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# A MODEL TO DEAL WITH UNCERTAINTY IN ELLSBERG'S PARADOX:
The Analytic Hierarchy Process with Feedback

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**Abstract**: We show that the supermatrix approach is a suitable method to deal with ambiguous situations such as the one described by Ellsberg's Paradox. Experimental results support this finding.

## Introduction

Theories of decision making that represent uncertainty with probabilities are based on Ramsey-Savage's proposal (Ramsey, 1931; Savage, 1954). Savage gave a set of postulates that any binary relationhip such as "not less probable than" should satisfy for it to be considered a qualitative probability relationship:

- **P1**: The gambles or actions must be completely ordered,
- **P2**: The choice between actions must be unaffected by the payoff values corresponding to events for which both actions have the same payoff,
- **P3**: All dominated actions must be rejected, and
- **P4**: The probabilities of the events and the resulting pay-offs must be independent.

Thus, the preference of a decision maker for an event because of its probability of occurrence should not depend on the payoff associated with its consequences.The problem with these axioms is that there are several types of uncertainty some of which cannot be represented solely with probabilities. Einhorn and Hogarth (1986) distinguish between ignorance, risk and ambiguity. To clarify these three concepts consider three urns, U1, U2 and U3, which contain 100 red (R) and black (B) balls. U1 has an unknown composition of balls, U1(?R,?B); U2 contains an even split of red and black balls; U2(50R,50B); and U3 contains balls all of which are the same color, but the color is unknown, U3(100R or 100B, ?color). The first urn is an example of ignorance because the distribution of the outcomes is unknown. The second urn is an instance of a situation involving risk because the distribution of outcomes is known, and finally, the third urn is the case of ambiguity because even if the distribution of outcomes is known, the color is unknown, that is, the consequences rather than their probabilities are unknown.

Another concept used when dealing with uncertainty is imprecision, a synonym of inexact, inaccurate or vague. Some authors have proposed vagueness (e.g., Heath & Tversky, 1991; Wallsten, 1990) when referring to ignorance. Vagueness is a concept that oscilates the entire spectrum from risk to ignorance. An example of an imprecise or vague choice would be an urn for which it is known that it contains 100 balls, but the number of balls of each color is not known. It could be some number between 30 and 40 red balls and hence between 60 and 70 black balls. According to Ellsberg, these types of uncertainties cannot be modelled with any theory based on Savage's axioms. In particular, his paradox deals with choices between risk and ignorance or ambiguity. Ellsberg (1961) questionned Savage's axioms, but he did not go as far as saying that they were not valid. He wrote (Ellsberg, 1961, p.645):

> "The propounders of these axioms tend to be hopeful that the rules will be commonly satisfied, at least roughly and most of them, because they regard these postulates as normative maxims, widely-acceptable principles of rational behavior. In other words, people should tend to behave in the postulated fashion, because that is the way they would want to behave. ... A side effect of the axiomatic approach is that it supplies, at last a useful operational meaning to the proposition that people do not always assign, or act "as though" they assigned , probabilities to certain events. The meaning would be that with respect to certain events they did not obey, nor did they wish to obey -- even on reflection -- Savage's postulates or equivalent rules. One could emphasize here either that the postulates failed to be acceptable in those circumstances as normative rules, or that they fail to predict reflective choices. ... I tend to be more interested in the latter aspect ..."

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Ellsberg's work shows (1961, p.646) that:

> "... there would be simply no way to infer meaningful probabilities for those events from their choices, and theories which purported to describe uncertainty in terms of probabilities would be quite inapplicable in that area (unless quite different operations for measuring probability were devised)."

In this paper we present a method to deal with uncertainty which considers Ellsberg's objections. It uses pairwise comparisons to express the preferences of decision makers. The comparisons are based on the consequences of the actions and on the probabilities of those consequences. The subjects of our experiments compared urns for their desirability according to a given color, and also compared the desirability of the colors according to a given urn. The result is a matrix of relative preferences which is then used to abstract the overall weight of the urns and the colors of an individual. We then compared these preferences with the preferences of the same individual without comparing urns or colors separately. Our hypothesis is that this way of measuring preferences captures people's attitude toward uncertainty.

