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## THE USE OF AHP METHODOLOGY FOR INDIRECT MEASUREMENT OF QUALITATIVE FACTORS USED IN THE DEA AND THE RSE ANALYSES

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**Abstract:** AHP is used to extend the ordinal ranking among the categories for a DEA input categorical variable. The AHP Fundamental Scale is used to provide a decision rule for the inclusion or exclusion of the categorical variable as an intangible input like "growing conditions". The AHP procedure provides an operational empirical test as to the severity of any detected ordinal inconsistency by eliciting more judgment data from the decision maker.

### Introduction

The use of qualitative factors in Data Envelopment Analysis (DEA) and Relative Spatial Efficiency (RSE) generally relies on dummy variables to capture the influence of intangible or nonmeasurable factors (Charnes, Cooper, Lewin and Seiford, 1994). Some qualitative factors are truly binary (i.e., race, sex, and etc.) or truly equally spaced integer variables like time. However, other qualitative factors are treated as binary as a matter of convenience like location (i.e., suburban or central city instead of miles from the downtown location). Other qualitative factors are truly "intangible" like customer "goodwill" which can be treated as binary (i.e., high or low) or as an equally spaced dummy variable (i.e., below normal, normal, above normal, etc.). Whenever a qualitative factor is NOT truly binary or truly equally spaced, then the assignment of dummy variable categories (i.e., high or low, etc.) reflects judgmental data supplied by the decision maker (DM). This same type of judgmental information can be more systematically extracted from the DM by use of the Analytic Hierarchy Process methodology (Saaty, 1988). This in fact will permit an alternative approach to test the appropriateness of the widely used dummy variable methodology within DEA and RSE. If the standard dummy variable assumptions are appropriate to represent qualitative factors then the results should be closely duplicated when the more powerful AHP methodology is employed. However, if the standard assumptions are not appropriate then the AHP methodology should be used to provide INDIRECT measurement of the given qualitative factor as illustrated in the rest of this paper.

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\*See *Fortune*, Vol. 130, October 31, 1994, p. 38 for a discussion of the ESI Crosspoint Evaluator (CE) software used to implement DEA and RSE analysis, and now with the AHP option as a new feature (Norton, 1994).

## DEA and RSE Overview

DEA is a linear programming (LP) technique originally developed by (Farrel, 1957) and further developed by (Charnes, Cooper, and Rhodes 1978). It measures relative performance by the use of appropriate comparisons to other Decision Making Units (DMUs). This is the key to the success of DEA as a tool of Pareto-Koopmans efficiency analysis (Charnes, Cooper, Golamy, Seiford and Stutz, 1985). A given DMU is really not efficient if another DMU or some combination of other DMUs are producing more of the same outputs with no more than the same inputs, or they are producing the same output with less of the same inputs. The DEA procedure calculates the percentage differences in inputs or in outputs needed for the DMU being evaluated to achieve a comparison or "peer" group level of efficiency. This does NOT require the specification of a particular type of production function. Also, it does not require the explicit specification of weights for different outputs to determine the resulting trade-off values. These are implicitly determined as multipliers (i.e., shadow prices) for the inputs and the outputs.

The relative technical efficiency score of the DMU under consideration is calculated as the weighted sum of outputs divided by the weighted sum of inputs. The DEA procedure effectively determines these weights (i.e., multipliers) so that the efficiency score of the DMU being evaluated is maximized. This is unity for an input efficient DMU and within the unit interval for an input inefficient DMU. The output efficient DMU will also have an efficiency score of unity, but an output inefficient DMU will have its score above unity. Clearly, a ranking among inefficient DMUs is possible using their DEA determined relative efficiency scores. A recent development in the DEA literature permits an implicit opportunity "shadow" cost ranking among input efficient and output efficient DMUs (Anderson and Peterson, 1993). These Anderson and Peterson (A/P) "superefficiency" DEA scores make it possible to now rank the relative efficiency of all DMUs (i.e., both efficient and inefficient). The peer group still only consists of efficient DMUs who form a conceptually composite operating unit to produce a higher level of output while using fewer resources and operating in the same environment.

