QUANTIFYING TASK OUTPUT AND PROJECT VALUE IN KNOWLEDGE-WORK CONTEXTS

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ABSTRACT

The central and challenging project planning responsibility of deriving realistic duration estimates founders particularly in cases where tasks lack naturally occurring output metrics. In this paper we show how AHP/ANP can be used to help measure task 'magnitude', especially in difficult to quantify areas such as research, design, analysis and writing. We generalise an approach used in Agile software development planning methods, where a number of 'story' points are attached to an identified base task, usually one of lowest apparent complexity, with point values then being assigned to other tasks in proportion to how their respective apparent complexities compare with that of the base.

The approach begins with the observation that tasks can frequently and usefully be described as a group of sub-tasks which do not warrant their own explicit representation in the WBS and whose individual influences on the task's duration are frequently neglected. By means of the AHP/ANP, we can prioritise the sub-tasks against a rich set of appropriate criteria such as magnitude, quality and complexity, interpret the priorities as relative contributions to aggregate task output, assign an arbitrary number of output units to a selected sub-task and finally, scale all the priorities up accordingly to obtain a quantitative description of the task magnitude. Duration estimation is then restored to the familiar process of identifying and applying resource productivity, corresponding effort and likely attendance levels to the computed task magnitude, adding appropriate time-driven factors such as interruptions and contingencies as needed.
The problem of the lack of output measures extends to the project implementation phase where resort is frequently made to inputs as the basis for progress assessment at higher levels of the WBS. For example the widely used Earned Value Methods methodology equates progress with the planned cost to produce, in alignment with the 'implicit' theory of value.

The paper will therefore also demonstrate the power of prioritisation in helping to define value in alignment with the 'subjective' theory, in this case as perceived by the end-user or client. This requires the development of an AHP/ANP model across the entities of the highest level of the WBS hierarchy, characterised by a variety of criteria such as re-usability, functionality, maintainability, sustainability, constructability and their sub-criteria. This will produce a set of priorities that describe the normalised contributions each entity makes to the production of the project deliverable. This can then be repeated for the sub-hierarchies down to an appropriate depth. Progress, both local and global, can then be defined as a proportion of genuine output value delivered. Finally, since these measures are tied to scheduled WBS entities, their accumulation across project time can be projected with a smoothness dependent upon the degree of decomposition. During implementation, we can, at any WBS level, define progress, attendance and productivity ratios *Actual Progress /Scheduled Progress*, *Actual Effort /Scheduled Effort* and the their quotient respectively, thereby providing a rational, output-oriented and value-based assessment of project health.

Keywords: value, projects, progress measures

1. General Context for Task Duration Estimation

As is well known, the determination of a project's duration involves the identification of its longest path, the critical path, comprising a series of critical tasks, each of which if delayed would delay the entire project. This property dictates that the duration of these tasks be estimated with particular care. It is the purpose of this paper to show how this might be done in difficult cases where natural output measures are not available.

Of course to identify the critical path and floats (its excess over other paths) initially requires that all task durations to be identified, even if only roughly, serving to direct attention to where the more careful analysis is required. In cases where paths may compete for criticality, so that floats are small, iterations involving successive refinement might be necessary until the critical path has been identified with sufficient confidence.

2. A General Framework for Task Duration Estimating

A general framework for duration estimating might recognize that the estimate is a function of three major components of the task. These are its magnitude, the capacity of the resources assigned to it (both human and material) and the external factors that affect it. The first of these provides a major difficulty for non-physical 'knowledge' or specialist type tasks such as analysis, design, research and software development where the notion of a measure is elusive. However, this can be overcome by recognizing that tasks, although conventionally regarded as the lowest level of the workbreakdown structure (WBS) into which the project is decomposed, nonetheless frequently comprise minor sub-tasks not warranting explicit representation in the WBS.

If we produce a prioritisation (Saaty, 1982) according to the perceived relative magnitudes of these sub-tasks, a set of numbers on a ratio scale emerges, the total of which can serve as a measure for the magnitude of the task itself.