## Ellsberg's Paradox

Ellsberg (1961) stated the following paradox: A decision maker is given a choice between two urns (U1 and U2) containing red and black balls. U1 contains 100 balls in unknown proportions while U2 contains 50 red and 50 black balls. Consider the following gamble: if you bet on a color and the color is drawn from the urn selected then you get a $100 payoff; otherwise, the payoff is $0. For U1, empirical evidence supports the idea that most people are indifferent between betting on a red or on a black ball, i.e., P{Red|U1} = P{Black|U1}. Thus, the subjective probabilities of obtaining red or black are equal. For U2, most people are also indifferent between the colors, i.e., P{Red|U2} = P{Black|U2}. What people are not indifferent, when asked to bet on a color, is the urn from which they would draw the ball. Most people select U2. Hence, the subjective probability of drawing a ball of a given color, e.g., red, from U2 must be greater than the equivalent probability in U1. That is,

$$P\{Red|U2\}>P\{Red|U1\} = 0.50 \tag{1}$$

and

$$P\{Red|U2\} = 0.5 > P\{Red|U1\} \tag{2}$$

which yields P{Red|U2}+ P{Black|U2} > 1 (superadditivity), or

$$P\{Black|U2\} = 0.5 > P\{Black|U1\} \tag{3}$$

and

$$P\{Black|U2\}>P\{Black|U1\} = 0.50 \tag{4}$$

which yields P{Red|U1} + P{Black|U1} < 1 (subadditivity). These results lead to a contradiction because (1) and (2) yield complementary probabilities that add to more than unity, while (3) and (4) yield complementary probabilities that add to less than unity. The main effect of these results is that probabilities cannot be used to measure the uncertainty of choices. In addition, Ellsberg also noted that people are more likely to draw balls from the urn whose composition is known (U2) than from the urn with an unknown composition (U1), but if the probability of winning is small, he conjectured that people would prefer to draw balls from the umbiguous urn.

A number of authors have developed models to incorporate ambiguity in models od choice. Fishburn (1991, p.3) gives a brief but rich account of the development of the concept of ambiguity and some of the models proposed to deal with it.

In this paper we use a generalization of the Analytic Hierarchy Process (Saaty, 1986, 1990) to systems with feedback known as the Supermatrix Approach (Saaty, 1990) that explains the behavior of individuals in ambiguous choices. The use of the supermatrix precludes the occurrence of subadditivity or superadditivity because preferences are measured in relative terms.

## The Supermatrix

In this model the elements of a system are represented as nodes of a network. Two nodes are connected by an arc if there is interaction between them. Saaty (1990) has shown that Hierarchic Composition is a particular case of this approach. The supermatrix is a natural extension of the concept of dominance on which the Analytic Hierarchy Process is built (see, for example, Saaty, 1981, and Saaty and Takizawa, 1986). It allows for dependencies between nodes and within the elements of nodes. For example, the matrix representation of a hierarchy with three levels is given by:

$$W = \begin{array}{r} \\ \textit{Goal } (G) \\ \textit{Criteria } (C) \\ \textit{Alternatives } (A) \end{array} \begin{array}{c} \begin{array}{ccc} G & C & A \end{array} \\ \begin{pmatrix} 0 & 0 & 0 \\ W_{21} & 0 & 0 \\ 0 & W_{32} & I \end{pmatrix} \end{array}$$

where $W_{21}$ and $W_{32}$ are matrices. $W_{21}$ represents the impact of the Goal on the criteria, and $W_{32}$ represents the impact of the criteria on the alternatives. If the criteria are dependent among themselves, then the (2,2) entry of W given by $W_{22}$ would be non-zero and we would have:

$$W = \begin{pmatrix} 0 & 0 & 0 \\ W_{21} & W_{22} & 0 \\ 0 & W_{32} & I \end{pmatrix}$$

This system can be represented by the network given in Figure 1.