DEA has a number of advantages over other analytical models (Seiford and Thrall, 1990). It easily handles multiple outputs and multiple inputs. Also, external conditions in the environment that impact the operating unit's performance are taken into account by demographic variables which allows valid comparisons. This introduces the notion of impartial fairness among all DMUs being evaluated because their common demographic conditions are being introduced into the DEA determination of their efficiency scores. Since DEA directly estimates the efficient level of inputs and outputs for a given DMU, then this can be interpreted as an "efficiency frontier" for those DMUs in the SAME demographic category. This is accomplished by the use of categorical variables (Banker and Morey, 1986a) which resemble dummy variables in regression analysis. Categorical variables require an ordinal scale instead of the cardinal scale of measurement for inputs and outputs. In contrast, dummy variables associate different DMUs with different demographic "qualitative" factors like gender, location, or goodwill. The conversion of a dummy variable to a categorical variable requires an ordinal ranking among the dummy variable categories. Therefore, a comparison of DMUs which are operating at equal or higher levels of the dummy variable is possible (Kamakura, 1988). Once an ordinal ranking can be elicited from the DM concerning demographic factors then the AHP methodology can be introduced by eliciting even more preference information from the DM. This is the principle focus of this paper.

RSE is a special application of DEA to the siting of a new DMU. The inputs are travel distance and population coverage for the spatial tradeoff of existing DMUs to potential DMUs using the centroids of zip code polygons for relative measurement. When the RSE score exceeds a threshold level then that potential DMU site is efficient; otherwise, it is considered inefficient (Charnes, Cooper, Lewin and Seiford, 1994, Figure 12-3, p. 245). The relative spatial efficiency of the potential DMU sites can be compared to the RSE scores for existing DMUs. Also, the spatial inputs can be used with economic inputs and outputs for a combined RSE and DEA evaluation. This permits the spatial proximity of an efficient DMU to influence whether or not it is chosen to be in the peer group of an

inefficient DMU. Finally, the DEA methodology permits the distinction between "discretionary" (i.e., controlled by the DM) inputs and outputs from "nondiscretionary" (i.e., uncontrolled by the DM) inputs and outputs (Banker and Morey, 1986b). The "free disposability" assumption for certain nondiscretionary inputs is appropriate; whereas, for other nondiscretionary inputs the utilization level can not be changed (Ray, 1988). This amounts to the treatment of nondiscretionary inputs as an inequality or as a strict equality condition depending upon the DM's judgment.

### DEA and RSE Numerical Example

The incorporation of AHP into DEA and RSE can be illustrated in the case of one output, two discretionary economic inputs, one categorical variable, and eight DMUs. The vectors of outputs, inputs, and DMU weights are:

$$Y_o = (Y_1, Y_2, \dots, Y_5, \dots, Y_8) = (1, 1, 1, 1, 1, 1, 1, 1) \text{ Crop Yield} \quad (1a)$$

$$X_o = (X_1, X_2, \dots, X_5, \dots, X_8) = (1, 2, 5, 2.5, 3, 6, 2, 5) \text{ Fertilizer Doses} \quad (1b)$$

$$(10, 6, 2, 7, 3, 2, 9, 4) \text{ Water Increments}$$

$$(0, 0, 0, 1, 1, 1, 0, 1) \text{ Growing Conditions}$$

$$w' = (w_1, w_2, \dots, w_5, \dots, w_8). \quad (1c)$$

The primal envelopment problem for DMU5 (i.e., point E) is:

$$\begin{aligned} \text{Min: } \Theta & \\ \text{Sub to: } \begin{bmatrix} Y_5 \end{bmatrix} & \leq \begin{bmatrix} Y_o \end{bmatrix} [\Theta] + \begin{bmatrix} w \end{bmatrix} \text{ and } w \geq 0 \text{ with } \Theta \text{ unrestricted.} \\ \begin{bmatrix} \Theta * (-X_5) \end{bmatrix} & \begin{bmatrix} -X_o \end{bmatrix} \end{aligned} \quad (2)$$

