PRODUCT METHOD OF AHP

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ABSTRACT

In this article, we give out a new method of AHP, that is the product method on which some new concepts and basic procedures are touched. We finally discuss some special characters of this mothod. The decision alternatives are independent of criteria, and the concepts of relative priority of alternatives and the alternative weight can be set apart. The addition or subtraction of decision alternative does not influence on the relative priority of the various decision alternatives, but a new optimum decision alternative derived from the convex combination of original decision alternatives can be obtained as well.

I. Introduction

The AHP method has succeeded in multicriteria and multiobjective decision-making. But some concepts in AHP method are not clear enough and should be revised.

First, the concepts of priority and weight. In AHP the unit eigenvectors of comparison matrices are emploied not only to describe the order of criteria in the same layer but also to calculate the criterion weight for the higher level. This means that the priority is equal to weight.

Second, the relation between alternatives and criteria. In the hierarchy structure, there is no difference between alternative layer and criterion layers. It deals with the alternative layer as a general criterion layer.

Based on these concepts we are restricted in a close system to solve problem because we use unit vector to represent the importance of alternatives respect with to criterion that means we need not add or delete any alternative. In fact, this not true. Usually we can not list all alternatives. Sometimes it is good enough to solve problems in a subset of alternatives. In this paper we give out a new method of AHP as an attempt to clarify these questions.

II. Concepts and Steps

We give out the concepts and steps of the new method as following:
Definition 1 Attribute Layer.

The attribute layer is the lowest layer of the subcriteria in hierarchy structure.

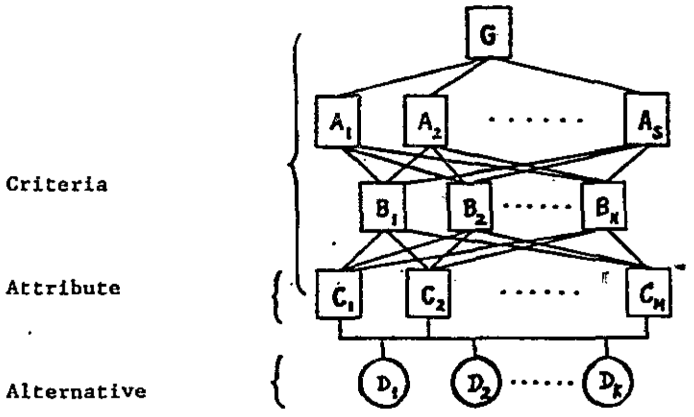

figure 1

Définition 2 Attribute Vector of Alternative

An attribute vector is a vector in which every component represents a quality of alternative.

Definition 3 Criterion Value (of An Alternative)

For a certain alternative, a criterion value (of this alternative) is a number which can only be calculated from its subcriterion values or it is a component of the attribute vector of this alternative if the criterion belongs to the attribute layer.

Definition 4 Alternative Importance

The alternative importance is the general criterion (the top criterion) value of this decision alternative.

Definition 5 Relative Priority of Alternatives

To order the alternative importances decreasingly, the relative relations of alternatives are their relative priority.

Definition 6 Alternative Weight

Let D_1, D_2, ..., D_k be decision alternatives which are concerned by us, if a new decision alternative $x_1D_1+x_2D_2+\ldots+x_kD_k$ takes the maximum general criterion value, for all $x_1+x_2+\ldots+x_k=1$ and $x_k \geq 0$, then we will name x_k being weight of alternative D_k.

The mew method contains 5 steps:

Step 1. Setting Up the Hierarchy

It is similar to the classical AHP method. But the decision alternative layer is quite different from the classical method. It is independent of the criterion structure. The criteria are rules which describe the process of analyzing problem. The decision alternatives are the methods which can be chosen to solve problems. We will see the difference later.

Step 2. Getting the Value of Attribute Vector

Every decision alternative has an attribute vector. If the component of the attribute vector can be measured (such as: size, velocity, temperature, etc.), we can use the value directly. If the component of the attribute vector can not be measured or we do not want to employ the measuring value, we can get the component value employing eigenvector of pairwise comparison matrix or other methods such as DELPHY method too. But it is not necessary to make the vector being a unit vector. In order to make that be convenient in practice, we restrict the components of attribute vector in non-negative numbers, the bigger the value is, the better the quality is. If some components do not agree with the appointment, we can transform the values to obey the stipulation.