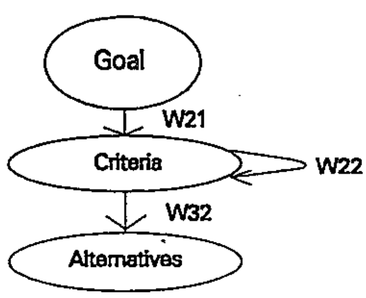


**Figure 1**

From W being a column stochastic matrix and from graph theory, it is known that the synthesis of all the interactions among the elements of this system is given by:

$$W^{\infty} = \lim_{k \to \infty} W^{k}$$

If the matrix is primitive irreducible then it suffices with raising the matrix to powers because the limit is unique and there exists a column vector $w^{\infty}$ for which:

$$W^{\infty} = w^{\infty} e^{T}$$

where $e^T = (1, ..., 1)$. However, if the matrix is reducible and the multiplicity of 1 is 1, then $W^{\infty}$ is given by:

$$W^{\infty} = (I - W)^{-1} \Psi(1) / \Psi'(1)$$

where $\Psi(\lambda)$ is the minimum polynomial of W and $\Psi'(\lambda)$ is its first derivative with respect to $\lambda$. For the example given in Figure 1 we have:

$$W^{\infty} = \lim_{k \to \infty} \begin{pmatrix} 0 & 0 & 0 \\ W_{22}^{k} W_{21} & W_{22}^{k} & 0 \\ W_{32}\left(\sum_{h=0}^{k-2} W_{22}^{k}\right) W_{21} & W_{32}\left(\sum_{h=0}^{k-1} W_{22}^{k}\right) & I \end{pmatrix}$$

From $|W_{22}| < 1$, $(W_{22})^k$ tends to zero as k tends to infinity, and the limiting contributions are given by:

$$W^{\infty} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ W_{32}(I - W_{22})^{-1} W_{21} & W_{32}(I - W_{22})^{-1} & I \end{pmatrix}$$

Thus, the contribution of the alternatives to the goal is given by the (3,1) entry of $W^{\infty}$.

**The Supermatrix Model of Ellsberg Paradox**

We now construct a supermatrix model consisting of two nodes, the urns and the colors that interact with each other (see Figure 3).

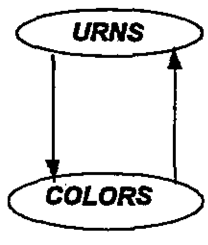


**Figure 2**

The urns can be compared according to their desirability depending on what color is selected, and in turn the desirability of the colors depend on the urn selected. Thus, we compare the colors with respect to the urns chosen and the urn with respect to the colors. The paired comparisons depend on the payoffs associated with the consequences, the probability distribution of the consequences, the competence of the decision maker (Heath & Tversky, 1991), the value added of the knowledge acquired during the decision making process, and many other subjective factors. Let us define the following matrix $\Omega$:

$$\Omega = \begin{array}{c} \\ R \\ B \\ U1 \\ U2 \end{array} \begin{array}{c} R\ B\ U1\ U2 \\ \left[ \begin{array}{cc} 0 & W_{12} \\ W_{21} & 0 \end{array} \right] \end{array}$$

where $W_{12}$ and $W_{21}$ are the relative priorities of the colors given the urns, and the relative priorities of the urns given the colors, respectively. These matrices are given by:

$$W_{ij} = \begin{bmatrix} w_{11}^{(ij)} & w_{12}^{(ij)} \\ w_{21}^{(ij)} & w_{22}^{(ij)} \end{bmatrix} \quad i,j=1,2$$

The columns of $W_{ij}$ are the relative priorities of the elements on the left of the matrix with respect to the elements on the top. Thus, the relative preferences of a decision maker for the two urns is given by the first column of $W_{21}$, if he decided to bet on red, and by the second column of $W_{21}$, if he decided to bet on black. These columns are the principal right eigenvectors of the pairwise reciprocal matrices given by:

$$\begin{array}{ccc} Red & U1 & U2 \\ U1 & 1 & R12 \\ U2 & R12^{-1} & 1 \end{array} \qquad \begin{array}{ccc} Black & U1 & U2 \\ U1 & 1 & B12 \\ U2 & B12^{-1} & 1 \end{array}$$