There are eight primal DEA problems that must be solved with the left-hand side (LHS) of (2) changing for each primal formulation. The dual multiplier problem for DMU5 (i.e., point E) is:

$$\begin{aligned} \text{Max: } \Theta & = u' Y_5 \\ \text{Sub to: } v' X_5 & = 1, \begin{bmatrix} Y_o \end{bmatrix} : \begin{bmatrix} -X_o \end{bmatrix} [\Theta] + \begin{bmatrix} u \end{bmatrix} \leq \begin{bmatrix} 0 \end{bmatrix}, \text{ and } \begin{bmatrix} u \end{bmatrix} \geq 0 \text{ with } \Theta \text{ unrestricted.} \\ \begin{bmatrix} v \end{bmatrix} & \begin{bmatrix} 0 \end{bmatrix} \end{aligned} \quad (3)$$

The categorical variable (i.e., "growing conditions") effectively organizes DMUs A, B, C, and G into one demographic group and DMUs D, E, F, and H into another demographic group. This is shown in Figure 1 with a "separate" efficiency frontier for each group (i.e., ABC and DEF, respectively). The partitioning of the original sample by use of the categorical variable for growing conditions yields 6 efficient DMUs and 2 inefficient DMUs (i.e., DMU G and DMU H). When the categorical variable is omitted and all 8 DMUs are evaluated together, then the "composite" efficiency frontier reveals that there are 4 efficient and 4 inefficient DMUs (DMU D, DMU F, DMU G, and DMU H). The UNCONDITIONAL use of the "growing conditions" categorical variable causes DMU D and DMU F to remain efficient. When the categorical variable is not used then DMU D and DMU F become inefficient and the "composite" efficiency frontier (i.e., ABEC) is different from each "separate" efficiency frontier. The AHP methodology can now be used to provide a CONDITIONAL use of the "growing conditions" categorical variable by eliciting additional judgmental data from the DM.

When the growing conditions are "normal" then a "zero" value is assigned to the categorical variable, and when growing conditions are "subnormal" then "unity" is the assigned value. Consequently, the DM prefers normal growing conditions to subnormal with the ordinal ranking over these two categories and not over the eight DMUs. The Saaty fundamental scale in Table 3 can be used to elicit a VERBAL scale response given by the DM to reveal the strength of preference for "normal" over "subnormal" growing conditions. The numerical scale entry in Table 3 corresponding to the verbal scale response is "a", and the resulting estimates for "p" and "1-p" to be used in the

categorical variable are:

$$p = a/(1+a) \text{ and } (1 - p) = 1/(1+a). \quad (4)$$

If the DM is "almost" indifferent between these two categories then the categorical variable should be eliminated and the "composite" efficiency frontier accepted. If the DM expresses a "substantially" SHARP preference for "normal" over "subnormal" growing conditions then the categorical variable should be retained and the "individual" efficiency frontiers used instead.

The rejection of the "composite" efficiency frontier in Figure 1 can be based on the derivation of a critical value of "p" (i.e.,  $\Pi$ ) that the judgment data estimate of "p" in (4) must exceed. This critical value of "p" is based on the (A/P) "superefficiency" DEA methodology. For example, point E in Figure 1 has an (A/P) superefficiency score of  $(1 + e)$  equals 1.238 and the critical value of "p" is:

$$\Pi = 1/(1 + e) \text{ and } \Pi = (1/1.238) = 0.81 \text{ in Figure 1.} \quad (5)$$

This corresponds to the determination of point  $\bar{E}$  on the BC line segment. Now  $\bar{E}$  is the "common point" between the ABC efficiency frontier for normal growing conditions and the "psuedo" efficiency frontier, D $\bar{E}$ F, for subnormal growing conditions. This occurs when point E is replaced with its (A/P) "superefficiency" point,  $\bar{E}$ , on the ABC efficiency frontier. The DM should express a sufficiently SHARP preference for "normal" over "subnormal" growing conditions so that the "psuedo" D $\bar{E}$ F efficiency frontier for the subnormal case can AT LEAST be implied. When "p" in (4) is less than  $\Pi$  in (5) then the composite ABEC efficiency frontier should be accepted and the categorical variable eliminated. However, when "p" exceeds  $\Pi$  then the "composite" efficiency frontier should be rejected and the categorical variable retained because individual efficiency frontiers are now warranted. The DM must express a strength of preference on the verbal scale for "normal" over "subnormal" growing conditions that exceeds  $\bar{a}$  to justify the use of this categorical variable:

