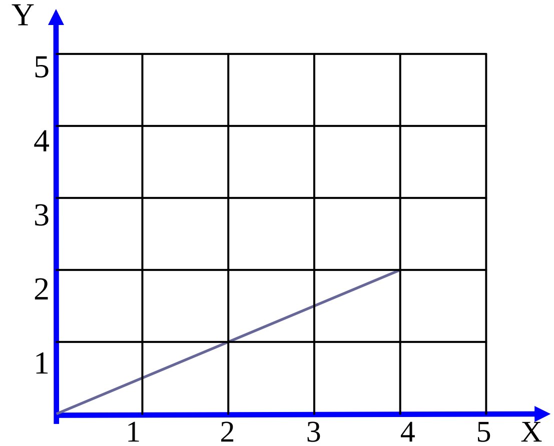

Figure 2b: X is twice important than Y (y=½x)
Coord. *x* is twice as preferable as *y*

As shown in figures 2a and 2b, the slope of the lines may represent the preference or importance that one axis has compared with the other, for instance, in the case of figure 2a, axis Y is as important as axis X, since following Y is just a fast (numerically) as going by X. In the case of figure 2b, the situation is different, the slope shows that going by the X axis is twice as fast (important) as going by Y.
Then, the slope (the ratio) between the axes is a simple way to represent the intensity or importance that one element may have over another, of course, the slope may change point to point (coordinate to coordinate) over the curve, that more general case have been addressed in the full document.

Inside this environment, a new compatibility index can be built which is able to work in this weighted environment. This index is created in order to measure the compatibility of two DMs, through the weighted projection of one vector over the other, considering that in a multidimensional space two normalized vectors are similar only if there are going in the same direction, and they are separated once they start to go in different directions and so becoming different and non compatible vectors.

3.- Why is Compatibility Important and Where can it be Applied:

The importance of this issue has many applications, first the possibility of knowing whether two decision makers or decision vectors that seem to be close are really close. It is necessary to measure the degree of closeness to establish if they are close enough to consider them compatible. So, if is possible to measure that closeness, then it would be possible to know when two DMs have actually similar points of view (in cardinal measure bases), this knowledge provides an interesting way of conflict resolutions when working in group decisions making problems.
It must be emphasized that having similar points of view is not synonymous to making the same decision or choosing the same alternative, or even having the same order of preference in a set of alternatives, (the last sentence might not be intuitive and is developed in few examples in the full document). This follows the main idea presented in this paper, that it is the intensity of the preference (dominance) among the elements (no matter if alternatives or criteria) that really matters. This is the key that gives us the measurement of closeness between DMs profiles or DMs pattern behaviors, just as in measuring the closeness or distance between two points in physics or engineering problems using metric topology.
The broad possible field applications for this compatibility index are:

1. **Medical** for diagnosis support; Benefit example: measuring the degree of matching between disease-diagnosis profiles.

2. **Buyer-Seller** matching profiles; Benefit example: measuring the degree of matching between house buyers and house sales projects.
3. **Conflict Resolution**; Benefit example: measuring how close are two different value systems (way of thinking) by measuring the discrepancies.
4. **Test of Quality for Created Metrics**; Benefit example: measuring which MCDM decision method builds a better metric.

When measuring compatibility between two DMs, the proximity of DM value systems is being measured, calculating the importance of their similarities and their differences, establishing when two persons are really thinking in a similar way. (See fig.1).
In this way, the degree of alignment among complex patterns or decision profiles conformed by many criteria and values is also being measured.

Next, a list with few examples of compatibility index applications in real life:
1.- In medical diagnosis process measuring the degree of matching among different disease profiles and the possible diagnosis profiles,
2.- Searching for employment, measuring the degree of matching between a person's profile with the desired profile,
3.- In curricula network design, measuring the degree of matching between the undergraduate student profile and society's needs,
4.- measuring the degree of matching between buyers and sellers in a housing market,
The last two types are known as two-sided matching market problems, (the people, buyer or seller, belong to only one side they cannot jump to or act in the other side)[4]
5.- The measurement of compatibility can be applicable in the classic sensitivity analysis process, measuring whether two created scenarios from sensitivity analysis are compatible or not, if they are not compatible, then is not possible to "replace" one scenario with the other, but if they are compatible, then it would be possible to study the more likelihood trending replacing different but at the same time compatible scenarios.
6.- And last but not least, to measure the quality of a metric built it in some MCDM method[5]. *(Note: this last application of compatibility is possible only when the real or actual vector of values is known).*