Let $u_k$, k=1,2 be the relative preference of the colors in Uk, k=1,2, respectively; and let $v_i$, i=1,2 be the relative preference of the urns with respect to betting on red and black, respectively, given by:

$$v_1 = \frac{R12}{1+R12} \quad and \quad v_2 = \frac{B12}{1+B12}$$

Thus, we have:

$$u_k = w_{1k}^{(12)}, \quad k=1,2.$$

$$v_k = w_{1k}^{(21)}, \quad k=1,2.$$

and,

$$\Omega = \begin{array}{c} \\ R \\ B \\ U1 \\ U2 \end{array} \begin{array}{c} \begin{array}{cccc} R & B & U1 & U2 \end{array} \\ \begin{bmatrix} 0 & 0 & u_1 & u_2 \\ 0 & 0 & 1-u_1 & 1-u_2 \\ v_1 & v_2 & 0 & 0 \\ 1-v_1 & 1-v_2 & 0 & 0 \end{bmatrix} \end{array}$$

Let $w_k$, k=1,2 be the limiting absolute priorities of the urns Uk, k=1,2, respectively, and let $t_k$, k=1,2 be the limiting absolute priorities of the colors red and black, respectively.

**Theorem 1**: $w_1 > w_2$ if and only if $2v_2 + (u_1+u_2)(v_1-v_2) > 1$

and

$t_1 > t_2$ if and only if $2u_2 + (v_1+v_2)(u_1-u_2) > 1$.

Proof: From $\Omega$ cyclic, with period 2, a reducible stochastic matrix, the priorities associated with the urns and the colors as they interact are given by:

$$\Omega^{\infty} = \frac{1}{2}(I + \Omega)(\Omega^2)^{\infty}$$

From

$$(\Omega^2)^{\infty} = \begin{bmatrix} (W_{12}W_{21})^{\infty} & 0 \\ 0 & (W_{21}W_{12})^{\infty} \end{bmatrix}$$

we have

$$\Omega^{\infty} = \begin{bmatrix} (W_{12}W_{21})^{\infty} & W_{12}(W_{21}W_{12})^{\infty} \\ W_{21}(W_{12}W_{21})^{\infty} & (W_{21}W_{12})^{\infty} \end{bmatrix}$$

Because $(W_{21}W_{12})$ is an irreducible column stochastic matrix, the limit of $(W_{21}W_{12})^k$ as k tends to infinity exists, and it is given by:

$$\lim_{k\to\infty}(W_{21}W_{12})^k = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}(1\ \ 1)$$

where the vector $(w_1\ w_2)^T$ is the principal right eigenvector of the matrix $(W_{21}W_{12})$. Hence, we have:

$$w_1 = \frac{v_2+u_2(v_1-v_2)}{1-(u_1-u_2)(v_1-v_2)}$$
$$w_2 = 1 - w_1 \qquad (5)$$

and the result follows from $1-(u_1-u_2)(v_1-v_2) > 0$.

Similarly, the priorities of the colors are given by the principal right eigenvector of the matrix $(W_{12}W_{21})$:

$$t_1 = \frac{u_2+v_2(u_1-u_2)}{1-(v_1-v_2)(u_1-u_2)}$$
$$t_2 = 1 - t_1 \qquad (6)$$

and as with the urns the result follows from $1-(u_1-u_2)(v_1-v_2) > 0$. ∎

## Experimental Design and Results

To test the model developed in the previous section we performed an experiment with a group of 205 Management students taking a course in Operations Research at the Faculty of Economics and Management, University of Zaragoza, Spain. They were told that the purpose of the experiment was to study the choices of individuals under nine scenarios. Each scenario is characterized by the payoff resulting from the choice made and the proportion of red/black balls in one of the urns. These nine scenarios are obtained by considering 3 different types of payoffs and 3 different urn compositions. We varied the payoffs from winning $100 and losing $0, to winning $0 and losing $100, to winning $100 and losing $100. Likewise, we considered three compositions of the urn U2: a 50/50 proportion, a 25/75 proportion, and a 1/99 proportion of red and black balls. In the first experiment we asked the subjects to choose in a given situation which urn and which color they would prefer and the intensity of their preference. This is basically Ellsberg's experiment.

