COMPARISONS OF THE AHP AND THE TRADITIONAL METHODS

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ABSTRACT

Usually, it's very difficult to make a decision based on merely the objective data except the intangible factors. There are many intangible factors such as preference or feeling to be considered.
We think that the AHP might be a powerful tool for such kinds of decision. However, many traditional methods have been applied for such decisions.
The purpose of this study is to compare the AHP with these traditional methods and to make clear the merits and demerits with respect to both methods.
Traditional methods We compared here are as follows:
(1) The Churchman's and Ackoff's weighting objectives (Churchman and Ackoff 1954). This methods has been applied mainly in a management science field so far.
(2) The Scheffé's method (Scheffé 1952). This methods is based on a paired comparison and a analysis of variance. This method has been applied mainly in a sensory test field.
Both the Churchman's and Ackoff's methods and the AHP were applied for a problem of the travel courses selection. Both the Scheffé's method and the AHP were applied for a problem of icecream brands selection.
The results of these comparisons almost coincided each other. However, in the examining the Churchman's and Ackoff's method a lack of consistency of result was observed. In the comparing with the Scheffé's method more hierarchical and structural advantage of the AHP was experienced.

A COMPARISON OF THE AHP AND THE CHURCHMAN'S AND ACKOFF'S METHOD

An object of a comparison

As an object of a comparison, a selection of the travel courses in Hokkaido was taken as shown in Figure 1.
This problem was to decide the preferable ranking among the following four courses.
The central course (Sapporo, Shakotan peninsula etc.)
The southern course (Hakodate, Okushiri island etc.)
The northern course (Asahikawa, Rebun island etc.)
The eastern course (Nemuro, Abashiri, Kushiro etc.)

Application of the AHP

The travel courses selection hierarchy of the AHP is shown in Figure 2. For this problem , three students compared respectively on each hierarchical items by means of the AHP.
Table 1 shows Mr. X's pairwise comparison and weights with respect to overall criteria. Table 2 shows Mr. X's pairwise comparison and weight

with respect to historical spots. Table 3 shows Mr. X's overall rating with respect to each courses. Mr.Y's and Mr. Z's similar tables are omitted in this paper. Table 4 shows the overall rating integrated above all student's judgments. From this rating, it became clear that the central course of Hokkaido was judged to have the highest preference.

Application of the Churchman's and Ackoff's method

The Churchman's and Ackoff's method was applied for same problem.
This method is summarized as follows:

Step 1 : Rank the criteria in their order of value. Let O_1 represent the most valued, O_2 the next most important, and O_m the least important.

Step 2 : Assign the value 100 to O_1 (i.e., v_1=100) and assign values that appear suitable to each of the other criteria. The values $(v_1,\dots,v_m)$ of each criteria $(O_1,\dots,O_m)$ have following relation.

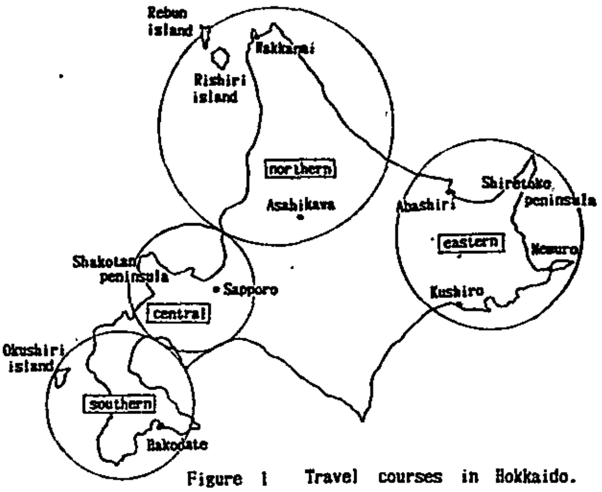

Figure 1 Travel courses in Hokkaido.

