ISAHP 1999, Kobe, Japan, August 12-14, 1999

DECOMPOSITION OF DATA AND VARIATION IN THE ANALYTIC HIERARCHY PROCESS

Shin'ya Nagasawa

College of Business Administration, Ritsumeikan University
1-1-1 Noji-kita, Kusatsu, Shiga 525-8577 Japan
snt01497@ba.ritsumei.ac.jp

Abstract: In order to improve Analytic Hierarchy Process (AHP), this paper proposes the decomposition of data in the AHP by the logarithmic linear model. Data are decomposed to a main effect, an order effect, a combination effect, personal interactions and errors by the proposed mode. The proposed model is considered to be useful to analyze details of decision making by the AHP.

In order to improve Saaty's Analytic Hierarchy Process (AHP) this paper proposes the decomposition of data and variation in the AHP by the following logarithmic linear model;

$$xijk = (\omega i/\omega j) \times (\omega ik/\omega jk) \times cij \times (d \times dk) \times eijk$$

where i,j : alternative
k : person
$xijk$: paired comparison data (ratio)
ωi : main effect (weight) $(\Pi\, \omega ik = 1)$
ωik : personal interaction of weight $(\Pi\, \omega ik = 1)$
cij : combination effect $(\Pi\, cij = 1)$
d : order effect
dk : personal order effect $(\Pi\, dk = 1)$
$eijk$: error

By logarithmic least squares method, data and variation are decomposed to a main effect, an order effect, a combination effect, personal interactions and errors by the proposed mode. The proposed model is considered to be useful to analyze details of decision making by the AHP.

Table 1 Inverse order AHP data by 3 persons

$i \backslash j$		1	2	3	4
	P_1		2^{0}	2^{-1}	2^{-1}
1	P_2	1	2^{0}	2^{-2}	2^{-2}
	P_3		2^{1}	2^{-1}	2^{-1}
	P_1	2^{3}		2^{0}	2^{0}
2	P_2	2^{0}	1	2^{-2}	2^{-2}
	P_3	2^{1}		2^{0}	2^{-2}
	P_1	2^{3}	2^{1}		2^{-1}
3	P_2	2^{3}	2^{1}	1	2^{-1}
	P_3	2^{2}	2^{1}		2^{-1}
	P_1	2^{3}	2^{3}	2^{1}	
4	P_2	2^{3}	2^{3}	2^{1}	1
	P_3	2^{2}	2^{1}	2^{0}	

Table 2 Analysis of geometric mean data (1)

$i \backslash j$	1	2	3	4	weight
1	1	$2^{-0.50}$	$2^{-2.00}$	$2^{-2.00}$	0.095
2		1	$2^{-0.83}$	$2^{0-1.83}$	0.143
3			1	$2^{0-0.83}$	0.294
4				1	0.467

C.I.=0.013, C.R.=0.014

Table 3 Analysis of geometric mean data (2)

$i \backslash j$	1	2	3	4	weight
1	1	$2^{0.33}$	$2^{-1.33}$	$2^{-1.33}$	0.095
2	$2^{1.33}$	1	$2^{-0.67}$	$2^{-1.33}$	0.143
3	$2^{2.67}$	$2^{1.00}$	1	$2^{-1.00}$	0.294
4	$2^{2.67}$	$2^{2.33}$	$2^{0.67}$	1	0.467

C.I.=0.430, C.R.=0.477

Table 4 Decomposition of Data in AHP

(a) ratio of main effect (weight) $\omega i/\omega j$

$i \backslash j$		1	2	3	4	weight
	P_1		$2^{-0.58}$	$2^{-1.62}$	$2^{-2.29}$	
1	P_2	1	$2^{-0.58}$	$2^{-1.62}$	$2^{-2.29}$	0.095
	P_3		$2^{-0.58}$	$2^{-1.62}$	$2^{-2.29}$	
	P_1	$2^{0.58}$		$2^{-1.04}$	$2^{-1.71}$	
2	P_2	$2^{0.58}$	1	$2^{-1.04}$	$2^{-1.71}$	0.143
	P_3	$2^{0.58}$		$2^{-1.04}$	$2^{-1.71}$	
	P_1	$2^{1.62}$	$2^{1.04}$		$2^{-0.67}$	
3	P_2	$2^{1.62}$	$2^{1.04}$	1	$2^{-0.67}$	0.294
	P_3	$2^{1.62}$	$2^{1.04}$		$2^{-0.67}$	
	P_1	$2^{2.29}$	$2^{1.71}$	$2^{0.67}$		
4	P_2	$2^{2.29}$	$2^{1.71}$	$2^{0.67}$	1	0.467
	P_3	$2^{2.29}$	$2^{1.71}$	$2^{0.67}$		

C.I.= 0 , C.R.= 0

(b) ratio of personal interaction $\omega ik/\omega jk$

$i \backslash j$		1	2	3	4	weight
	P_1		$2^{0.67}$	$2^{-0.12}$	$2^{-0.21}$	
1	P_2	1	$2^{0.33}$	$2^{-0.38}$	$2^{-0.46}$	0.25
	P_3		$2^{0.33}$	$2^{0.50}$	$2^{0.67}$	
	P_1	$2^{0.67}$		$2^{0.54}$	$2^{0.46}$	
2	P_2	$2^{-0.33}$	1	$2^{-0.71}$	$2^{-0.79}$	0.25
	P_3	$2^{-0.33}$		$2^{0.17}$	$2^{0.33}$	
	P_1	$2^{0.12}$	$2^{-0.54}$		$2^{-0.08}$	
3	P_2	$2^{0.38}$	$2^{0.71}$	1	$2^{-0.08}$	0.25
	P_3	$2^{-0.50}$	$2^{-0.17}$		$2^{0.17}$	
	P_1	$2^{0.21}$	$2^{-0.46}$	$2^{0.08}$		
4	P_2	$2^{0.46}$	$2^{0.79}$	$2^{0.08}$	1	0.25
	P_3	$2^{-0.67}$	$2^{-0.33}$	$2^{-0.17}$		