In the second experiment the subjects of the first experiment answered questions pertaining to the supermatrix model we built. Given an urn, the subjects had to select the color on which they would prefer to bet and the intensity of their preference. Also, given a color they had to select an urn and express the intensity of their preferences.

Both experiments were performed sequentially. Every subject answered both questionnaires in the same session and the results were paired. Table 1 gives the results for the first experiment, and Tables 2 gives the results for the second experiment.

**Table 1**

| | G=100 L=0 | G=0 L=100 | G=100 L=100 |
|---|---|---|---|
| | U1 I U2 | U1 I U2 | U1 I U2 |
| **50 - 50** | 57 34 114 | 53 46 106 | 58 46 101 |
| **25 - 75** | 20 0 185 | 19 1 185 | 15 3 187 |
| **1 - 99** | 10 0 195 | 13 1 191 | 10 1 194 |
| | W1 W2 W1/W2 | W1 W2 W1/W2 | W1 W2 W1/W2 |
| **50 - 50** | 0.405 0.595 0.682 | 0.406 0.594 0.685 | 0.426 0.574 0.742 |
| **25 - 75** | 0.202 0.798 0.253 | 0.229 0.771 0.297 | 0.207 0.793 0.261 |
| **1 - 99** | 0.137 0.863 0.158 | 0.149 0.851 0.175 | 0.143 0.857 0.167 |

Table 1 gives the preferences of the subjects in the nine scenarios presented to them. We give the proportions (counts) of individuals who prefer urn U1 to urn U2, who are indifferent between them, and who prefer urn U2 to urn U1. The lower portion of the table gives the average of the priorities the individuals assigned to the urns. This average priority was obtained by taking the geometric mean of the judgments of all the respondents. We note that the average priority obtained in this manner was almost the same as the arithmetic mean of the priorities obtained for each of the individuals separately.

Table 2 gives the preferences of the individuals for the colors under the assumption that the ball was going to be drawn from the ambiguous urn U1, and that the urn U2 had a specific composition.

In the first experiment, a contingency test performed on the data given in Table 1 imply that the payoffs appear to have no influence on the choice of the urn, although this result could be a consequence of our choice of monetary values. On the other hand, the urn composition (red/black proportion in U2) does influence an individual's choice.

In the second experiment, we find that when U2 has a 50/50 composition, people choose U2 over U1, and that when the probability of winning decreases the individuals choose the ambiguous urn more (see Tables 4 and 5). These results coincide with those obtained by Becker and Brownson (1964), Curley and Yates (1985), Gärdenfors and Sahlin (1982, 1983) and Yates and Zukowsky (1976) for gains, and with those obtained by Hogarth and Kunruther (1985) and Einhorn and Hogarth (1986) for loses.

In general, one would expect, when an individual is required to bet on a color, given that the ambiguous urn U1 has been chosen, that both colors are equally preferred. However, we find (see Table 2) that although the majority remains indifferent, the number of respondents that selected a color when the number of balls of the other color was increased, almost doubled. That is, there is a significant difference in the choice of color in U1 when the composition of U2 varies. Thus, as the composition of U2 changes in favor of one color or another, the choices in U1 vary in the opposite direction. For example, if drawing a red ball represents a win, and the number of red balls in U2 is decreased, people's preference of red over black increase rather than remaining indifferent. This is a consequence of the mental process of the individual when he/she takes into account the scarcity or abundance of one of the colors. The priorities an individual assigns to the urns and the colors reflect this phenomenon. Heath and Tversky (1991) point out that this phenomenon is not cognitive but rather motivational.