$$v_1(=100) > v_2 > \cdots > v_{m-1} > v_m$$

Step 3 : Compare O_1 versus $O_2 + O_3 + \cdots + O_m$.
(1) If O_1 is preferable to $O_2 + O_3 + \cdots + O_m$, adjust (if necessary) the values of v_1 so that $v_1 > v_2 + v_3 + \cdots + v_m$. In this adjustment, as in all others, attempt to keep the relative values of the adjusted group (v_2, v_3 etc.) invariant. Proceed to Step 4.
(2) If O_1 and $O_2 + O_3 + \cdots + O_m$ are equally preferred, adjust (if necessary) the values of v_1 so that $v_1 = v_2 + v_3 + \cdots + v_m$. Proceed to Step 4.
(3) If O_1 preferred less than $O_2 + O_3 + \cdots + O_m$ adjust (if necessary) the values of v_1 so that $v_1 < v_2 + v_3 + \cdots + v_m$. Then compare O_1 versus $O_2 + O_3 + \cdots + O_{m-1}$, $O_2 + O_3 + \cdots + O_{m-2}$, etc. until either O_1 is preferred to the rest or until the comparison of O_1 versus $O_2 + O_3$ is completed.

Step 4 : Compare O_2 versus $O_3 + O_4 + \cdots + O_m$ and proceed as in Step 3.

Step 5 : Continue until the comparison of O_{m-2} versus $O_{m-1} + O_m$ is completed.

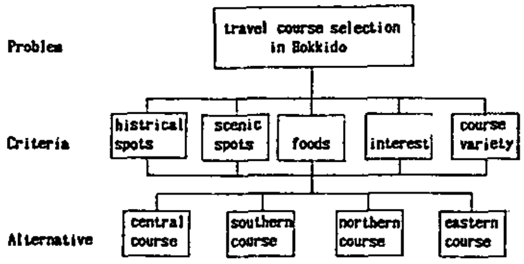

Figure 2 Course selection hierarchy.

Step 6 : Convert each v_j into a normalized value v_j', dividing it by Σv_j. Then $\Sigma v_j'$ should be equal to 1.00.

As described above, the additivity assumption is essential to this method.
That is , the total value for all criteria should be equal to the sum of the value of each criterion.
The criteria of this problem : a richness of historical spots, scenic spots, and a sort of foods, a intensity of interest, and a variety of course etc. are independent each other. The additivity assumption on this problem seems to be true.
Now the rating for criteria by individual members was done with above steps. The criterion at the head of the list was assigned a value of 100 and the remainder placed in a descending order with numerical values proportionate with the first criterion. This assessment is shown as Table 5. Next, from these value for each criterion by each individuals the average was taken as the value of group.
Dividing by summation of these averages, a normalized value for each criterion was obtained. The most valued one was assigned the value 100 as shown in Table 6.
New variables (A,B,C,

TABLE 1 Mr.X's Pairwise Comparison Matrix and Weights

	Historical spots	Scenic spots	Foods	Interest	Variety	Weights
Historical spots	1	1/3	1/6	1/7	1/8	0.033
Scenic spots	3	1	1/3	1/6	1/7	0.060
Foods	6	3	1	1/3	1/6	0.127
Interest	7	6	3	1	1/3	0.264
Variety	8	7	6	3	1	0.516

Consistency index = 0.084 < 0.1

TABLE 2 Mr.X's Weights about Histrical Spots

	Central	Southern	Northern	Eastern	Weigts
Central	1	1/3	6	3	0.271
Southern	3	1	7	5	0.562
Northern	1/6	1/7	1	1/3	0.052
Eastern	1/3	1/5	3	1	0.115

Consistency index = 0.042 < 0.1

TABLE 3 Mr.X's Overall Rating

Criteria / Weights / Courses	Historical	Scenic	Foods	Interest	Variety	Total rate
	0.033	0.060	0.127	0.264	0.516	
Central	0.271	0.595	0.517	0.569	0.570	0.544
Southern	0.562	0.252	0.055	0.264	0.054	0.139
Northern	0.052	0.059	0.081	0.061	0.102	0.085
Eastern	0.115	0.094	0.347	0.106	0.274	0.223

