A SIMPLE ALGORITHM OF THE DYNAMIC PRIORITIES AND AN APPROACH TO CHINA'S FORESTRY PRODUCTION STRUCTURE FOR THE FUTURE

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ABSTRACT

When judgments change over time, the solution to the eigenvalue problem $A(t)w(t)=\lambda_{max}(t)w(t)$ should be a time-dependent function. In this case, the use of AHP is concerned with dynamic priorities. A simple algorithm of dynamic priorities, suitable for the dynamic judgment matrices with total consistency, is presented in this paper. Using the dynamic priorities of AHP and the algorithm we analysed and forecast variations in the ratios of three major factors of forestry production structure (FPS) in China to the year 2000.

INTRODUCTION

The Analytic Hierarchy Process (AHP) developed by Thomas L. Saaty, as a useful tool in decision making and in planning, has been applied to various decision areas and has proven to be of great value.

The AHP is a multicriteria decision method that uses hierarchic or network structures to represent a decision problem and then develops priorities for the alternatives based on the decision-maker's judgments throughout the system. The AHP assigns numerical values with which to evaluate the relative importance of each factor in the structure by a procedure of paired comparisons. Once the matrix of pairwise comparisons is obtained, the priority vector of the corresponding factors is given by the solution to the eigenvalue problem

$$Aw= \lambda_{max}w \quad (1)$$

When judgments change over time, the solution to the eigenvalue problem, which is shown by

$$A(t)w(t)= \lambda_{max}w(t) \quad (2)$$

should be a time-dependent fuction. In this case, the use of AHP is concerned with dynamic priorities. Saaty discussed the analytic solution of problem (2) (Saaty, 1980). Xu Shubo provided a new improved dynamic priorities model (Xu Shubo, 1986). In this paper, we provide a simple algorithm of dynamic priorities suitable for the dynamic judgment matrices with the total consistency. The charateristics of the algorithm are: it is much easier to calculate the analytic solution to the eigenvalue problem of the dynamic judgment matrices than Saaty's and Xu Shubo's, however, it is suitable only for the totally consistent dynamic judgment matrices.

Forestry is playing an important role in China's modernization. It can be easily seen that social developments in China are reflected to some degree in forestry development. The change in the forestry production structure

(FPS) is proving to be the main expression of the changes in forestry advancement. In a macroscopic sense, the change in FPS is mainly shown by the variations of the ratios of three major factors---timber production, forest product processing, and diversification. So, to analyse and forecast these ratios in the future has great practical significance for controlling the development of China's forestry. Undoubtedly, these ratios change over time, and must be regarded as a problem of dynamic priorities. Using the dynamic priorities of AHP and the algorithm presented in this paper we have analysed and forecast the changes in the ratios of the three factors in China's forestry production structure to the year 2000.

A SIMPLE DYNAMIC PRIORITIES ALGORITHM

The algorithm presented in this paper provides a simple method of calculating the analytic solution of the eigenvalue problem of the totally consistent dynamic judgment matrices. To explain it, let us first review a definition of total consistency of the dynamic judgment matrices (Xu Shubo, 1986).

<u>Definition</u>: A dynamic judgment matrix $A(t)=(a_{ij}(t))_{n\times n}$ is totally consistent at $[t_0,t_1]$ if

$$a_{ij}(t)=a_{ik}(t)/a_{jk}(t) \qquad (i,j,k=1,2,3,\ldots,n;\quad t_0\leqslant t\leqslant t_1) \qquad (3)$$

For $A=(a_{ij})_{n\times n}$, consistency and total consistency are equivalent.

With regard to the dynamic judgment matrices with the total consistency, we have the following theorem.

<u>Theorem</u>: If a dynamic judgment matrix $A(t)=(a_{ij}(t))_{n\times n}$ is totally consistent at $[t_0,\ t_1]$, then its principal eigenvalue $\lambda_{max}(t)$ equals n, and the normalized principal eigenvector corresponding to $\lambda_{max}(t)$ is given, for any $k\in\{1,2,3,\ldots,n\}$, by

$$w(t)=(w_1(t),w_2(t),w_3(t),\ldots,w_n(t))^T, \qquad t_0\leqslant t\leqslant t_1 \qquad (4)$$

where

$$w_i(t)=a_{ik}(t)/(a_{1k}(t)+a_{2k}(t)+\ldots+a_{nk}(t)) \qquad (5)$$

<u>Proof</u>: Since A(t) is totally consistent at $[t_0,t_1]$, it implies that A(t) is consistent for any $t\in[t_0,t_1]$. Hence, rank(A(t))=1. This shows that the principal eigenvalue $\lambda_{max}(t)$ of A(t) equals n, and all other eigenvalues are 0.