Table 3 combines both experiments to test for differences and/or similarities of results. To show that the results of the two experiments support the hypothesis that the supermatrix models human behavior in ambiguous situations, we tested if the distribution of choices among the individuals is the same in the two experiments. We did this by testing if the proportion of individuals whose choices remain unchanged from one experiment to the other is significant. We simultaneously look at an individual's choice in the two experiments under the three different urn compositions, and the three types of payoff structures. The first block of this table assumes 50/50 composition of U2 and a gain of $100 if the color selected is drawn, but no loss. The first row of this block gives the number of individuals that selected U1 in the first experiment and then selected U1, were indifferent or selected U2 in the second experiment. The choices of the individuals in the second experiment were determined using the priorities of the urns given by (6) from the supermatrix. We find that for blocks (2,1) through (3,3), the supermatrix choices and the first experiment results coincide. However, for the 50/50 composition of U2, we observe that the payoffs do not appear to influence an individual's choice. Although the results are not conclusive, indifference appears to increase in the second experiment. Some individuals who selected U1 or U2 during the first experiment had a tendency to become indifferent during the second experiment.

A chi-squared test of independence of the rows and columns to Table 2 reveals that for the 50-50 case, the choices in the second experiment are dependent on the choices in the first experiment. That is, the selections made using the supermatrix are not independent of the selections made without it, and hence, the dependence is not attributable to chance. For the other two cases, even if the general rule that the expected frequencies must be at least 5 is violated, and recognizing the limited accuracy of the chi-squared approximations, we still reject the null hypothesis that the choices in the 1st and 2nd experiments are independent. This offers strong evidence in support of our claim that the supermatrix helps to measure the preferences of individuals in ambiguous situations.

## Table 2

| | G=100 L=0 | | | G=0 L=100 | | | G=100 L=100 | | |
|---|---|---|---|---|---|---|---|---|---|
| U1 | R | I | B | R | I | B | R | I | B |
| "50-50" | 23 | 166 | 16 | 14 | 172 | 19 | 21 | 171 | 13 |
| "25-75" | 40 | 134 | 31 | 35 | 146 | 24 | 30 | 153 | 22 |
| "1-99" | 38 | 142 | 25 | 37 | 144 | 24 | 36 | 148 | 21 |
| U2 | R | I | B | R | I | B | R | I | B |
| "50-50" | 18 | 172 | 15 | 14 | 177 | 14 | 15 | 172 | 18 |
| "25-75" | 3 | 0 | 202 | 5 | 3 | 197 | 5 | 1 | 199 |
| "1-99" | 5 | 0 | 200 | 3 | 0 | 202 | 2 | 1 | 202 |
| R | U1 | I | U2 | U1 | I | U2 | U1 | I | U2 |
| "50-50" | 51 | 61 | 93 | 53 | 60 | 92 | 49 | 69 | 87 |
| "25-75" | 175 | 5 | 25 | 164 | 10 | 31 | 168 | 8 | 29 |
| "1-99" | 187 | 3 | 15 | 180 | 7 | 18 | 191 | 5 | 9 |
| B | U | 1 | U2 | U1 | I | U2 | U1 | I | U2 |
| "50-50" | 49 | 60 | 96 | 46 | 61 | 98 | 40 | 73 | 92 |
| "25-75" | 5 | 0 | 200 | 4 | 4 | 197 | 6 | 0 | 199 |
| "1-99" | 6 | 0 | 200 | 4 | 1 | 200 | 4 | 1 | 200 |