$$\bar{a} = \Pi/(1 - \Pi) \text{ and } \bar{a} = (0.81/(1 - 0.81)) = 4 \text{ in Table 3.} \quad (6)$$

The (A/P) primal problem can be solved by deleting the Y5 and X5 columns from  $Y_0$  and  $X_0$  in (1) that is used in (2) so that  $\Theta 5$  will now equal  $(1 + e)$  as desired. The application of AHP to the data points of individual DMUs instead of the intangible input categories used to group them must now be explored.

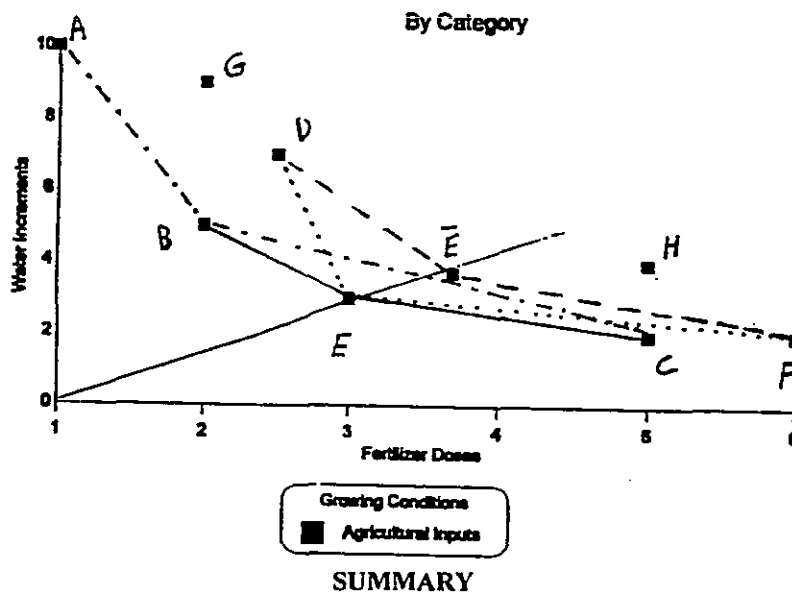
### Textbook Example

The simplest production relationship used in all introductory econometric textbooks concerns the crop yield produced by the farmer with a controllable input like fertilizer and an uncontrollable input like rainfall (Pindyck and Rubinfeld, 1981). All of these variables are measurable reflecting tangible inputs and a tangible output. However, an intangible input like "growing conditions" can be introduced into this production relationship to cover the influence of "all other factors". If this intangible input is treated as a binary dummy variable (i.e., good or not good growing conditions) then this represents a certain amount of judgment data provided by the DM. Of course, if this same intangible input is treated as an equally spaced multivalued integer dummy variable (i.e., below normal, normal, and above normal growing conditions) then this represents even more judgment data provided by the DM. Clearly, the more judgment data that can be consistently elicited from the DM, then the more INDIRECT measurement as to the nature of "growing conditions" in each time period can be achieved for this intangible input. It will be illustrated shortly that the AHP methodology provides this consistent method of elicitation from the DM desired (Wedley, 1990).