According to the power method, we let

$$w_0(t)=w(t) \qquad \text{for any } t\in[t_0,t_1] \qquad (6)$$

then

$$w_1(t)=A(t)w_0(t) \qquad (7)$$

so

$$w_1(t)=A(t)w(t)=\lambda_{max}(t)w(t)=nw(t) \qquad (8)$$

and

$$(w_1(t))_i/(w_0(t))_i=n=\lambda_{max}(t) \qquad i=1,2,3,\ldots,n \qquad (9)$$

Thus, $w_1(t)$ is the principal eigenvector of A(t) corresponding to $\lambda_{max}(t)$.

Once we normalise it, the theorem is proved.

Obviously, a simple algorithm of dynamic priorities suitable for the totally consistent dynamic judgment matrices is given by the above theorem. That is, the dynamic priority vector w(t) can be obtained with any normalized column vector of A(t).

THE DYNAMIC PRIORITIES OF THE MAJOR FACTORS IN THE FORESTRY PRODUCTION STRUCTURE

China's forestry is gradually developing from a labour intensive forestry to a technology intensive one. With regard to FPS, it is gradually developing from single timber management to multiple resources management. FPS is composed of a sector structure, a forest type structure and a product structure, with the sector structure being the most important. The sector structure consists of three major factors including timber production, forest product processing and diversification. So, in a macroscopic sense, the change in FPS is mainly shown by the variations of the ratios of these three factors. Obviously, the ratios change over time, and it can be regarded as a problem of dynamic priorities.

The Leizhou Forestry Bureau in Guangdong Province is a representative forestry bureau in China, on which our research is based. According to the dynamic priorities of AHP and limited information we made pairwise comparisons in output values of timber production (TP), forest product processing (FPP) and diversification (D). We have the following dynamic judgment matrix:

$$\begin{array}{c} \\ TP \\ FPP \\ D \end{array} \begin{array}{c} \begin{array}{ccc} TP & FPP & D \end{array} \\ \begin{pmatrix} 1 & 5e^{-0.1t} & 16e^{-0.1t} \\ e^{0.1t}/5 & 1 & 16/5 \\ e^{0.1t}/16 & 5/16 & 1 \end{pmatrix} \end{array}$$

where t denotes the range from 1 to 21 years corresponding to the period between 1980 and 2000.

We took exponential functions as the elements of the matrix because TP, FPP and D output values all change exponentially.

Obviously, the matrix is totally consistent. According to the algorithm presented in this paper, the normalised principal eigenvector of A(t) is given by

TP FPP D

$$w(t)=(5e^{-0.1t}/s(t),\ 1/s(t),\ 5/(16s(t)))^T \quad (10)$$

where

$$s(t)=21/16+5e^{-0.1t} \quad (11)$$

On the basis of AHP, w(t) is the priority vector of the three factors (TP, FPP and D) in FPS from year 1980 to year 2000. We have the computed result in Table 1.

Table 1: The relative importance of major factors in the forestry production structure

Year	1980	1982	1984	1986	1988	1990	1992	1994	1996	1998	2000
TP	0.78	0.74	0.70	0.66	0.61	0.56	0.51	0.46	0.41	0.36	0.32
FPP	0.17	0.20	0.23	0.26	0.30	0.34	0.37	0.41	0.45	0.49	0.52
D	0.05	0.06	0.07	0.08	0.09	0.10	0.12	0.13	0.14	0.15	0.16

According to the calculated results, the importance of forest product processing and diversification appear to increase, whereas timber production becomes less important over time. It implies that greater importance should be attached to forest product processing and diversification, specifically, by increasing relative investment in technology-intensive FPP and D, and reducing the share of investment in labour-intensive TP.

CONCLUSION

In this paper we introduce a simple algorithm of dynamic priorities which is suitable for totally consistent dynamic judgment matrices. Using the dynamic priorities of AHP and the algorithm, we analysed the development trend of the major factors in China's forestry production structure. The results obtained in this study are valuable for the formulation of an ideal forestry policy for the future.

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