THE METHOD FOR MIXED-ATTRIBUTES GROUP AHP AND ITS SOFTWARE IN COMMON USE

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ABSTRACT

This paper discusses the method for group AHP including different attributes qualitative, quantitative, and probable in detail and gives a software in common use. It has been proved that the method is effective and practical in solving a large-scale and complex group AHP decision making problems.

I. Introduction

As a measure of social economic system and a simple and practical method in the realm of decision making, AHP has been paid attention to by more and more people. But with the extending of its application extends the problems as follows will be found frequently in decision analysis: 1. For a complex decision analysis problem, a group of experts will often be employed to construct the comparison matrices so as to avoid mistakes and make the result more accuracy. But how to ensure the results of analysis to be convergent and reasonable and enable decision makers not to step back because of the complicated computation? 2. One of the advantage of AHP is that it is covenient to deal with "quantitative factors" and "qualitative-factors" at the same time. How to do that in a large hierarchy structure, which usually includes both of these two factors in application, and make it "succession and link up" in computation? 3. For important but undetermined criteria, including probable and "section-definite", such as the geologic conditions in coal mine construction etc., how to construct the comparison matrices under these criteria, and what is the difference between probable judgment and Fuzzy judgement if we introduce probable factors into comparison? In dealing with the problems above, the author has done some beneficial study and combining with his practical experience and given a simple instruction of "software in common use for mixed-attribute group AHP".

II. The construction of comparison matrix in group AHP

A group AHP includes two steps generally. One is to construct a hierarchy for a problem and the other is to make comparison matrix. For the first step we can refer to reference [3] and here we will only discuss the construction method of the matrix. There are several ways at present.

1. Geometrical mean for elements of comparison matrix. [5]
Supposing the matrices given by n experts are:

A = (a ,k), k=1,2,3,...n; (1)

Then let synthetic comparison matrix

A = (a) (2)

and the elements of A are:

a = a (3)

= 1

The number is the priority weight of the kth expert according to his ability. then apply the eigenvetor method to compute the priority of A:

2. Geometrical mean for priority weights

There are geometrical mean for both simple weights and composite weights, here means the former.
Supposing the priority vector given by the kth expert is:

W = (w , w ,...., w) k=1,2....n; (4)

Then let the synthetic priority vector:

W = (w , w ,w ,...., w) (5)

and W = W , , i=1,2,...m; (6)

The meaning of is as above.

3. Take the weighted arithmetical mean instead of geometrical mean in 1. That is

a = (7)

4. Take the weighted arithmetical mean instead of geometrical mean in 2. That is

W = w i=1,2,3,.....,m (8)

Although these methods have the advantages of more perfect theory [2] and larger information capacity, they have the problems as follows to different degrees in application.

1. Because the 1st and the 3rd method cover the fact that the comparison matrices given by each expert may not meet the requirement of consistency (some experts may far deviate from the consistency, but the synthetic matrix may meet consistency index), so that although the eigenvalue method has been proved the best method to bring out the priority weights [1], it cannot be used in this case simply. Moreover, when some synthetic matrices in the model be found not meeting the consistency index, the computation of all the model must be started again.

2. For the 2nd and the 4th method, if some comparison matrices given by experts are unconsistent, the computation is complicated, it is difficult to repeat on computer.

3. One expert can only make model one time no matter which one of these methods be employed, and the feedback is not considered, so that if an expert has done some mistaken judgment because he is not familiar with AHP or other cause, there will be no chance to make it correct and may cause the final result not converging to its objective priority sequence (This objective priority always exists in the inner link of the bject, what can we do is to make the computation priority converging to this objective value as near as possible), Therefore these above methods can't fully develop the superiority of group AHP to obtain information from experts to the greatest extent.

In order to make up for the insufficiency, the author suggests Delphi Method be used to construct the matrices, which involve mainly steps as follows:

Step 1. Determine the number n of experts according to the scale and property of the question, and notice their different fields and rank.

Step 2. Give the weights to each expert,, k=1,2,...,n.