Exhibit 1: Hypothetical Data for Figure 1

	A	B	C	D	E	F	G	H	$\bar{E}$
Crop Yield	1	1	1	1	1	1	1	1	1
Fertilizer Doses	1	2	5	2.5	3	6	2	5	3.7
Water Increments	10	6	2	7	3	2	9	4	3.7
Growing Conditions (Normal = 0 and Subnormal = 1)	0	0	0	1	1	1	0	1	1

Figure 1: DEA Efficiency Frontiers



- ABC is Efficiency Frontier Given "Normal" Growing Conditions.
- DEF is Efficiency Frontier Given "Subnormal" Growing Conditions.
- ABEC is "Composite" Efficiency Frontier ignoring the categories for Growing Conditions.
- $D\bar{E}F$  is "Pseudo" Efficiency Frontier for "SHARP" Distinction Between Intangible Categories on Growing Conditions where Normal is 0 and Subnormal is 1 when  $p \geq \Pi$ ; yet when  $p < \Pi$  then Ignore the Categorical Variable and Use the "Composite" Efficiency Frontier.

The formal representation of this textbook example consists of the following variables:

- o  $Y_t$  = Crop Yield (measurable output),
- o  $X_t$  = Fertilizer Doses (controllable and measurable input),
- o  $Z_t$  = Rainfall (uncontrollable and measurable input), and
- o  $W_t$  = all other factors that influence "growing conditions" (nonmeasurable inputs).

Although  $W_t$  could be represented by types of labor, machinery, seed, and other tangible inputs, there would remain an "intangible" factor that influences "growing conditions" that can not be directly measured. This "intangible" factor can represent X-inefficiency consisting of the level of effort beyond the mere number of manhours, the level of the farmer's managerial skill, and the level of coordination of the means of production to achieve the possible crop yield beyond that actually observed (Leiberstein and Maital, 1992). In a step-wise regression strategy the initial formulations are:

$$Y_t = a_1 + b_1 * X_t \tag{7a}$$

$$Y_t = a_2 + c_1 * Z_t \tag{7b}$$

$$Y_t = a_3 + b_2 * X_t + c_2 * Z_t \tag{7c}$$

In (7a) the simple relationship between crop yield ( $Y_t$ ) and fertilizer doses ( $X_t$ ) is established by regression techniques ignoring all uncontrollable inputs like rainfall ( $Z_t$ ) and all intangible inputs like "growing conditions" ( $W_t$ ). The estimates for the intercept and the slope (i.e.,  $a_1$  and  $b_1$ ) will be biased unless the influence of the uncontrollable input, rainfall, is included in (7c). The multiple regression estimates for the intercept and two slopes (i.e.,  $a_3$ ,  $b_2$ , and  $c_2$ ) will be different from ( $a_1$  and  $b_1$ , or  $a_2$  and  $c_1$ ). Due to the lack of independence between these two tangible and measurable inputs ( $X_t$  and  $Z_t$ ) it requires that (7c) be used because the results in (7a) and (7b) will be biased.

This same lack of independence among regressors is extended to the intangible input  $W_t$  that can not be directly measured and is represented by the following linear regression equation:

$$Y_t = a + b * X_t + c * Z_t + d * W_t. \tag{8}$$

When  $W_t$  is represented by a 0-1 dummy variable (i.e., 0 indicates "poor" growing conditions and 1 denotes "good" growing conditions) then the following interpretation of (8) results:

$$Y_t = a + b * X_t + c * Z_t \quad \text{when } W_t = 0 \quad \text{and} \tag{9a}$$

$$Y_t = (a + d) + b * X_t + c * Z_t \quad \text{when } W_t = 1. \tag{9b}$$

The use of a 0-1 dummy variable for this "Qualitative factor" (i.e., growing conditions), implies that the full impact of the change in qualitative status is absorbed by the operational intercept (i.e., "a" in (9a) and "(a + d)" in (9b)) without biasing the slope coefficients "b" and "c". Of course, when  $W_t$  is a multivalued equally spaced dummy variable (i.e., 0 for "poor", 1 for "good", 2 for "excellent", etc. growing conditions) then more judgment data is needed from the DM. The more categories that  $W_t$  can be defined with and used on the database then the closer to an indirectly measured  $W_t$  variable for "growing conditions" can be obtained. A more systematic methodology for eliciting this judgment data and an established cardinal scale is now required.