TABLE 4 Overall Rating

Students / Weights / Courses	X	Y	Z	Total rate	Rank
	0.333	0.333	0.333		
Central	0.554	0.126	0.357	0.346	1
Southern	0.139	0.390	0.170	0.233	3
Northern	0.085	0.384	0.044	0.171	4
Eastern	0.223	0.099	0.428	0.250	2

D,E) for each criterion were defined in Table 6.
As before, a series of comparisons for new variables were done until all possible combination for comparison were exhausted. These comparisons and decisions are shown in Table 7. In these judgements, three individuals were slightly different as to the relative importance. The collective decision by group was accomplished by accepting majority rule on each comparison.

TABLE 5 Initial Assignment by Individual

Students / Criteria	X	Y	Z
Variety	100	50	100
Interest	85	100	45
Scenic	55	55	75
Foods	70	25	55
Histrical	45	40	35

TABLE 6 Initial Assignment by Group

Variable	Criteria	Value
A	Variety	100
B	Interest	92
C	Scenic	74
D	Foods	60
E	Histrical	48

The final adjusted rating is shown in Table 8. Next,three member's task was to assess the each course in terms of each criteria.
Refering to a guidebook etc., they decided a composite judgement provided some number between 0 and 1, where "0" means that the richness of criterion is very low. "1" means that is very high as shown in Table 9. Multipling the value of criteria (Table 8) by the richness of each course (Table 9), the overall rating was obtained as shown in Table 10.
Thus it was concluded that the central course of Hokkaido was judged to have the highest preference too. This conclusion completely coincides with the result of the AHP. However, Comparing Table 4 and Table 10, the second and the fourth of ranking changes the places, because the Churchman's and Ackoff's method gives an order scale with certain constraints placed upon the distances between items. This constraints are not sufficient to guarantee an interval scale (Hall 1962). Then many alternative combinations of values satisfing the condition of Table 7 are considered. One of this example is shown in Table 11. From this rating Table 12 is obtained as an alternate overall rating. Comparing Table 10 and Table 12, the first and the second of ranking changes the place with a slight difference.
While the AHP based on a ratio scale gives

TABLE 7 Comparisons by Criteria Combinations

Comparison	Decision			Majority decision
	X	Y	Z	
A>(B+C+D+E)	×	×	×	×
A>(B+C+D)	×	×	×	×
A>(B+C+E)	×	×	O	×
A>(B+D+E)	×	×	Δ	×
A>(C+D+E)	×	×	×	×
A>(B+C)	×	Δ	×	×
A>(B+D)	×	O	×	×
A>(B+E)	×	O	×	×
A>(C+D)	×	×	×	×
A>(C+E)	O	O	O	O
A>(D+E)	O	O	O	O
B>(C+D+E)	×	×	×	×
B>(C+D)	×	×	×	×
B>(C+E)	O	×	×	×
B>(D+E)	O	O	O	O
C>(D+E)	×	Δ	×	×

(注) O :Yes ×:No Δ:Equal

TABLE 8 Adjusted Rating

Variable	Criteria	Value	Normalized
A	Variety	100	0.32
B	Interest	81	0.26
C	Scenic	58	0.19
D	Foods	45	0.14
E	Histrical	27	0.09

definite scales for intensity of importance (Saaty 1980).
Therefore, the Churchman's and Ackoff's method have some arbitrariness on the weighting of values. This means a lack of consistency on solution. On the contrary, the AHP is superior to the above one in terms of a consistency on solution.