Step 3. Give the AHP model and its blank comparison matrices (or designed it to a "0-1" alternative form) to each of the n experts to judge. So for each attribute we can receive n pieces of different comparison matrices and give it a serial number. Then do the data processing below:

a a (9)

(a -a) (10)

a is the judging value for element in the ith row and jth column of the matrix given by the kth expert, and a , is respectively the weight arithmetical mean and the standard deviation of the value. Taking the error for the a with , we think it meet the requirement of convergence and take it as the corresponding element of the synthetica comparison matrix. Otherwise, for the a with , rejudgement must be done.

Step 4. The comparison matrices which have been treated in above steps are given again to the experts and the elements which meet the requirement should not be filled, but the a and of the elements, which do not meet the requirement, must be filled in the corresponding positions of the matrices to provide a feedback information to the experts, so that they can correct their judgment according to the means and the deviations.

Step 5. Return to step 3.

The process will be repeated until the requirement is met and then we can obtain the synthetica comparison matrices with accuracy and consistency being raised greatly.

I find in practice adopting Delphi Method to construct the synthetica comparison matrix can basically compensate the weaknesses of four methods mentioned above. First, because Delphi Method is set up based on experts group judgment and information feedback, it can receive information as much as possible. Next, the unconsistency of the experts can be corrected frequently according to weighted means and standard diviation feedbacked, and make the synthetica comparison matrix to trend towards (only "to close") its limit ---- complete consistency. Moreover, it can avoid leading "pseudconsistency" of synthetica matrix by the unconsistency of some experts. I have made 40 synthetica comparison matrices with this method., employing 15 experts. None showed that C.I. 0.1. Third, the whole process can be carried out by computer and only one operation is required to the whole model.

III. The construction of the comparison matrix under quantitative attributes

If all of the attributes in a hierarchy model are qualitative then our problems have been solved. But it is possible that both qualitative attributes and quantitative attributes exist in the same level in a practical model. For example, when we select construction plan for a coal mine, there are many attributes in the criteria level, such as "security" and "speed", "quality" and "cost", ect. How to deal with them in a model at the same time and keep up "succession and link up " in quantity in calculation? I think that the qualitative attributes in such a problem can also be solved with group AHP as mentioned above, but for the quantitative attributes there are two of alternatives described below. The first alternative is to make the unitized measured-value of the quantitative attribute as the simple priority result under the criteria, so we can "leap over" calculation the comparison matrix under the criteria. The second is to adopt the following formula to change the measured-value into the form of comparison matrix and then calculate the priority vector. the practical moves are as follows:

1. Assume B is a measurable attribute under a criteria A, and the value for each B , i=1,2,...,n, to be ;

2. Let

max[/i=1,2,...,n] (11)

min[/i=1,2,...,n] (12)

calculate

$$= \quad /9 \quad (13)$$

3. Let

$$= \text{------------} \quad i,j=1,2,\ldots,n \quad (14)$$

Then the value of element b in the comparison matrix correspond to the comparison between arbitrary B and B is taken as follows: when !r ! 1, the b =1; when r 1, then b =r ; when r -1, then b = -1/r . In addition, according to the character of comparison matrix, we can obtain

$$b = 1/b \quad , b = 1.$$

Example 1: (1) Assume the quantitative attributes concerned in speed S are plans P P , the relative value is V =43.5, V=59.2, V =63.3, V = 65.0, V =71.2;

(2) Let

$$V = \max [V / i=1,\ldots,5] = 71.2,$$

$$V = \min [V / i=1,\ldots,5] = 43.5,$$

then

$$= \frac{V - V}{\quad} = 3.08,$$

(3) Calculate element S in the "comparison matrix":

$$r = \frac{V - V}{\quad} = \frac{43.5 - 59.2}{3.08} = -5.1 \quad -1.$$

$$S = -1/r = 1 / 5.1$$

The others are on the analogy of this. Finally we can obtain the "Comparison matrix" as shown in Table 1.

Table 1.