#### Numerical Illustration

The regression in (8) can be estimated with at least "n" equals 5 observations as shown in Table 1. The 5

observations for  $Y_t$ ,  $X_t$ , and  $Z_t$  are all directly measured because the variables are tangible. However, the 5 values for  $W_t$  require judgment data. The use of a 0-1 dummy variable for  $W_t$  implies a dichotomous ranking between "poor" growing conditions and "good" growing conditions with "good" preferred to "poor" by the DM. The use of a multivalued equally spaced dummy variable implies a more general ranking with 0 for "poor", 1 for "good", and 2 for "excellent" indicating a trichotomous ranking among the observations for "growing conditions".

Table 1: Sample Data

OBS	$Y_t$	$X_t$	$Z_t$	$W_t$
1	Y1	X1	Z1	W1
2	Y2	X2	Z2	W2
3	Y3	X3	Z3	W3
4	Y4	X4	Z4	W4
5	Y5	X5	Z5	W5

The general ordering for a DM's judgment data with respect to "growing conditions" can be summarized in the following pairwise comparison Table 2. If the "growing conditions" in the time period corresponding to the  $W_1$  "state" is preferred to that in the  $W_2$  "state" then place a "+" in the ( $W_1$  row,  $W_2$  column) and place a "-" in the ( $W_2$  row,  $W_1$  column). All diagonal elements have a "0" to indicate indifference. This pairwise comparison process is continued until all the entries in Table 2 are filled with "+" for preference, "0" for indifference, and "-" for not preference and not indifference. The number of "+" entries for each row are recorded in the row sum with the DM appearing to indicate that the "growing conditions" are "best" in  $W_2$ , "good" in  $W_1$  and  $W_3$ , and "poor" in  $W_4$  and

Table 2: Pairwise Comparisons

To	$W_1$	$W_2$	$W_3$	$W_4$	$W_5$		SUM	
From	<hr/>							
$W_1$	0	+	+	-	0		2	
$W_2$	-	0	+	+	+		3	
$W_3$	-	-	0	+	+		2	
$W_4$	+	-	-	0	-		1	
$W_5$	0	-	-	+	0		1	

$W_5$ . This would appear to correspond to a trichotomous ranking over the 5 observations with  $W_t$  a multivalued equally spaced dummy variable. However, Table 2 provides MORE information about the DM than the simple use of a dummy variable which is why a pairwise comparison table is so valuable. This DM is ordinally INCONSISTENT because  $W_1$  is preferred to  $W_2$ ,  $W_2$  is preferred to  $W_4$ , and consistency would require that  $W_1$  be preferred to  $W_4$ . However, Table 2 reveals that the DM has  $W_4$  preferred to  $W_1$ . This is called a "cycle" over the ( $W_1$ ,  $W_2$ ,  $W_4$ ) triple and is analogous to the child's game of Rock, Sissors, Paper (i.e., Rock breaks Sissors, Sissors cuts Paper, but Paper wraps around Rock so no dominant choice exists). If the DM created a  $W_t$  dummy variable based on the SUM values shown in Table 2 and used this in the regression in (8), a serious BIAS would be introduced BECAUSE THE DM'S JUDGMENT DATA IN CONSTRUCTING THE DUMMY VARIABLE REFLECT ORDINALLY INCONSISTENT PREFERENCES. This constitutes an "operational test" for the construction of a dummy variable to represent an intangible input like "growing conditions" in this example.

### AHP Methodology Applied To Table 2

An obvious remedy to the situation when the DM is found to be ordinally inconsistent would be to bring this fact to the DM's attention for clarification. This is certainly better than reducing the amount of information elicited from the DM by merely asking that the DM assign the descriptors "poor", "good", or "excellent" to each of the 5 observations! Another more general remedy is to elicit a verbal descriptor from the DM each time that "preference" is indicated in Table 2. This can be accomplished by using the Saaty fundamental scale in Table 3. Now when the pairwise comparisons are elicited the "+" entries above and below the diagonal are replaced with a numerical scale entry from Table 3 based on the VERBAL scale response given by the DM to reveal the strength of preference! This AHP remedy to determine the actual severity of the ordinal inconsistency by eliciting more information from the DM is shown in Table 4 with "0" entries replaced with the number 1 and "-" entries replaced with the RECIPROCAL of the corresponding "+" entries.

