ON SETTING GOAL-PROGRAMMING WEIGHTS USING THE AHP

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ABSTRACT

A procedure that uses the Analytic Hierarchy Process framework and the Expert Choice PC software for determining a large set of consistent goal-programming objective function weights is described.

INTRODUCTION

Multicriteria decision problems have been and will continue to be an outstanding difficult area of research. Competing technologies for solving such problems include multiattribute utility theory, [Keeney and Raiffa, 1976] and the Analytic Hierarchy Process (AHP), [Saaty , 1980]. The AHP has proven to be a versatile methodology that has been used to structure a wide class of decision problems, whereas the utility theory approach, although succssful in many important areas, has not attained as wide a popularity of use as one would have surmised. In our view, the main reason for this is the inability of the general analyst to determine, in a straight-forward manner, the form and structure of the requisite utility function. This is contrasted to the AHP process by which the analyst can, without much difficulty, structure the elements of a decision problem rather rapidly, and refine the analysis with ease.

Apparent limitations of the AHP are the need to make numerous pairwise comparisons and the ability of the analysts (decision makers) to process such comparisons effectively. In his seminal paper, Miller (1956) demonstrates that "... the capacity of people to transmit information ..." when making unidimensional judgements is limited to seven categories plus-or-minus-two, i.e., most persons when presented with stimuli can differentiate them correctly when there are about seven alternatives. Saaty in his writings notes this empirical rule and suggests that when the number of criteria or alternatives are numerous (say, greater than nine) they be grouped into fewer homogeneous classes that are compared and then subdivided further, as required. Harker(1987a,1987b) describes processes that can be used to reduce the tedium of making pairwise comparisons when the criteria and/or alternatives are many. These apparent operational difficulties of the AHP appear to be overcome successfully in practice.

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goal-programming weights into a small set of categories and used the AHP to calculate the system's underlying weights or priorities . We then showed how these weights could then be used to produce the individual weights for the many thousands of goal programming deviation variables. Here we propose a different approach that uses the new ratings procedure that has been incorporated into the AHP software package Expert Choice.

The Goal-Programming Problem

The basic goal-programming problem of interest is a multiyear army manpower planning problem in which the variables are indexed by time(t), grade(g) and skill(s). The original problem has 9060 equations, 28730 variables, with 6950 of the equations goal constraints containing 13900 deviation (positive and negative) variables. The goal-programming constraints are target conditions in that each represents a function of the variables (e. g., gains, losses, promotions, skill inventories) that is set equal, in a goal-programming sense, to a target (goal) value. A typical constraint can be written (functionally) as

$$f[X(t,g,s)] + GP(t,g,s) - GN(t,g,s) = T(t,g,s)$$

where $f[X(t,g,s)]$ is a function of decision variables such as separations or promotions or inventories in time period t, grade g and skill s; $GP(t,g,s)$ and $GN(t,g,s)$ are the goal-programming under- and overachievement deviation variables for the target constraint, respectively, and $T(t,g,s)$ is the target goal for the function. Each $GP(t,g,s)$ and $GN(t,g,s)$ appears in the linear objective function multiplied by a weight $WP(t,g,s)$ and $WN(t,g,s)$, respectively. These weights are meant to indicate the importance of meeting the associated target and are to reflect the decision maker's explicit and implicit tradeoffs in selecting a particular solution to implement (or to use as a basis for further planning). It should be clear that the element that makes this problem difficult is that there is no true single optimizing solution and the selection process is one of compromise and satisficing. In sum, the analysis problem reduces to selection values of the thousands of weights $WP(t,g,s)$ and $WN(t,g,s)$ such that the solution produced in optimizing (here minimizing) the objective function subject to the target constraints would be a compromise solution acceptable to the decision maker. We do not mean to imply that a single setting of the weights wounld produce such a solution. But, the process described below yields, we feel, a systematic and rational way of eventually calibrating the model's weights to produce such an acceptable solution.

How the weights are to be determined and varied (for sensitivity studies) were the issues we attempted to address in Gass (1986). The reader is referred to that paper for a direct application of the AHP and a suggested procedure for modifying the AHP weights to goal-programming weights. Here we use the Expert Choice AHP software ratings module to produce the goal-programming weights. We emphasize two things: (1) As the problem is large-scale and to goal-programming weights. Here we use the Expert Choice AHP software ratings module to produce the goal-programming weights. We emphasize two things: (1) As the problem is large-scale and involves many goal constraints, much effort must go into any procedure that will produce consistent weights (there is no free lunch), and (2) The aim is to develop a procedure that will enable the analyst, working with the decision maker, to generate weights that can be calibrated, in a systematic and rational manner, to the decision maker's objectives.