Step 3. Calculating the Criterion Values

We can get every criterion value of a alternative step by step. We describe the process recursively. For a certain criterion A_j, we know its subcriteria $B_1, B_2, \cdots, B_N$. Comparing the importance of these subcriteria with classical AHP method: making the pairwise comparison matrix, unit eigenvector can be got. Let $(\alpha_1, \alpha_2, \ldots, \alpha_N)$ be unit eigenvector, be the value of subcriterion B_i, then the value of criterion A_j can be got from the following formula:

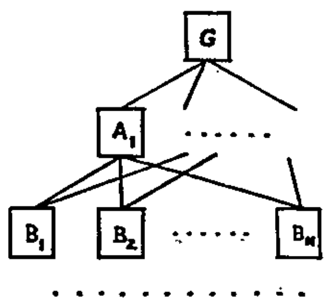

figure 2

$$a_j = \prod_{i=1}^{N} \alpha_i \qquad (1)$$

After we get the attribute vector, we can calculate the criterion values layer by layer up to the top--general criterion.

Assuming there are M attributes $C_1, C_2, \ldots, C_M$. For kth decision alternative, let vector

$$(f_1^k, f_2^k, \cdots, f_M^k)^T$$

be its attribute vector of D_k. The general criterion value will be

$$F^k = \prod_{i=1}^{M} (f_i^k)^{\beta_i} \qquad (2)$$

This formula is a composition of functions got from formulas (1). In fact, to calculate the general criterion value, we should get every unit eigenvector first, then get exp-vector

$$(\beta_1, \beta_2, \cdots, \beta_M)^T$$

through matrices product. It is not nacessary to calculate every subcriterion values layer by layer.

Step 4. Getting the Relative Priority of Decision Alternatives

From Step 3, we can get every value of decision alternative preference. Arranging the alternative priorities decreasingly we can get the relative priority of decision alternatives.

Step 5. Getting the Weight of a Decision Alternative

Let

$$D_1, D_2, \cdots, D_K$$

be a set candidate decision alternatives, x_k be weight of D_k, solving the following programming problem

$$\max \ \left(\sum_{k=1}^{K} x_k f_1^k \right)^{\beta_1} \cdots \left(\sum_{k=1}^{K} x_k f_M^k \right)^{\beta_M}$$

$$\text{s.t.} \quad x_1 + x_2 + \cdots x_K = 1$$

$$x_k \geqslant 0 \qquad (k=1,2,\ldots,K)$$

we get the values of x_k.

Usually this programming problem is a nonlinear programming, but it can be solved by Personal Computer employing MONTE CARLO or other methods easily.

III. Some Characters of the Product Method

There are some characters of the new method.

1. Independence of the Hierarchy Structure

Here the word "independence" contains two meannings. First, the decision alternatives are independent of all criteria, so it does not change relative priority of alternatives to add or delete decision alternatives. Second, the criterion values of the same layer are independent. If we add or delete some criteria in a layer and do not change their subcriteria, the criterion values in this layer will not change. These can be verified by formulas in step 3.

2. The Independence of the Attribute Values

It does not matter that we can choose any unit as a standard of measuring attribute value, i.e. it does not change the results to increase or decrease any kind of attribute values in proportion. So when we use comparison matrix calculate attribute values, we needn't make the eigenvector be unit vector.

3. Distinguish the Concepts of Priority and Weight Strictly

There are some relationship between relative priority and weight, but they are not equal. Sometimes we can get relative priority from weights, sometimes we can not, becaase different alternatives can have the same weight (see example 1). On the other hand. if we have the relative priority only, we can not get the weights directly. The new concepts of relative priority and weight are more suitable for practice.

4. An Optimum Decision Alternative Can Be Got

If the linear combination of decision alternatives can be allowed, we can get a new alternative which makes the benefit of the whole system maximum. The optimum decision alternative is very useful. It gives us not only the best result but also expand the range in which alternative can be selected. In optimum alternative, the weight of a origenal alternative can be explained as per cent of this alternative in random decision problems. So in random decision problems we can give mixed strategy with these probabilities.

IV. EXAMPLES

Example 1.