4.- Application Case:

Finally, a simple application case of the quality of a metric built it in some MCDM method, (in this case comparing AHP with Fuzzy sets), the metric quality is measured through the *G* function measuring compatibility of both methods compared with actual data, using a 2008 paper of Thomas Saaty and Liem Tran, called "*Fuzzy Judgments and Fuzzy Sets*" [6]. There are many examples in this paper of the unnecessary (and sometime dangerous) use of fuzzy set theory for AHP/ANP when building metric in decision-making. The idea is to measure the quality of the new metric, this is done through the assessment of the compatibility between the actual and the new built metric, measuring how close (or far) is the fuzzy theory compared with AHP regarding actual values when building metrics. Do they get better results? Or is there no gain on using a fuzzy set. Or even worse, are we getting farther from actual results than with an AHP outcome?

This first example is well known: "which country is more wealthy?" *(Dr. Saaty and a colleague analyzed this example during a long airplane flight in 1972).*

[4] *Two-Sided Matching. A study in game theoretical modeling and analysis. Alvin E. Roth & Marilda A. Oliveira Sotomayor. Econometric Society Monographs, 1990*
[5] *All the application examples presented are real case applications of compatibility index G.*

[6] *All the figures and tables of the examples have been take it from the paper "Fuzzy Judgments and Fuzzy Sets"*

The pairwise comparison matrix performed for the seven countries considered was:

$$\begin{pmatrix} & U.S & U.S.S.R & China & France & U.K & Japan & W.Germany \\ U.S & 1 & 4 & 9 & 6 & 6 & 5 & 5 \\ U.S.S.R & 1/4 & 1 & 7 & 5 & 5 & 3 & 4 \\ China & 1/9 & 1/7 & 1 & 1/5 & 1/5 & 1/7 & 1/5 \\ France & 1/6 & 1/5 & 5 & 1 & 1 & 1/3 & 1/3 \\ U.K & 1/6 & 1/5 & 5 & 1 & 1 & 1/3 & 1/3 \\ Japan & 1/5 & 1/3 & 7 & 3 & 3 & 1 & 2 \\ W.Germany & 1/5 & 1/4 & 5 & 3 & 3 & 1/2 & 1 \end{pmatrix}$$

The Results from AHP and actual result are:

	Normalized Eigenvector	Actual GNP (1972)	Normalized GNP Values
U.S	.427	1,167	.413
U.S.S.R	.23	635	.225
China	.021	120	.043
France	.052	196	.069
U.K	.052	154	.055
Japan	.123	294	.104
W. Germany	.094	257	.091

And the Results from the fuzzy model was:

	US	USSR	China	France	UK	Japan	W. German	Normalized Eigenvector
US	1.00	3.63	10.31	5.67	6.09	4.78	4.93	0.468
USSR	0.28	1.00	8.00	4.81	4.67	2.79	3.31	0.184
China	0.10	0.13	1.00	0.19	0.19	0.15	0.21	0.030
France	0.18	0.21	5.40	1.00	0.96	0.39	0.39	0.063
UK	0.16	0.21	5.20	1.04	1.00	0.36	0.38	0.060
Japan	0.21	0.36	6.74	2.59	2.78	1.00	2.18	0.107
W. German	0.20	0.30	4.70	2.54	2.65	0.46	1.00	0.087
				Eigen value:	7.468			
				Inconsistency index:	0.078			

The results seem to be more or less the same when compared with actual results showing some numbers close to the actual results and other far removed. The priority vector should be measured to know which is closer to actual value in a global measure way, to do so, the *G* function is required (and also the best) to determine numerically which is the closest built metric; the AHP or the Fuzzy Sets, and also how much closer (better) one is compared to the other. Applying 1-*G* for incompatibility index then:

- 1 - *G*(AHP-Actual)= 7.39%, saying that AHP's metric vector of priorities is compatible with the real or actual values (less than 10%).
- 1 - *G*(Fuzzy-Actual)= 11.80%, saying that Fuzzy Set's metric vector of priorities is slightly incompatible with real values (slightly greater than 10%).

5.- Conclusion

This simple example shows that the use of AHP gets better results (measurable better) than the use of Fuzzy Sets, this means that the quality of the metric created with AHP is greater than the one created by Fuzzy Sets. There are many other examples whit similar results (and even larger), using *G* function to predict the quality of any new "*created*" decision metric.

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