Consider the following expression for the duration of a task within a project

$$Duration = \sum_{j}^{<>} \{\ Duration(j)\} + External\ Factors \qquad 1)$$

where

- j enumerates the sub-tasks into which each critical task is decomposed,
- $\{\sum_{j}^{<>}\ \}$ is a function that spans its operand in time, i.e. returns the working time between the earliest start and latest finish of the sub-tasks and
- *External Factors* reflect the total additional time across these sub-tasks due possibly to interruptions, lead-times and contingencies

Further we define

$$Duration(j) = \frac{Effort(j)}{HRA(j)} \qquad 2)$$

where

$$Effort(j) = \frac{Task\ Magnitude(j)}{PD(j) * MRE(j)} \qquad 3)$$

is the quantity of human input required for the sub-task, and where

HRA(j) is the average human resource attendance for sub-task j, measured in full time equivalents (FTEs),
PD(j) is the average productivity of the resources assigned to sub-task j and
MRE(j) is the average productivity enhancement made to PD(j) by a material resource and is set to unity if no such effect is present.

Equations (1) to (3) represent a formalization of a common expression used to combine task outputs measured in convenient units (e.g. m^2 if painting a room) and productivity (m^2 per unit hour worked) and possibly material resource enhancement (e.g. the effects of particular types or lengths of paint roller brushes on productivity) to produce effort (therefore measured in work hours) which is then converted to duration by examining the number of people available to participate (attendance).

3. Defining a Measure for Task Magnitude in Knowledge work projects

We have stated that output units for a task can be defined and quantified by means of a prioritisation process. This claim is founded upon the following observations:

- The magnitude of the task can be computed by first establishing a suitable AHP hierarchy (or ANP network) comprising criteria and sub-criteria perhaps involving size, scope, complexity, quality requirements and risk, and then using this to prioritise the sub-tasks.

- It is usually easier and safer to assess the effort required for smaller or simpler sub-tasks than for larger or more complex ones. They are easier to conceive, less heterogeneous and more accessible for analysis.

Therefore, if a task in the project is truly judged to be elemental and possesses a natural measure of output, the effort required to complete it should be calculated based upon its size and an average or selected productivity rate using equation (3). If it possesses no natural unit of output, an arbitrary value (perhaps unity) should be assigned and the effort required for it directly assessed using a reference resource skill level. This allows a productivity rate PD to be identified. In this way, values for effort, productivity and output become available for this task.

If a task is not elemental and warrants subdivision, an elemental 'anchor' sub-task should be identified with a corresponding quantity of magnitude, effort and productivity (denoted by as M_a, E_a and PD_a respectively) derived as suggested above. The possibly normalized relative magnitudes of the sub-tasks along with the corresponding efforts can be established by scaling according to the prioritization ratios. Thus if Pr_a is the normalized priority value for the anchor task, then its magnitude measure can be defined as

$$PD_a = M_a / E_a \qquad 4)$$

and the magnitudes of the sub-tasks j can be found by scaling, i.e.

$$M_j = \frac{Pr_j}{Pr_a} * M_a \qquad 5)$$

or

$M_j = \frac{Pr_j}{Pr_a} * PD_a * E_a$, where Pr_j are the prioritisation values for the sub-tasks, j= 1 to N

And if we denote Effort(j) as E_j we have

$$E_j = \frac{M_j}{PD_j * MRE_j} \qquad 6)$$

where the PD_js could reflect either different resources assigned to sub-task j, or a different skill level of the same person when attending to it.

Note that when comparative judgments are made directly against a single and therefore top-level criterion (i.e. a single node degenerate hierarchy), there is no need to normalize the results and it may be convenient to choose output units which are numerically equal to effort quantities, implying a productivity rate of 1. When a more detailed set of criteria is required, an AHP/ANP synthesis will yield a normalized result which will require equation 4 in order to determine the effective productivity rating. We shall see examples of both situations in section 3 below. However, it is important to retain a conceptual difference between output units and effort. The purpose of defining the former is to quantify magnitude independent of resource skills with productivity rates being modifiable later to reflect potential differences in these. In any event it is highly desirable to use output measures that are linearly related to effort.