**Table 3**: Comparison of Urn Selection in Experiment 1 and Experiment 2

| | | G=100, L=0 | | | | G=0, L=100 | | | | G=100, L=100 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | U1 | I | U2 | | U1 | I | U2 | | U1 | I | U2 | |
| 50-50 | U1 | 24 | 17 | 16 | 57 | 20 | 17 | 16 | 53 | 21 | 16 | 21 | 58 |
| | I | 6 | 17 | 11 | 34 | 8 | 21 | 17 | 46 | 8 | 28 | 10 | 46 |
| | U2 | 20 | 26 | 68 | 114 | 20 | 24 | 62 | 106 | 14 | 28 | 59 | 101 |
| | | 50 | 60 | 95 | | 48 | 62 | 95 | | 43 | 72 | 90 | |
| 25-75 | U1 | 2 | 0 | 18 | 20 | 3 | 0 | 16 | 19 | 3 | 0 | 12 | 15 |
| | I | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 2 | 3 |
| | U2 | 7 | 0 | 178 | 185 | 4 | 4 | 177 | 185 | 2 | 1 | 184 | 187 |
| | | 9 | 0 | 196 | | 7 | 4 | 194 | | 6 | 1 | 198 | |
| 1-99 | U1 | 2 | 0 | 8 | 10 | 1 | 0 | 12 | 13 | 1 | 1 | 8 | 10 |
| | I | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| | U2 | 5 | 3 | 187 | 195 | 3 | 3 | 185 | 191 | 3 | 2 | 189 | 194 |
| | | 7 | 3 | 195 | | 4 | 3 | 198 | | 4 | 3 | 198 | |

## Conclusions

Hogarth and Kunruther (1992) point out that

"It is difficult to distinguish between "distortion" in probability due to the ambiguity and the genuine differences in beliefs about underlying probabilities."

Using the supermatrix we have tested that when individuals are faced with ambiguous situations as in Ellsberg's paradox, the ambiguity of choice is eliminated. The next step in this research is to show that the supermatrix approach can be generalized to other ambigous situations.

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## EXPERIMENT I

Consider two urns U1 and U2 which contain 100 red (R) and black (B) balls. Urn U1 contains an unknown number of each color, and urn U2 contains X red balls and Y black balls, and X+Y=100.

You now select a color and a ball is drawn at random from the urn of your choice. If the colors match you win an amount W and if they do not match you lose an amount L. The idea is to select a color and an urn from which to draw the ball. In addition, we would also like to know how strongly you prefer your selections. To express this intensity we would like you to use the following intensities:

1 - Equal; 3 - Moderate; 5 - Strong; 7 - Very Strong; 9 - Extreme
2, 4, 6, 8 are used to express intensities between the categories given above.

You are going to make selections in nine situations in which the payoffs and the composition of the urns will be different. For example, a situation could be the following:

| SITUATION 0 | Which one? | How much? |
|---|---|---|
| W=$100; L=$0<br>X=50R; Y=50B | U1 U2<br>R B | 5<br>3 |

Circle the urn you would prefer under "Which one?" for example U2, and in the box under "How much?" select an intensity from the scale given above. For example, if your preference is that U2 is strongly more preferred than U1 then write a 5 under "How much?". Do the same with the colors. Select a color and write the intensity of your preference, for example, if moderate write a 3. Do this for the situations given in the next page.



## EXPERIMENT I (cont.)

| SITUATION 1 | Which one? | How much? |
|---|---|---|
| W=$100; L=$0<br>X=50R; Y=50B | U1 U2<br>R B | |

| SITUATION 2 | Which one? | How much? |
|---|---|---|
| W=$100; L=$0<br>X=25R; Y=75B | U1 U2<br>R B | |

| SITUATION 3 | Which one? | How much? |
|---|---|---|
| W=$100; L=$0<br>X=1R; Y=99B | U1 U2<br>R B | |

| SITUATION 4 | Which one? | How much? |
|---|---|---|
| W=$0; L=$100<br>X=50R; Y=50B | U1 U2<br>R B | |

| SITUATION 5 | Which one? | How much? |
|---|---|---|
| W=$0; L=$100<br>X=25R; Y=75B | U1 U2<br>R B | |

| SITUATION 6 | Which one? | How much? |
|---|---|---|
| W=$0; L=$100<br>X=1R; Y=99B | U1 U2<br>R B | |

| SITUATION 7 | Which one? | How much? |
|---|---|---|
| W=$100;L=$-100<br>X=50R; Y=50B | U1 U2<br>R B | |

| SITUATION 8 | Which one? | How much? |
|---|---|---|
| W=$100;L=$-100<br>X=25R; Y=75B | U1 U2<br>R B | |

| SITUATION 9 | Which one? | How much? |
|---|---|---|
| W=$100;L=$-100<br>X=1R; Y=99B | U1 U2<br>R B | |

## EXPERIMENT 2

Consider two urns U1 and U2 which contain 100 red (R) and black (B) balls. Urn U1 contains an unknown number of each color, and urn U2 contains X red balls and Y black balls, and X+Y=100.