C.I.= 0 , C.R.= 0

(c) combination effect ci

$i \backslash j$		1	2	3	4	weight
	P_1		$2^{0.08}$	$2^{-0.38}$	$2^{0.29}$	
1	P_2	1	$2^{0.08}$	$2^{-0.38}$	$2^{0.29}$	0.25
	P_3		$2^{0.08}$	$2^{-0.38}$	$2^{0.29}$	
	P_1	$2^{-0.08}$		$2^{0.21}$	$2^{-0.12}$	
2	P_2	$2^{-0.08}$	1	$2^{0.21}$	$2^{-0.12}$	0.25
	P_3	$2^{-0.08}$		$2^{0.21}$	$2^{-0.12}$	
	P_1	$2^{0.38}$	$2^{-0.21}$		$2^{-0.17}$	
3	P_2	$2^{0.38}$	$2^{-0.21}$	1	$2^{-0.17}$	0.25
	P_3	$2^{0.38}$	$2^{-0.21}$		$2^{-0.17}$	
	P_1	$2^{-0.29}$	$2^{0.12}$	$2^{0.17}$		
4	P_2	$2^{-0.29}$	$2^{0.12}$	$2^{0.17}$	1	0.25
	P_3	$2^{-0.29}$	$2^{0.12}$	$2^{0.17}$		

C.I.=0.013, C.R.=0.014

(d) order effect d

$i \backslash j$		1	2	3	4	weight
	P_1		$2^{0.44}$	$2^{0.44}$	$2^{0.44}$	
1	P_2	1	$2^{0.44}$	$2^{0.44}$	$2^{0.44}$	0.25
	P_3		$2^{0.44}$	$2^{0.44}$	$2^{0.44}$	
	P_1	$2^{0.44}$		$2^{0.44}$	$2^{0.44}$	
2	P_2	$2^{0.44}$	1	$2^{0.44}$	$2^{0.44}$	0.25
	P_3	$2^{0.44}$		$2^{0.44}$	$2^{0.44}$	
	P_1	$2^{0.44}$	$2^{0.44}$		$2^{0.44}$	
3	P_2	$2^{0.44}$	$2^{0.44}$	1	$2^{0.44}$	0.25
	P_3	$2^{0.44}$	$2^{0.44}$		$2^{0.44}$	
	P_1	$2^{0.44}$	$2^{0.44}$	$2^{0.44}$		
4	P_2	$2^{0.44}$	$2^{0.44}$	$2^{0.44}$	1	0.25
	P_3	$2^{0.44}$	$2^{0.44}$	$2^{0.44}$		

C.I.=0.360, C.R.=0.400

(e) personal order effect dk

$i \backslash j$		1	2	3	4	weight
	P_1		$2^{0.47}$	$2^{0.47}$	$2^{0.47}$	
1	P_2	1	$2^{-0.28}$	$2^{-0.28}$	$2^{-0.28}$	0.25
	P_3		$2^{-0.19}$	$2^{-0.19}$	$2^{-0.19}$	
	P_1	$2^{0.47}$		$2^{0.47}$	$2^{0.47}$	
2	P_2	$2^{-0.28}$	1	$2^{-0.28}$	$2^{-0.28}$	0.25
	P_3	$2^{-0.19}$		$2^{-0.19}$	$2^{-0.19}$	
	P_1	$2^{0.47}$	$2^{0.47}$		$2^{0.47}$	
3	P_2	$2^{-0.28}$	$2^{-0.28}$	1	$2^{-0.28}$	0.25
	P_3	$2^{-0.19}$	$2^{-0.19}$		$2^{-0.19}$	
	P_1	$2^{0.47}$	$2^{0.47}$	$2^{0.47}$		
4	P_2	$2^{-0.28}$	$2^{-0.28}$	$2^{-0.28}$	1	0.25
	P_3	$2^{-0.19}$	$2^{-0.19}$	$2^{-0.19}$		

C.I.= 0 , C.R.= 0

(f) error $eijk$

$i \backslash j$		1	2	3	4	weight
	P_1		$2^{0.25}$	$2^{0.21}$	$2^{0.29}$	
1	P_2	1	$2^{0.00}$	$2^{0.21}$	$2^{0.29}$	0.287
	P_3		$2^{0.92}$	$2^{0.25}$	$2^{0.08}$	
	P_1	$2^{0.92}$		$2^{-0.62}$	$2^{0.46}$	
2	P_2	$2^{-0.33}$	1	$2^{-0.62}$	$2^{0.46}$	0.256
	P_3	$2^{0.58}$		$2^{0.42}$	$2^{-0.75}$	
	P_1	$2^{-0.04}$	$2^{-0.21}$		$2^{-1.00}$	
3	P_2	$2^{0.46}$	$2^{-0.71}$	1	$2^{-0.25}$	0.222
	P_3	$2^{0.25}$	$2^{0.08}$		$2^{-0.58}$	
	P_1	$2^{-0.12}$	$2^{0.71}$	$2^{-0.83}$		
4	P_2	$2^{0.38}$	$2^{0.21}$	$2^{-0.08}$	1	0.235
	P_3	$2^{0.42}$	$2^{-0.75}$	$2^{-0.92}$		

C.I.=0.038, C.R.=0.042

Nagasawa, S.(1993) *Pcoceedings of the Autumn Conference of ORSJ*, pp.220-221.(in Japanese)