9/9 - 9/1 SCALE METHOD OF AHP

Da Ma and Xuequing Zheng
Dept. of System Engg. & Maths.
Changsha Institute of Tech.
Changsha, China

ABSTRACT

The scale values of AHP stand for classes of language. The suitability of the scale should be measured by the consistency between the scale and the language. In this paper we put forward the 9/9 - 9/1 scale method (scale values: $s(k) = 9/(10-k)$, $k=1, 2, \ldots,9$) and then it is compared with the 1 - 9 scale method where $s(k) = k$, $k=1, 2, \ldots, 9$. Contradictions between the scale and the language are greatly improved with the new scale. As an example of a relatively poor scale, the 10/10 - 18/2 scale method ($s(k)=(9+k)/(11-k), k=1,2,\ldots.9$) is given.

1. Introduction

For the purpose of expressing the relative importance of two things, the scale values stand for the classes of language in the Analytic Hierarchy Process (AHP). The suitability of a scale is the fundamental base for a correct decision.

The 1-9 scale is in common use (see Table 1). People often use many classes of language represented by judgments, such as equal, moderate, strong, very strong and extreme. The extreme of psychology in numbers is 9. There are many reasons for using scale values from 1 to 9. The 1-9 scale is intuitive and makes natural sense. In this scale, the relationship between the scale values and the language classes is given by:

$$S(k) = k, \quad k = 1,2,\ldots,9.$$

One of the first experiments using the 1-9 scale was performed by T.L. Saaty. The experiment consisted on measuring the relative luminosity of a set of objects located at a given distace from a source of light. We don't think that the experiment is effective enough.

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Here we discuss the problem of how to arrange the scale values in the range of 1 to 9 and study the 9/9 - 9/1 and the 10/10 - 18/2 scales (see Table 1). The contradiction of the scale and the language is greatly improved by using the new scale systems, especially by using the 9/9 - 9/1 scale.

Table 1. Three Scale Systems

Class	Language	1-9	10/10-18/2	9/9-9/1
1	Equal	1	10/10	9/9
3	Moderate	3	12/8	9/7
5	Strong	5	14/6	9/5
7	Very Strong	7	16/4	9/3
9	Extreme	9	18/2	9/1
k		k	(9+k)/11-k)	9/10-k)

2. Scale Methods

The range of the scale values is still from 1 to 9. Let S(k) express a scale value on the k class.

There are two meanings when a judgment matrix is consistent: (1) The judgments of the people are consistency, and (2) The scale values used to express the judgments are consistent. This means that when the judgments are consistent, the scale values are also consistent. It requires that the scale agrees with language.

Assume we are given alternatives whose weights are w(i),i=1,1...., n. The scale values of the paired comparisons between alternatives i and j are given by a(i,j). For a consistent

matrix we have:

$$a(i,j) = w(i)/w(j), \tag{1}$$

and

$$a(i,j) = a(i,1)/a(j,1). \tag{2}$$

If $a(i,j) = S(k)$ and $w(i)>w(j)$, we define the relative importance difference as follows:

$$\Delta(k) = 2\,[w(i)-w(j)]/[w(i)+w(j)]$$

$$= 2\,[S(k)-1]/[S(k)+1] \tag{3}$$

Thus, we have:

$$S(k) = [2 + \Delta(k)]/[2 - \Delta(k)] \tag{4}$$

$S(k)$ is a monotone function of $\Delta(k)$ and k. Suppose now that the k is a continuous parameter, and that

$$d^2\Delta(k)/dk^2 = 0 \tag{5}$$

Then, if

$$\Delta(1) = 0 \tag{6}$$

and

$$\Delta(9) = 8/5 \tag{7}$$

from (3) and (4), we have:

$$S(k) = (9+k)/(11-k) \tag{8}$$

This is the 10/10-18/2 scale (see Table 1).

Let us assume that the judgments are transitive. Let A be moderately more preferred than B and that B is moderately more preferred than C. Experiments have shown that if we compare A with C, the result is a judgment between moderate and strong. Let x express the class (continuous) of language, and assume that:

$$\Delta(x) = 2/3\ \Delta(4) + 1/3\ \Delta(5) \tag{9}$$

and that

$$d^2\,\Delta(k)/dk^2 = \text{constant} \tag{10}$$

According to (3), (4), (6), (7), (9), and (10), we have:

$S(1) = 1$, $S(3) = 9/7.3$, $S(5) = 9/5.1$, $S(7) = 9/2.9$ and $S(9) = 9$.

These results are close to the scale value $S(k) = 9/(10-k)$, $k=1,2,3,....,9$. This is the 9/9 - 9/1 scale (See Table 1).

3. Comparisons

The following comparisons could be done by the reader.

1. Correspondence with language

In a psychology test, we choose two data (the sum is 1) to correspond with the language of judgments. Then we get the results of the same language from the three scales. All of these are shown in Table 2.

Table 2

Language	Psychology test	1-9	10/10-18/2	9/9-9/1
Moderate	0.55:0.45	0.75:0.25	0.6:0.4	0.56:0.44
Strong	0.65:0.35	0.83:0.17	0.7:0.3	0.65:0.35
Very strong	0.75:0.25	0.88:0.12	0.8:0.2	0.75:0.25

According to (1), it is known by comparing the results that the 9/9-9/1 scale method is the best one, 10/10-18/2 scale method is relatively poor, and the 1-9 scale method is the worst.

2. Consistency of scale values

The results obtained in the previous experiment are now represented in a matrix of paired comparisons for the three scales. We have:

1-9	A	B	C
A	1	3	4
B		1	3
C			1

10 18 / 10 2	A	B	C
A	1	12/8	13/7
B		1	18/2
C			1

9-9 / 9-1	A	B	C
A	1	9/7	9/6
B		1	9/7
C			1

The consistency indices of these matrices are given by:

C1(1-9) = 0.0367

C1(10/10-18/2) = 0.00205

and C1(9/9-9/1) = 0.0005.

We could do many experiments like this. Most of them show that it's difficult for the 1-9 scale to get closer to consistency than it is for the other two scales.

3. The Peculiarity of the judgments

Study the peculiarity of the judgments. The larger the relative important difference, the rougher the judgement. So when $k_2 > k_1$, then we should have $\Delta(k_2) > \Delta(k_1)$ (See Figure 1).

Where I, II and III express the 1-9 scale, the 10/10-18/2 scale, and the 9/9-9/1 scale, respectively. III appears to be the best. II falls between I and III.

Figure 2

4. Conclusion

There are many scales that we did not discuss in this paper. But the idea developed in this paper could be used as a starting point. The suitability of a scale should be measured by the consistency between the scale and the language.

We can perform experiments to check the character of the scale. It is worthwhile of note that the weights we discussed here are the psychological feelings. It is no used to check physical laws. Many experiments show that there are contradictions between the scale and language and that they are improved by using the 9/9-9/1 scale and the 10/10-18/2.

Reference

[1]. Saaty, T.L. The Analytic Hierarchy Process, McGraw Hill, Inc., 1980.

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