S	P	P	P	P	P
P	1	1/5.1	1/6.43	1/7	1/9
P	5.1	1	1/1.33	1/1.88	1/4
P	6.43	1.33	1	1	1/2.6
P	7	1.88	1	1	1/2
P	9	4	2.6	2	1

The first alternative appears simple and practical. But compared with the second alternative, it includes a lot of unreasonable factors. First of all, as a comparisonable quanlitative attributes, the values of the attributes have reflected their importance. If we only make the single scale comparison judgment, the first alternative is useful. But as there both quantitative and qualitative attributes, the result of the first alternative would lead to confussion at times and does not reflect the real objective priority. Next, because the scale of quantitative attribute has its own physical meaning, the priority calculated according to the first alternative also has practical meaning, which reflects the proportional relation among attributes in quantity directly. On the other hand, the scale of qualitative attribute in AHP is only a prescriptive scale, and has no practical meaning, so its lead-scale, namely priority weights, also has no practical proportional meaning [1]. There is no "succession and link-up" in calculation between the first alternative and other qualitative attributes. But the second alternative adopts relative conversion formula to change the quantitative scale into the form of "comparison matrix" corresponding to qualitative attribute, so it has "succession and link-up" in calculation. Third, from the point of view decision making, if the value in some plans are very close to each other and we have to select one or two plans from them, the result given by the second alternative is more helpful to the decision maker than the result given by the first alternative. Let us take the priority of the 5 plans in Example 1 as a example, the results of priority given by both the first and second alternatives separately are shown in Table 2.

Table 2

	P	P	P	P	P
The 1st alternative	0.1439	0.1959	0.2095	0.2151	0.2356
The 2nd alternative	0.0330	0.1313	0.1897	0.2170	0.4291

Obviously, the interval of priority weights in the 2nd alternative is extended, that is much more helpful to the decision maker. when we conduct a complex multilevel composite priority, the conclusion is significant. Finally, let us take an initial approach in keeping-rank. Because there is no comparison matrix in the 1st alternative, and the "comparison matrix" in the 2nd alternative is complete consistent (strictly speaking, the matrix is not complete consistent and has a little perturbation because of taking approximate value), so the keeping-rank under single criteria for the both alternatives is tenable. But there is a certain difference between the 1st and 2nd alternatives for the keeping-rank of composition priority. In theory, the greater the difference of importance among plans, the better the keeping-rank. If the difference of the factors is little, the difference of importance (priority weights) of the factors given by the 1st alternative is also little. But the intensity of importance can be extended when using the 2nd alternative, so keeping-rank of composition priority of the 2nd alternative is better than the 1st. When doing an experiment employing a new plan, I found the rank reversal occur if the 1st alternative was adopted. but if the 2nd alternative was adopted, the rank reversal didn't occur.

As mentioned above, if the attribute in a problem is quantitative, we can change it into the form of comparison matrix with the 2nd alternative and calculate the priority vector. The process has been developed into the software in common use discussed below.

People would think that it is better to treat the quantitative attribute as a qualitative directly, for example. there are Plan A, B and C, the economic results are 40, 30, and 10 thousand yuan respectively. We can say that the economic results of Plan A is slightly better than Plan B by a value 3, Plan B is obviously better than Plan C by a value 5, and so on. The author think that AHP mainly solve the measure problem among the cannot-be-measured attributes by means of comparing each two attributes, and there are some errors in the comparing always. If some attributes have had concrete measured-value, and it is assurable that this measured-value has "succession and link-up" with other attributes in calculation, then why do not we use the more accurate information directly? Why do we make it fuzzy?

IV. Dealing with undetermined attributes

UNdetermined attributes mainly include two kinds of attributes as follows: One is probable, that is the factors which under an attribute appearing in probable form. The water inflow given by the geological report is an example. Another is section-definite attribute, that is the factors which under an attribute having different value in different section. The rock conditions given by geological drill-log is an example. We often meet with these two kinds of attributes in decision analysis of social economic system, optimum planning, and alternative selection, etc. If they are treated as definite attribute, there will be mistakes. Take the selection of mine construction plan as an example. It is not unusual that at first people select a construction plan based on little water inflow, but the real water inflow in construction is much more, so the plan selected is not suitable, leading to enforced idleness or doing the work again.