Table 3: Fundamental Scale for Making Judgments

Numerical Scale	Verbal Scale	Explanation
1	Equal importance of both elements.	Two elements contribute equally to the property.
3	Moderate importance of one element over the other.	Experience and judgment favor one element over the other.
5	Strong importance of one element over the other.	An element is strongly favored.
7	Very strong importance of one element over the other.	An element is very strongly dominant.
9	Extreme importance of one element over the other.	An element is favored by at least an order of magnitude.
2, 4, 6, 8	Intermediate values between the above adjacent values.	Used for compromise between two judgments.
<b>NORMAL SCALE</b>	Used to compare elements of the same order of magnitude. In general, you should attempt to group elements so they are within one order of magnitude of each other.	



Table 4: AHP Pairwise Comparison Data

From	To	W1	W2	W3	W4	W5	Wt (Weights)	Summary	Statistics
W1		1	5	7	1/2	1	0.3542		
W2		1/5	1	4	8	9	0.2980	LAMBDA =	8.7076
W3		1/7	1/4	1	5	7	0.1598	C.I. =	0.9269
W4		2	1/8	1/5	1	1/2	0.1067	R.I. =	1.12
W5		1	1/9	1/7	2	1	0.0813	C.R. =	0.08276
SUM							1.0000		

Notice that the "growing conditions" in the time period corresponding to the W1 "state" is now "strongly favored" to that in the W2 state because the "+" in the (W1 row, W2 column) of Table 2 is now replaced with the numerical scale value of "5" in the (W1 row, W2 column) of Table 4 using the fundamental scale in Table 3. Also, the "-" in the (W2 row, W1 column) of Table 2 is now replaced with the RECIPROCAL numerical scale value which is "1/5" in the (W2 row, W1 column) of Table 4. This simple transformation of the pairwise comparisons information in Table 2 by eliciting the "strength of preference" in Table 3 produces the AHP pairwise comparison data in Table 4. Now the relative weights reveal the relative importance of each Wt value to the DM. The "growing conditions" are most favorable in state W1 and least favorable in state W5. Also, these AHP weights have a cardinal interpretation that the DM "strongly favors" the "growing conditions" in state W1 over state W5 BECAUSE the ratio (.3542/.0813) = 4.5 which is almost 5 on the numerical scale in Table 3. This intensity of preference interpretation is NOT possible with the original simplistic-dummy variable formulation of this intangible input variable, Wt.

Finally, the original ordinal inconsistency from Table 2 is still to be found in Table 4 because the underlying pattern of pairwise comparisons is unchanged. However, with this additional "strength of preference" information elicited from the DM the severity of this ordinal inconsistency may have diminished to such a degree that the Wt "weights" in Table 4 may now be acceptable judgment data for this intangible input to be used in the desired regression analysis. The "operational test" for the AHP based construction of this dummy variable, Wt, to represent the "growing conditions" intangible input is based on the construction of a critical ratio (C.R.) which is less than ten percent (0.10). This requires that the lambda statistic ( $\lambda_t = 8.7076$ ) associated with the INDIRECTLY measured intangible variable, Wt, for "growing conditions" satisfy the following:

$$A * Wt = (\lambda_t) * Wt \tag{10}$$

where A is the elicited AHP pairwise comparison data in Table 4. Now the consistency index (C.I.) associated with the lambda statistic (i.e.,  $C.I. = (\lambda_t - n)/(n - 1)$ ) is divided by a random index (R.I.) for n = 5 observations and the resulting consistency ratio (C.R.) (i.e.,  $C.R. = C.I./R.I.$ ) should be less than 0.10 for the Wt variable to be acceptable. The summary statistics associated with Table 4 reveal that C.I. = 0.9269, R.I. = 1.12 (as determined from the Expert Choice software) and the resulting C.R. = 0.08276 (i.e.,  $(0.9269/1.12) = 0.08276$ ) is LESS than 0.10 as desired (Forman, 1995). Consequently, despite the observed ordinal inconsistency in Table 2, this DM is overall sufficiently consistent to provide an acceptable INDIRECT measurement for the intangible input variable, Wt, based on the AHP data in Table 4 using the Saaty scale in Table 3. Next the multiple linear regression in (8) can be implemented but now by an INDIRECT procedure.

