PROBABILISTIC DIMENSION OF THE AHP APPROACH TO INPUT-OUTPUT ANALYSIS: A NOTE

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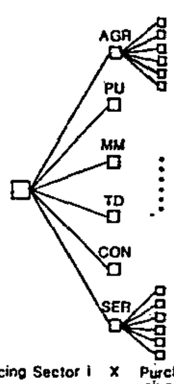
ABSTRACT

A probabilistic dimension is here noted to lie at the kernel of the priority theory approach to Input-Output analysis. Practical implication of this assimilation is noted alternatively for a direct survey and/or a non-survey approach to the construction of the table of technical coefficients using the analytic hierarchy process.

INTRODUCTION

Recent priority theory contributions to Input-Output (I/O), beginning with a seminal work by Saaty and Vargas (1979) in which a table of technical coefficients for a national economy was reproduced with remarkable accuracy, and in the absence of direct survey information, have offered certain new insights (e.g., Steenge 1986). A recent work (Banai 1987) applied the analytic network approach (ANP) to deal explicitly with the phenomena of interactions, non-linearity and sectoral feedbacks in input-output analysis. Here we note a probabilistic dimension which is inherent in the AHP approach to input-output estimation of technical coefficients, depicted in Figure 1.



Purchasing

Producing	AGR	PU	MM	TD	CONS	SERV	Pi = ΣPij
AGR	.0079	0	.2331	.0008	.0699	0	.3117
PU	.0009	0	.0130	.0075	0	.0033	.0247
MM	.0041	0	0	.0089	.0379	.0037	.0546
TD	.0691	.1694	.1281	0	.1115	.0153	.4934
CONS	0	0	0	0	0	.0546	.0546
SERV	0	.0117	.0224	.0224	.0039	.0004	.0608
							Σ = 9998

Prob. [Producing Sector i X Purchasing Sector j, given i = Joint Production i,j]

Figure 1. A probabilistic representation of the AHP approach to input-output analysis

The table contains the estimates of input-output coefficients reproduced from Saaty and Vargas (1979). First, we note that the row totals of the matrix of technical coefficients, Σ_j Pij, here called by the marginal probabilities, Pi, reproduce the vector of the relative importance of sectors originally estimated in Saaty and Vargas (1979), repeated here in Table 1:

Table 1. The index of relative importance vs. marginal probability

Sectors*:	AGR	PU	MM	TD	CONS	SERV
Index of Relative Importance:	0.3108	0.0248	0.0546	0.4934	0.0546	0.0608
Marginal Probability (Pi):	0.3117	0.0247	0.0546	0.4934	0.0546	0.0608

Agriculture(AGR); Public Utilities(PU); Mfg. & Mining(MM); Transport. & Distribution(TD); Construction(CONS); and Services (SERV).

The matrix of technological coefficients can be interpreted as joint probabilities of the sectoral interactions. Now consider the logical and practical implications of this interpretation, specifically from the probability operation,

marginal probability (Pi) x conditional probability (Pj/i) = joint probability (Pij),

also shown on the probability tree (Figure 1).

Next, the conditional probabilities (Pj/i) can be derived. We define Pj/i as the (conditional) probability of an individual sector, drawn at random, will be a sector i purchasing from a producing sector j. For example, we have the following conditional probabilities from the relation Pj/i = Pij/pi:

Purchasing sector (AGR), if the producing sector is also AGR: 0.0079/0.311 = 0.025.
Purchasing sector (AGR), if producing sector is PU: 0.0009/0.0247 = 0.0364.
Purchasing sector (AGR), if producing sector is MM: 0.0041/0.0546 = 0.075.

Similarly, we can obtain the remaining conditional probabilities. Now we turn to practical implications for the construction of I/O table with this interpretation.

CONCLUSION

Previously, we noted the development of an alternative approach to deriving the relative importance of sectors, the marginal probabilities Pi, in which the nonlinearities involving sectoral feedbacks, or interactions, can be accounted for by using the ANP. The vector of marginal probabilities, Pi, can now be weighted by the vector of conditional probabilities, Pj/i, to determine the table of technical coefficients, the joint probabilities, Pij. The conditional probabilities can be estimated probabilistically, or deterministically by using the AHP. Further research can pursue these possibilities in practical application, in either a direct survey or a non-survey approach to input-output analysis. In a direct survey approach, information of each sector's input which is purchased from a producing sector's output provides a basis to estimate the conditional probabilities. Alternatively, conditional probabilities can be obtained from the marginal and joint probabilities of a previous estimate and thereby adjusted (weighted) by the information of marginal probability (Pi's), to obtain the adjusted table of technical coefficients.

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