To demonstrate the use of the Expert Choice ratings module, we consider a simplified hierarchy as shown in Figure 1. We illustrate the problem for three time periods (years or grouping of years) and first use the AHP process to determine the importance weights for meeting each time period's total manpower target, Level 2. Then, for each time period, AHP comparisons are made to develop the associated weights for meeting of the promotion, separation and inventory targets, Level 3, The level 3 targets are really indexed by grade g and skill s, while the Level 2 total manpower targets are indexed only by time t. However, as the grade/skill combinations are many (here g = 7 and g = 33 for a total of 231 combinations), the Level 3 weights are determined without regard to grade and skill. That is, we assume a generic grade/skill when determining the Level 3 comparisons and weights.

The process described up to now is standard AHP methodology. What we need to do is to factor in the impact of having each promotion, separation and inventory target indexed by grade and skill. Under each Level 3 target we must determine the importance of a particular grade and skill and then convert the result into a number that can be used as weights for the corresponding deviation variables in the goal-programming objective function. The ratings module requires the analyst to indicate that the corresponding grade/skill combination is either (1) Extremely Important, (2) Very Important, (3) Important, (4) Moderately Important, and (5) Not Important to the meeting of a Level 3 target for the corresponding year. (A finer breakdown of categories can be used.) The resultant hierarchy for the ratings module is shown in Figure 2. The five importance measures are given normalized weights either by an AHP pairwise analysis or subjective judgments. These weights are then multiplied accordingly by the Level 2 and Level 3 weights in the usual AHP fashion. Figure 3 shows the final weights for ten grade/skill combinations for promotions in year 1. Year 1/promotions has an AHP hierarchical weight of 0.507 • 0.262 = 0.1332 (rounded accuracy). The ratings module multiplies this number by the weight of the grade/skill importance factor to produce the final weights. For example, in Figure 3, as alternative 1 (g = 1, s = 1) is extremely important (weight of 0.338), the resultant weight is .1332 • 0.338 = 0.045. Note that the weights range from a high of 0.045 to a low of 0.009 (not important). Promotion over- and underachievement deviation variables for year 1 should have objective function weights that vary between these limits for all grade/skill combinations. Specific weights can be adjusted for calibration purposes, but any adjusted year 1 promotion weight should lie in this range in order to maintain consistency of the analysis. (We suggest using the broader range of 450 to 90.) The standard sensitivity procedures of the AHP and Expert Choice allow for further analyses and fine-tuning calibration to take place. Much effort must go into this procedure. The 231 grade/skill combinations have to be entered for each Level 2 and Level 3 pair. However, the total process can be automated into one analysis system in which the goal-programming model is automatically fed the AHP weights, with the AHP model imbedded into the system by an appropriate user interface.

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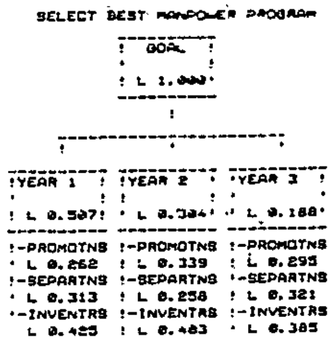
FIGURES



Figures 1. Hierarchy for manpower priorities

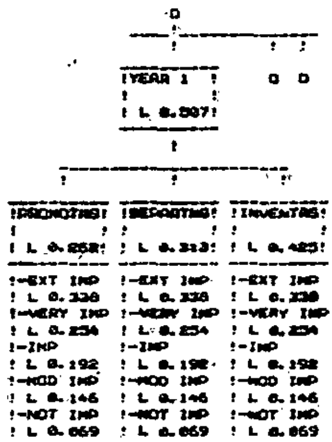


Figure 2 Ratings module hierarchy

ALTERNATIVES	YEAR 1 PROMOTNS .1332	YEAR 1 SEPARTNS .1585	YEAR 1 INVENTRS .2156	TOTAL
1 1 1	EXT IMP			0.045
2 1 2	VERY IMP			0.034
3 1 3	IMP			0.026
4 2 1	MOD IMP			0.019
5 2 2	NOT IMP			0.009
6 3 3	NOT IMP			0.009
7 3 4	MOD IMP			0.019
8 3 5	IMP			0.026
9 3 6	VERY IMP			0.034
10 4 1	EXT IMP			0.045

Figure 3 Ratings module output (year 1/promotions