TABLE 9 Richness of Criteria on Each Courses

Courses \ Criteria	Variety	Interest	Scenic	Foods	Bistrical
Central	0.5	0.6	0.5	0.7	0.4
Southern	0.6	0.4	0.8	0.3	0.5
Northern	0.4	0.5	0.5	0.3	0.1
Eastern	0.7	0.3	0.5	0.6	0.2

TABLE 10 Overall Rating

Courses	Total rate	Rank
Central	0.545	1
Southern	0.535	2
Northern	0.404	4
Eastern	0.499	3

TABLE 11 Adjusted Rating

Variable	Criteria	Value	Normalized
A	Variety	100	0.31
B	Interest	75	0.23
C	Scenic	72	0.23
D	Foods	47	0.15
E	Histrical	28	0.08

TABLE 12 Overall Rating

Courses	Total rate	Rank
Central	0.545	2
Southern	0.547	1
Northern	0.407	4
Eastern	0.507	3

A COMPARISON OF THE AHP AND THE SCHEFFÉ'S METHOD

An object of a comparison

An experiment on preference of the icecream brands was taken as an object of a comparison as follows :

Icecream : 50 Yen/piece, 2 brands (A_1, A_3)
100 Yen/piece, 2 brands (A_2, A_4)

Experimenter : 5 students (O_1, O_2, O_3, O_4, O_5)

This experiment was made by paired comparison. The brands were not informed to experimenters in advance.
All combinations of pair (A_1, A_2), (A_1, A_3), (A_1, A_4), (A_2, A_3), (A_2, A_4), (A_3, A_4) were tested by each experimenter only once.
There are a difference between the AHP and the Scheffé's method with respect to the scale of pairwise comparisons for importance of preference. These scale differences are shown in Tabel 13.

An outline of the Scheffé's method

The Scheffé's method had been developed in the sensory test field originally. This method is based on a paired comparison and an analysis of variance. Each experimenter states his preference and then this result is converted to a numerical scale.

Now the preference variable between the pair i and j in the order (i,j) of the kth judge is defined x_{ijk}, which constitutes the following equation. This equation is the expansion of the Scheffé's method by Ms. Nakaya (Miura 1973).

$$x_{ijk} = (\alpha_i - \alpha_j) + (\alpha_{ik} - \alpha_{jk}) + \gamma_{ij} + \varepsilon_{ijk}$$

where, α_i and α_j are the prefernce effects of A_i and A_j respectively, α_{ik}, α_{jk} are the individual effects respectively, γ_{ij} is the combination effects A_i and A_j, ε_{ijk} is the experimental error.
The data obtained by this method is showen in Table 14. The analysis of variance of this data is shown in Table 15. From the above analysis, it became clear that both the main effects and the individual effects are significant at the 0.01 level, and the preference ranking is in order of A_2, A_4, A_1, A_3 as shown in Table 16.

TABLE 13 Scale Comparison

Definition	Scheffé	AHP
Equal	0	1
Slight	1	3
Moderate	2	5
Strong	3	7
Absolute	4	9
Reverse comparison	minus of above	reciprocal of above

TABLE 14 Scheffé's Paired Comparison

Experimenter / Sample	O_1	O_2	O_3	O_4	O_5	Total
A_1 A_2	2	-3	-2	-1	-1	-5
A_1 A_3	-1	2	1	-2	0	0
A_1 A_4	1	-2	1	-1	-1	-2
A_2 A_3	-1	3	2	1	1	6
A_2 A_4	1	4	2	2	1	10
A_3 A_4	2	-2	-1	-2	-2	-5

Application of the AHP

The icecream preference hierarchy of the AHP is shown in Figure 3. The overall rating integrated with a geometric mean for each experimenter's judgement is shown in Table 16. The conclusions of both methods completely coincide each other. However the Scheffe's method made clear the main effects and the individual effects. On the other hand, the AHP made clear the preference ranking of brands and the difference among a taste, a smell, a feeling of the tongue. The weights of preference as shown in Table 16 are in order of a taste, a feeling, a smell.