Then how should we deal with these two kinds of attributes? Obviously, these attributes are different from so-called fuzzy factor (the intension and the extension of fuzzy factor are both fuzzy, they do not obey a certain distribution and are not section-definite), so adopting fuzzy judging is neither suitable nor necessary, on the contrary, it bring about another link. The methods of dealing with these two kinds of attributes combining with real example are discussed respectively as follows:

1. Probable attribute

(1) When we deal with an uncontinuous probable attribute, for

example, the distribution law of the water inflow X of a mine is shown in Table 3. We take the probability P as the result of simple priority of the "comparison matrix" under the water inflow criteria, and need not build the "matrix" of the criteria. It should be noted that this is different from the 1st alternative of quantitative attribute. Because the construction of the comparison matrix reflects people's subjective judgment to the objective reality, the subjective judgment should in certain degree reflect the objective reality correctly. So we can say that, in fact, the priority result is the subjective probability to a objective reality. [4] Now we replace subjective probability with objective probability, it is simple and reasonable.

Table 3.

x	x 10	10 x 25	25 x 40	x 40	m /h
P	0.1	0.5	0.3	0.1	

(2) When we deal with a continuous probable attribute, obeying a certain distribution, for example, the degree X of air pollution varies as the distance to the pollution source obeying a distribution: X f(x), for the sake of simplification, we can make the continuous distribution dispersed, that is to devide the field of definition into n sub-intervals (x , x), i=1,2,...,n, and to calculate the probable value p in each sub-interval to X:

$$P = P [x \quad X \quad x] = \quad f(x)dx, \quad i=1,2,...,n; \tag{15}$$

then, solve it using the method dealing with the uncontinuous attribute.

2. Section-definite attribute

We sometimes deal with an attribute whose measured value varies as time goes on and space changes. For example, the properties of the rock in a mine shaft are different at different depths, so the rock conditions are also different at different depths. According to what kinds of condition should we select the construction plan and equipment? We can classify the rock conditions into several types, add up the height of each type of rock conditions according to the geological drill-log (see Table 4), then take the height as the values of the rock conditions respectively and treat them with the 2nd alternative to quantitative attribute mentioned earlier. Thus we can select construction plan and equipment more comprehensive and more reasonable than according to only one condition.

Table 4.

Rock conditions	Very good	Good	General	no good	Shaft depth
Adding up Height (m)	50	150	200	50	450 (m)

V. The functions and instructions of "the software in common use".

On the basis of discussion above, I have developed "a software in common use for mixed-attribute group AHP", which is called "SMAG-AHP" for short, on the IBM microcomputer. Because of the limit of the length of the paper, the details of the SMAG-AHP are not introduced hear. The readers who are interested in it may contact with the author directly.

VI. An application example

Let us take the optimizing construction method of the shaft of a coal mine as a practical example, its simplified hierarchy model is shown in Fig. 1.

Fig. 1 Optimizing for Shaft Construction

Selecting the Method A

B1 Multiple Profit

B2 Restraints

Speed Cost Quality Safety Water Inflow Rock Conditions Shaft Feature

much middling small no water most steady general not steady depth diameter

mixed-operations no-parallel operation parallel on reverse direction parallel on same direction others

The model consists of five levels. The highest level (A) is the target level, that is to optimize the construction method. The lowest level (P) is the alternative level. There are alternatives to be selected. The other three levels are criteria level and restrain level. In the model, C and C are quantitative attributes (for the construction speed and cost of each alternative, we can refer to " the quota of time limit" for a project), C (water inflow of the shaft) and C (rock conditions) are probable attribute and section-definite attribute respectively (given by the geologic report), the others are qualitative attributes.

The author optimized the construction method of the ventilation shaft of Chen Zhuang Coal Mine, Shanxi Province, using the methods given by this paper and "SMAG-AHP" software. The composite priority weights of the alternatives are: P (0.3839) P (0.2156) P (0.1627) P (0.1328) P (0.1050). Obviously, the composite priority weight of P is the heaviest, so we should give a priority to select the mixed-operation method (p) to construct the shaft.

This paper summarizes my experience in application of AHP in recent years. It is not systematically theorized yet. There must be some mistakes in the study. The readers' advice is earnestly requested.

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