### Indirect Estimation of Equation (8)

The  $W_t$  weights in Table 4 reflect "relative" measurement NOT "absolute" measurement of the UNOBSERVED values for  $W_1, W_2, W_3, W_4,$  and  $W_5$  in Table 1. Now the observed values for  $Y_t, X_t,$  and  $Z_t$  in Table 1 all constitute "absolute" measurement. One remedy would be to scale each  $Y_t$  value by its sum,  $n \cdot Y_{\text{bar}}$ , each  $X_t$  value by its sum,  $n \cdot X_{\text{bar}}$ , and each  $Z_t$  value by its sum,  $n \cdot Z_{\text{bar}}$ , as:

$$Y_t \cdot (1/(n \cdot Y_{\text{bar}})) = a \cdot (1/n) + b \cdot X_t \cdot (1/(n \cdot X_{\text{bar}})) + c \cdot Z_t \cdot (1/(n \cdot Z_{\text{bar}})) + d \cdot W_t \quad (11)$$

where each transformed variable now has a unit sum just like  $W_t$ . However, this simple rescaling procedure changes the standard deviations for each rescaled variable and the interpretation of each regression coefficient. A more traditional remedy is to transform (8) so that "normalized" regression coefficients are being estimated instead of "regular" regression coefficients (Pindyck and Rubinfeld, 1981, formula (4.18), p.90):

$$Y_t = \beta \cdot (X_t - X_{\text{bar}}) \cdot (S_y/S_x) + \gamma \cdot (Z_t - Z_{\text{bar}}) \cdot (S_y/S_z) + \delta \cdot (W_t - W_{\text{bar}}) \cdot (S_y/S_w) \quad (12)$$

where  $S_y, S_x,$  and  $S_z$  are the sample standard deviations. There must be some inequality in the  $W_t$  weights or else  $\delta$  will not be estimatable due to implicit perfect multicollinearity with the intercept. The "normalized" regression coefficients all have unit variance and zero mean so that their relative importance in influencing crop yield can be easily determined. For a given  $S_x$  "distance" in  $X_t$  the  $\beta$  coefficient indicates the amount of  $S_y$  "distance" determined in the dependent variable. Also, for a given  $S_z$  "distance" in  $Z_t$  the  $\gamma$  coefficient reveals the amount of  $S_y$  "distance" explained. Now replacing  $W_{\text{bar}}$  in (12) with  $1/n$  as an "operational hypothesis" with the actually estimated  $S_w$  value still used, then for a given  $S_w$  "distance" in  $W_t$  the  $\delta$  coefficient provides the amount of  $S_y$  "distance" explained by this "intangible" input variable. The reverse transformation from "normalized" regression coefficients to the "regular" regression coefficients with the interpretation in (8) can be easily obtained as follows (Johnson, 1971, formula (5.31) p. 133):

$$b = \beta \cdot (S_x/S_y); \quad c = \gamma \cdot (S_z/S_y); \quad d = \delta \cdot (S_w/S_y) \quad \text{and} \quad (13a)$$

$$a = Y_{\text{bar}} - b \cdot X_{\text{bar}} - c \cdot Z_{\text{bar}} - d \cdot W_{\text{bar}}. \quad (13b)$$

### Summary

This paper has shown that AHP can be used to extend the ordinal ranking among the categories for a DEA input categorical variable to provide a decision rule for its use as an intangible input (i.e., growing conditions). Also, this paper has demonstrated that the traditional treatment of intangible inputs by the use of dummy variables can be arbitrary and misleading when the DM is ordinally inconsistent. The AHP procedure elicits more judgment data from the DM in such a way as to provide "operational" empirical tests as to the severity of any detected ordinal inconsistency. The resulting indirect measurements for the intangible inputs can be used in regression, DEA, and RSE analysis MORE effectively than the dummy variables that are conventionally used!

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