You now select a color and a ball is drawn at random from the urn of your choice. If the colors match you win an amount W and if they do not match you lose : an amount L. The idea is to select A COLOR and AN URN from which to draw the ball. In addition, we would also like to know how strongly you prefer your selections. To express this intensity we would like you to use the following intensities:

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1 - Equal; 3 - Moderate; 5 - Strong; 7 - Very Strong; 9 - Extreme
2, 4, 6, 8 are used to express intensities between the categories given above.

You are going to make selections in nine situations in which the payoffs and the composition of the urns will be different. For example, a situation could be the following:

| SITUATION 0 | Given | Which one? | How much? |
|---|---|---|---|
| W=$100; L=$0 | U1 | R B | 5 |
| X=50R; Y=50B | U2 | R B | 3 |
| | R | U1 U2 | 2 |
| | B | U1 U2 | 7 |

There are two sets of comparisons in this table. First, given an urn, for example U1, decide which color you prefer by circling your choice under "Which one?" and then decide what the intensity of this preference by selecting a value from the scale given above, for example, if you strongly prefer Red to Blue, then write a 5 under "How much." Similarly, Given a color, for example Red, if you had to bet on Red which urn would you prefer, circle one of them under "Which one?" and write under "How much" the intensity of this preference. Do this for every situation given below.

### EXPERIMENT 2 (cont.)

| SITUATION 1 | Given | Which one? | How much? |
|---|---|---|---|
| W=$100;L=$0 | U1 | R B | |
| X=50R;Y=50B | U2 | R B | |
| | R | U1 U2 | |
| | B | U1 U2 | |

| SITUATION 2 | Given | Which one? | How much? |
|---|---|---|---|
| W=$100;L=$0 | U1 | R B | |
| X=25R;Y=75B | U2 | R B | |
| | R | U1 U2 | |
| | B | U1 U2 | |

| SITUATION 3 | Given | Which one? | How much? |
|---|---|---|---|
| W=$100;L=$0 | U1 | R B | |
| X= 1R;Y=99B | U2 | R B | |
| | R | U1 U2 | |
| | B | U1 U2 | |

| SITUATION 4 | Given | Which one? | How much? |
|---|---|---|---|
| W=$0;L=-100 | U1 | R B | |
| X=50R;Y=50B | U2 | R B | |
| | R | U1 U2 | |
| | B | U1 U2 | |

| SITUATION 5 | Given | Which one? | How much? |
|---|---|---|---|
| W=$0;L=-100 | U1 | R B | |
| X=25R;Y=75B | U2 | R B | |
| | R | U1 U2 | |
| | B | U1 U2 | |

| SITUATION 6 | Given | Which one? | How much? |
|---|---|---|---|
| W=$0;L=-100 | U1 | R B | |
| X= 1R;Y=99B | U2 | R B | |
| | R | U1 U2 | |
| | B | U1 U2 | |

| SITUATION 7 | Given | Which one? | How much? |
|---|---|---|---|
| W=100;L=-100 | U1 | R B | |
| X=50R;Y=50B | U2 | R B | |
| | R | U1 U2 | |
| | B | U1 U2 | |

| SITUATION 8 | Given | Which one? | How much? |
|---|---|---|---|
| W=100;L=-100 | U1 | R B | |
| X=25R;Y=75B | U2 | R B | |
| | R | U1 U2 | |
| | B | U1 U2 | |

| SITUATION 9 | Given | Which one? | How much? |
|---|---|---|---|
| W=100;L=-100 | U1 | R B | |
| X= 1R;Y=99B | U2 | R B | |
| | R | U1 U2 | |
| | B | U1 U2 | |