We can now apply equations 1, 2 and 3 to provide an overall duration for the task. The spanning operation allows for overlaps in time. When these are absent it reduces to summation.

4. Example of Task Magnitude Definition

Consider the task of writing a scientific report, occurring within a larger research project. Suppose that the sub-tasks identified are given in the table below.

Table 1: High-level task decomposition

No	Sub-Tasks	Output Units	Effort
1	Literature Survey	1	8
2	Data Gathering	2	16
3	Data Review	1	8
4	Formatting	1	8
5	Writing	6	48
6	Proofing	3	24
7	Final Review	2	16
Total		16	128

If a single output unit M_a is assigned to an anchor task (say sub-task 1 – 'Introduction'), and an 'n-1' set of comparisons are made according to the criterion of likely magnitude, we might see a set of output units as shown in the third column of the table. This is similar to the use of 'story' points used in Agile software development contexts (Chandramouli, Dutt, 2012). There is no need to normalize these values as the scale is arbitrary. We would declare the tentative magnitude of the task to be 16 points subject to further refinement and revision. If we further assess that 8 work hours of effort is required to the anchor sub-task, the implied productivity rate is 0.125, in accordance with equation 3, with material resource effects being ignored (i.e. $MRE_a = 1$). We can now compute the efforts of the sub-tasks using equation 6 (with productivity kept constant in this case), with results shown in the final column of the table.

The 'Writing' sub-task represents the highest contribution to the effort and so warrants further decomposition. Suppose that the sizing prioritisation for this decomposition requires a more detailed set of criteria and sub-criteria, represented by the decision hierarchy shown in in Figure 1.

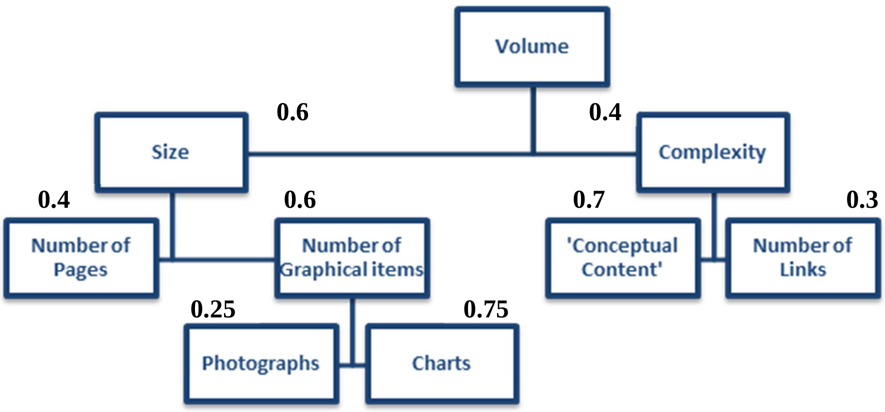

Figure 1: Hierarchy for Task Magnitude Prioritisation

Suppose further that the sub-tasks associated with this decomposition and their relative normalized magnitudes resulting from an AHP synthesis are shown in the table below.

Table 2: Report Decomposition

Sub-Task #	Report Sections	Output Units	Expected Effort (work-hours)
1	Introduction	0.099	4
2	Methodology	0.107	5.0
3	Results (Text)	0.065	3.0
4	Results (Graphics)	0.336	14.0
5	Discussion and Interpretation	0.302	13.0
6	Conclusion	0.092	4.0
			43

We will select 'Introduction' to act as the anchor and judge a typical resource person expected to perform this type of work to spend approximately four work-hours to complete this sub-task. The ratio of 4 work hours to the corresponding priority value M_a = 0.099 provides a productivity rate of 40.55 in accordance with equation 3, with MRE set to 1. Again using equation 6 and a constant productivity to generate effort values for the others, we arrive at the quantities shown in the last column of Table 2. The total of 43 work hours constitutes a refinement on the 48 work hours first identified in the in the high level analysis shown in Table 1, providing an overall effort of 123 work hours for the task.

