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Error Estimation by Bootstrap in AHP

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abstract

The bootstrap[Efrom,1979] is an error estimation method originated in statistics. It is especially useful in the field with complicated structured models.

The weights of objects (alternatives or criteria) in AHP analyses[Saaty,1980] are mainly calculated through eigen vectors, and its analytical error estimation are usually very difficult or almost impossible. So we propose its error estimation methods by bootstrap.

The essence of bootstrap is stated through group decision process in AHP.

Firstly we describe group decision in AHP. n persons evaluate m objects through AHP. Person k constructs the $m \times m$ comparison matrix $\boldsymbol{A}(k) = [a_{ij}(k)](k = 1 \sim n)$, from which we calculate the averaged weight vector $\bar{\boldsymbol{u}}$.

There are three group decision methods to have $\bar{\boldsymbol{u}}$.

Method I : Let $\bar{\boldsymbol{A}} = [\bar{a}_{ij}]$ where $\bar{a}_{ij}$ is the geometric mean of $a_{ij}(1), \cdots, a_{ij}(n)$, and $\bar{\boldsymbol{u}}$ be the principal eigen vector of $\bar{\boldsymbol{A}}$.

Method II : Let $\boldsymbol{u}(k)$ be the principal eigen vector of $\boldsymbol{A}(k)(k = 1 \sim n)$ and $\bar{\boldsymbol{u}}$ be the component-wise geometric mean of $\boldsymbol{u}(1), \cdots, \boldsymbol{u}(n)$.

Method III : Let $\boldsymbol{u}(k)$ be the principal eigen vector of $\boldsymbol{A}(k)$. And let $\hat{w}_k$ be $w_k(> 0)(k = 1 \sim n)$ minimizing $\sum_i \sum_k (w_k - \sum_l w_l u_i(l)/n)^2$ subject to $w_1 + \cdots + w_k = 1$, and $\bar{\boldsymbol{u}} = \sum \hat{w}_k \boldsymbol{u}(k)$ [Nakanishi,1999].

The error estimation by bootstrap is as follows;

(1) For each $(i, j)(i, j = 1 \sim m, i < j)$ select randomly one of $a_{ij}(1), \cdots, a_{ij}(n)$ and denote it r_{ij}. Then construct $\boldsymbol{R}$=$[r_{ij}]$ (for $i > j$, $r_{ij} = 1/r_{ji}, r_{ii} = 1$).

(2) Repeating process (1) by n times, we have n matrices like $\boldsymbol{R}$, which are denoted by $\boldsymbol{R}(1)$, $\cdots$, $\boldsymbol{R}(n)$. $\boldsymbol{R}$=$\{\boldsymbol{R}(1), \cdots, \boldsymbol{R}(n)\}$ is a kind of random sample from $\boldsymbol{A}$=$\{\boldsymbol{A}(1), \cdots, \boldsymbol{A}(n)\}$.

(3) Further repeating process (2) many (T) times, we have T sets of R-type matrices $\boldsymbol{R}_1, \cdots, \boldsymbol{R}_T$.

(4) For each $\boldsymbol{R}_t(t = 1 \sim T)$ we calculate averaged weight vector $\bar{\boldsymbol{u}}_t$ by one of the above mentioned averaging methods.

(5) Let $\bar{\bar{\boldsymbol{u}}} = \sum_t \bar{\boldsymbol{u}}_t/T \ (= [\bar{\bar{u}}_1, \cdots, \bar{\bar{u}}_m]^T)$, then $\bar{\bar{u}}_i$ is an estimate of weight u_i of *i-th* object $(i = 1 \sim m)$. And

$$s_{u_i} = \sqrt{\sum_t (\bar{u}_{ti} - \bar{\bar{u}}_i)^2/T}$$

($\bar{u}_{ti}$ is *i-th* component of $\bar{\boldsymbol{u}}_t$) is the standard error of $\bar{\bar{u}}_i (i = 1 \sim m)$.

By simulation we compare the above methods I, II and III. Firstly we set the virtual value of u_i and

$$a_{ij}(k) = \frac{u_i}{u_j} e_{ij}(k)$$

as (i, j) element of comparison matrix $\boldsymbol{A}(k)$, where $e_{ij}(k)$ is a log normal random number with $V(\log e_{ij}(k)) = \sigma^2$.

And for each of methods of I, II and III, we estimate true u_i and stand error σ by the bootstrap method. From the results of the simulation we can conclude that method III is the best group decision method of the above mentioned three methods.

References

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