A university select students according to two scores. One is the intelligence, another is health. The standards are: excellent, 5; good, 4; prety, 3. If there are five students want to enroll the university, their scores are listed in table 1:

Student Score	A	B	C	D	E
Intelligence	5	4	3	3	4
health	3	3	3	5	5

table 1

We make the hierarchy strictire as figiue 3.

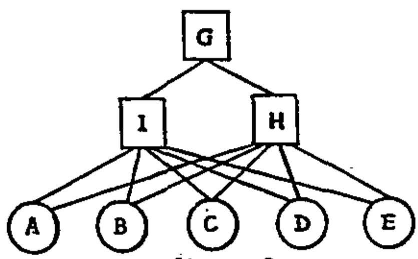

figure 3.

There are two layers of criteria. G is the general criterion. I and H are G's subcriteria, they belong to attribute layer. Every student has a attribute vector listing in table 1 (column vector).

Assuming the pairwise comparison matrix and its results are listing in table 2.

	I	H	unit eigenvector
I	1	1.5	0.6
H	1/1.5	1	0.4

table 2

We use (i,h) to describe a student's attribute vector, then $i^{0.6}h^{0.4}$ is its importance. We can get these values of students A, B, C, D, E as 4.178, 3.849, 3.485, 3.837, 4.201 respectively. The relative priority of the five students are as E, A, B, D, C. The university should accept the students according to the order. If A, B, C, D, E are not five students. They are five kinds of students coming from different schools. And in the same school there is no difference between any two students. How can we select students to make the new classes that be both intelligent and health?

Let x_i be per cent which we want to select from ith school then we can solve the progamming problem:

$$\max\ (5x_1+4x_2+3x_3+3x_4+4x_5)^{0.6}(3x_1+3x_2+3x_3+5x_4+5x_5)^{0.4}$$
$$\text{s.t.}\quad x_1+x_2+x_3+x_4+x_5=1$$
$$x_i \geqslant 0 \qquad (\ i = 1,2,\dots,5\ )$$

We got $x_1= 0.35$, $x_5= 0.65$, $x_2=x_3=x_4= 0$.

So we should accept 35% students from 1st school, 65% students from 5th school. This is the best method in these kinds of problems.

Example 2

Assuming we get a hierarchy structure like figure 4. For every criterion we get unit eigenvectors as

$A, (0.6,\ 0.4)^T$; $B_1, (0.1,0.3,0.2,0.4)^T$; $B_2, (0.2,0.5,0.2,0.4)^T$

Then the alternative importance formula is

$$F = (f_1)^{\beta_1}(f_2)^{\beta_2}(f_3)^{\beta_3}(f_4)^{\beta_4} \qquad (3)$$

In this formula

$$\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\\ \beta_4\end{pmatrix} = \begin{pmatrix}0.1 & 0.2\\ 0.3 & 0.5\\ 0.2 & 0.1\\ 0.4 & 0.2\end{pmatrix}\begin{pmatrix}0.6\\ 0.4\end{pmatrix} = \begin{pmatrix}0.14\\ 0.38\\ 0.16\\ 0.32\end{pmatrix}$$

(f_1,f_2,f_3,f_4) is an attribute vector of an alternative.

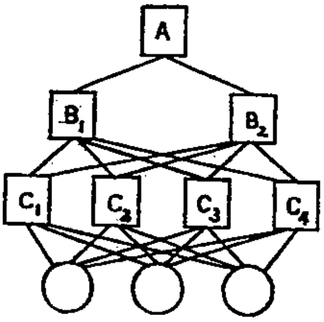

figure 4

V. CONCLUSION

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In this paper we have already given out a new method for AHP, listed its steps and discussed its characters. Through the new method we can distinguish the alternative layer from criterion layers in hierarchy structure, and make differences between relative priority of alternatives and alternative weight. We can get the best alternative in an expanded scope.

The new method was born, yet we need more experience in its applications.

Some questions should be paid attention to, when we use this method. First, for a certain alternative the criterion values are got from product of its subcriterion values. If a subcriterion value is zero the criterion value will be zero too. So the alternative importance (i.e. general criterion value) will be zero. If we do not want such condition, we should use a very small positive number instead of the zero criterion value. Second, the scale for pairwise comparison of the new method should be distinguished from classical method.

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