5. Converting Sub-Task Efforts to Total Task Duration

Each of the effort quantities calculated in Table 1 can be converted to durations by means of equation (2) above. This requires the identification of the number (or proportion) of human resources available. These durations can be considered 'effort driven'. However, some aspects of them may involve 'time-driven' effects such as interruptions, lead-times, waiting for approval and contingencies.
These need to be identified and combined to form a single 'External Factors' quantity expressed in terms of duration and eventually added to the total in equation 1.

In separating the operations in equations 2 and 3, there is a danger that the diminishing returns effect can be lost. Since productivity can be a function of attendance, growing with team size at small numbers due to synergetic effects but falling way due to inefficiencies at larger ones, these factors should be incorporated when present.

Finally, the total task duration can be found by spanning the calculated sub-task durations, i.e. allowing for overlaps or lead-time gaps. In the general case, some sub-tasks will sit on a 'critical path' within the overall task, giving rise to the notion of 'critical' priority values emanating from the AHP calculations. It is the critical values which determine the overall duration of the task and which therefore need most accuracy and possible refinement via its own hierarchy, much as was done with the 'Writing'. This is the same argument made at the task level in section 1.

6. Combining incommensurate sub-tasks

In cases where task components are incommensurate in the sense of not yielding to comparisons under the same set of AHP criteria, these should be grouped in sub-sets whose elements do qualify for comparison. Each such sub-set can be dealt with as described above, and the various efforts so derived then used to determine corresponding durations which will be combined by a spanning operation using equation (1).

7. Defining Value for Progress Reporting

The output units defined in the sections above not only make possible a rational estimate of the total duration and effort of a task, but can also be used as a measure of progress during the implementation stage of the project management process. For example, if completed sub-tasks correspond to K output points out of a total of K, then the proportion

$$P = \frac{K}{N} \qquad 7)$$

provides an output oriented quantification of the completeness status of the task. This is a significant advance because project managers have historically found it challenging to define progress in knowledge tasks. This in turn has led to difficulties in computing a meaningful ratio analyses for factors like productivity and schedule. For example, Earned Value Methods (PMI 2008) allows for many definitions of the contributed value of partially completed tasks and then rolls these up to higher levels of the WBS hierarchy. This is achieved by means of a weighted average, where the weights are provided by the ratios of the tasks' planned costs to that of their common parent, a process repeated all the way up the hierarchy. This is in accord with the so-called 'implicit' or 'labour' theory of value.

By using AHP to establish relative value, we have the tools necessary to employ a 'subjective' theory of value (Taylor, 1980), whereby the value identified for each WBS entity is defined by the team to meet its desiderata. Specifically, entities at the highest level of the WBS can be compared pairwise in relation to a decision hierarchy reflecting and combining a variety of suitably weighted criteria and sub-criteria. Suppose there are n entities at the top level of the WBS and that these are prioritized in terms of the value they contribute to the overall project deliverable, yielding a normalized prioritization M_i. If progress measures P_i for each of these entities were expressed as a proportion and derived from lower WBS levels (see below), then we could define the project progress P toward final delivery as a weighted average of these measures where the weights are provided by the prioritization. Thus

$$P = \sum_{i=1}^{n} P_i \, V_i$$

In turn, the P_i values are found similarly by performing an AHP prioritization on the 'children' of each top-level entity i according to the value their deliverables contribute to that entity. If this value is denoted M_{ij}, $j = 1$ *to* r_i then we would have for i = 1 to n

8)

$$P_i = \sum_{j=1}^{r_i} P_{ij} M_{ij}$$

where P_{ij} represents the reported progress of the j^{th} child of entity i, j = 1 to r_i, i = 1 to n. This process can be continued to the bottom level of the WBS hierarchy where equation 7 would apply.