TABLE 15 Analysis of Variance

Factor	Sum of squares	Degree of freedom	Unbiased variance	F ratio	Significance
Main effects	31	3	10.33	10.35	high 0.01
Individual effects	48.5	12	4.04	5.10	high 0.01
Combination effects	7	3	2.33	2.94	
Error	9.5	12	0.79		
Total	96	30			

F ratio : $F^{3}_{12}(0.01) = 5.95$
$F^{12}_{12}(0.01) = 4.16$

After this result was obtained the analysis of variance for the effect of criteria was applied. From this analysis a taste and a feeling were

significant at the 0.01 level.
This conclusion completly coincides with the results of the AHP. This means that if we would apply only the Scheffé's method for this problem, we might be caught by the only effect of brands preference.

CONCLUSIONS

The merits that the AHP is superion to the other traditional methods became clear through the application to practical problems on the following main points.
(1) The AHP has a merit of consistency on solution.
(2) The hierarchy analysis of criteria is very useful for the purpose of formulating the problem.
However, a difference of the field developed it should be considered. The Scheffé's method has been developed for the sensory test field, so besides a main effect, an individual effect is a matter of importance.
On the other hand, the AHP has been developed for the political and the managerial decision making field, so a group decision is rather more important. However, we believe that the analytic hierarchy process is a useful tool even in the sensory test field.
The author would like to thank Mr. Satoh S. and Mr.Nozaki M. for their help in experiments and computations.

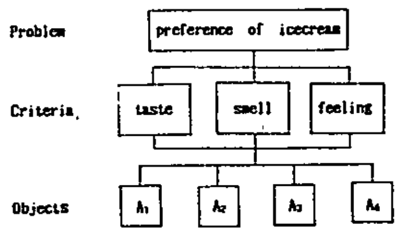

Figure 3 Icecream preference hierarchy.

TABLE 16 Overall Rating

Criteria / Weights / Sample	Taste	Smell	Feeling	Rating		Scheffé's result	
	0.57	0.09	0.34				
A_1	0.17	0.35	0.11	0.17	3	-0.35	3
A_2	0.44	0.25	0.46	0.43	1	1.05	1
A_3	0.15	0.22	0.12	0.15	4	-0.55	4
A_4	0.24	0.19	0.31	0.26	2	-0.15	2

TABLE 17 Subdivision of Total Sum of Squares

Factor	Sum of squares	Degree of freedom
Main effects	S_α	(t-1)
Individual effects	$S_{\alpha(n)}$	(t-1)(n-1)
Combination effects	S_γ	(t-1)(t-2)/2
Error	S_e	(t-1)(t-2)(n-1)/2
Total	S_t	t(t-1)n/2

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APPENDIX

The analysis of variance for paired comparisons developed by Scheffé can be calculated by means of the following equations. If there are t samples as an object of comparison and n judges, the estimates of the various parameters are given by:

The average preferences $\hat{\alpha}_i = \frac{1}{tn} x_{i..}$

The individual effect of preference $\hat{\alpha}_{i(k)} = \frac{1}{t} x_{i.k} - \hat{\alpha}_i$

The combination effect $\hat{\gamma}_{ij} = \frac{1}{n} x_{ij.} - (\hat{\alpha}_i - \hat{\alpha}_j)$

where, $x_{i..} = \sum_{j=1}^{t} \sum_{k=1}^{n} x_{ijk}$, $x_{i.k} = \sum_{j=1}^{t} x_{ijk}$, $x_{ij.} = \sum_{k=1}^{n} x_{ijk}$

The sum of squares of above estimates are given by:

$$S_\alpha = \frac{1}{tn} \sum_i x_{i..}^2$$

$$S_{\alpha(k)} = \frac{1}{t} \sum_k \sum_i x_{i.k}^2 - S_\alpha$$

$$S_\gamma = \frac{1}{n} \sum_i \sum_{j>i} x_{ij.}^2 - S_\alpha$$

$$S_e = S_t - S_\alpha - S_{\alpha(k)} - S_\gamma$$

$$S_t = \sum_k \sum_i \sum_{j>i} x_{ijk}^2$$

The degree of freedom of each sum of squares are shown in Table 17.