Having established a rational basis for achieved progress, we can now define ratios (Sharp, 2012) similar to those used in EVM which provide indications of the health of entities at any level of the WBS. These are given for the second level in table 3 below.

Table 3. Diagnostic Ratios for Project Health Across Several Dimensions

Factor Ratio	Dimension	Definition	Abbreviation
P_{ij}	Progress	$Actual\ Progress$	
DF_{ij}	Duration Factor	$\frac{Actual\ Progress}{Scheduled\ Progress}$	AP/SP
AF_{ij}	Attendance Factor	$\frac{Actual\ Effort}{Scheduled\ Effort}$	AE/SE
EF_{ij}	Efficiency Factor	DF_{ij} / AF_{ij}	(AP/AE)/(SP/SE)
PF_{ij}	Punctuality Factor	$\frac{Scheduled\ Progress}{Planned\ Progress}$	SP/PP
SF_{ij}	Schedule Factor	$DF_{ij} * PF_{ij}$	AP/PP

where;

Actual Progress	denotes progress recorded to date from the Actual Start
Scheduled Progress	denotes progress expected[1] to have been recorded to date from the Actual Start
Planned Progress	denotes progress expected to have been recorded to date from the Planned Start
Actual Effort	denotes work hours recorded to date from the Actual Start
Scheduled Effort	denotes work hours expected to have been recorded to date from the Actual Start

These definitions allow for non-linear behaviour of progress with respect to time. Performance matching expectations relative to the original baseline show values of unity in the five ratios defined above while those exceeding (or lagging) them show values greater than (or less than) unity.

For higher levels of the WBS, these ratios are found using the same formulae shown in the third column of Table 3, with quantities aggregated from children values – progress measures by weighted averages (like that shown in equation 8) and effort by arithmetic sums.

A numerical example of this is shown in the tables below.

[1] 'Expected' implies according to the original plan

Tasks	Current Date	Weight	Actual Start	Actual Finish	Planned Start	Planned Finish	Planned Duration	Planned Effort
Parent	5		2	0	0	10	10	20
Task 1	5	0.40	2		0	10	10	10
Task 2	5	0.60	3		0	10	10	10

Tasks	APAS	PPAS	AEAS	PEAS	PPPS	DF	AF	EF	PF	SF
Parent	18%	24%	3	5	50%	0.75	0.60	1.25	0.48	0.36
Task 1	15%	30%	2	3	50%	0.50	0.67	0.75	0.60	0.30
Task 2	20%	20%	1	2	50%	1.00	0.50	2.00	0.40	0.40

8. Conclusions

Estimating is central to good project planning yet finds particular difficulty with the sizing of intellectually oriented work. The process of decomposition, prioritisation (according to demand for resources) and linkage to real quantities such as effort (through the assessment of requirements of just one component, perhaps the smallest one, following by proportional scaling for the others), has the potential for finally bringing some rationality to a difficult area. Similarly, we have seen that prioritisation techniques provide the means for quantifying progress toward abstract project goals such as strategic, scientific or political value.

AHP/ANP can make valuable contributions to project management, a profession that has traditionally dealt with tangible and measurable entities such as those found in construction and engineering but one that has struggled with the challenges presented by those arising in knowledge work such as scientific research, software development and design.

REFERENCES

Subramanian Chandramouli; Saikat Dutt (2012). PMI Agile Certified Practitioner—Excel with Ease. *Pearson Education Media*

PMI (2008). A Guide to the Project Management Body of Knowledge (PMBOK® Guide). *Project Management Institute*

Thomas L Saaty, (1982). Decision Making for Leaders: The Analytical Hierarchy Process for Decisions in a Complex World. *Lifetime Learning Pub.*

S. Sharp (2012). Advanced Project Management. (Courseware), *Numerix Pty. Ltd*

Thomas C. Taylor, (1980). An Introduction to Austrian Economics. *Cato Institute*