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The Structure Of Qualitative Decision-Making: Implications for the Analytical Hierarchy Process

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Abstract: The implications of the structure of qualitative-decision-making for multi-criteria decision analysis are considered. It provides a basis for understanding qualitative distinctions between criteria and, consequently, their proper structuring and grouping to facilitate comparisons. The AHP approach of using relative comparisons is justified. The concept of a qualitative decision function is introduced and shown to correspond to the Cobb-Douglas or power function. It is explained why it is inappropriate to use the AHP to synthesise qualitatively identical attributes of an alternative. This should be done using the multi-attribute utility technique. The two techniques are proven to be complimentary and can be synthesised. Normalisations other than the geometric mean are shown to be unnecessary. The common concerns about rank reversal are dealt with. The importance of developing the thinking process of the decision-maker is discussed. The results of experiments with the proposed structuring approach are outlined.

Introduction

This article develops from work on the structure of qualitative decision-making (Brugha 1995a, 1995b and 1995c) and on its implications for multicriteria decision-aid science (Brugha, 1996a). The structure of qualitative decision-making is determined by nomology i.e. the science of the laws of the mind. Out of this research come two themes which will be developed here in the context of the Analytical Hierarchy Process: (1) the basis for how people think in terms of qualitative differences and how these can put put into an ordered hierarchy, and (2) increasing understanding of a complex human situation such as a multi-criteria goal is a development process.

To provide focus for these themes two illustrations will be used: Saaty's (1990a) house example and Schoner and Wedley's (1989) car example. In the house example the following were the criteria: size of house, location to bus lines, neighbourhood, age of house, yard space, modern facilities, general condition, and financing available. In the car example they were price ($) maintenance ($/yr.) and fuel (gal./mi.). In this case scale factors were known and were 1, 5 and 75,000. These arose because the car was expected to last five years, to be driven 10,000 miles a year and at a cost of $1.50 per gallon.

A most important question which challenges operations researchers in the field of multicriteria decision-aid science is to determine if and how multiple criteria can be synthesised. It has been shown (Brugha, 1995c) that there are distinct levels of activity and these form a hierarchy, a shortened version of which is physical, political, economic, social and cultural/higher. This hierarchy should be looked on as a problem or criteria classification system. Obviously, most operational research applications fall mainly into the economic category. The temptation, therefore, is to treat non-economic aspects in a broadly similar way, for example to convert them into units which are tradable for money. A significant contribution of the Analytical Hierarchy Process is that it offers a powerful method of synthesis of criteria which cross different levels of activity which does not require the decision-maker to reduce higher level attributes to a least common denominator measure such as money. Obviously this would be helpful in the case of the house example.

From a nomological point of view it would be important to take into account any qualitative distinctions which the decision-maker has identified, i.e. to respect the structure of human decision making processes. Hence, a first step would be to identify if the decision-maker's goals can be confined or not to one level of the hierarchy of activities. This should help to determine the appropriate direction for modelling the process. Such an identification is not a trivial issue. Most people would classify college education as an economic activity because its main objective is preparation for a job. But how about art education, religious education, literature (cultural education) or the political "re-education" that used to occur in some communist regimes? So, the context, and consequently the decision-maker, probably best determine the nature of the decision process and the degree to which qualitative distinctions must be taken into account in synthesising estimates of value for different alternative solutions.

To complicate the issue even further, the various levels of human activities are quite inter-related (Brugha, 1995c). For example, if one were considering alternative positions for a new town, a building or a bridge, one would have to give priority to those alternatives that were physically feasible. For instance, one should not build on sandy or swampy ground, etc. Obviously there is a fuzziness about what goal this affects: certainly the danger of its falling down is a physical issue; but, with lots of money (construction engineering, etc.) it could be converted to an economic issue. Next, there are questions of political acceptability. Many operational researchers have anecdotes about long hours fine-tuning solutions only to find the requisites of the political process superseding the solution. At the economic level there can be budgetary constraints. At the social level there can be issues about simple unpopularity or inconvenience of the location of the construction. Culturally there may be issues to do with displacing native communities.

Some of these issues can be handled simply by including constraints so that the goal is confined to the economic level, or by using an interactive approach. On the other hand, the purpose of Multi-Criteria Decision Aid science is to aid decision-makers. This can mean helping them to understand the underlying dynamics of the decision making process and its nomological foundations so that they grow to realise their wishes most fully. The nomological contribution with regard to the constraint aspect suggests that the hierarchy of activities runs from needs at the lowest level through preferences to values at the highest level. The nature of *importance* of the activities changes as one rises through the hierarchy. At the lowest level there are simple needs, e.g. that the bridge not fall down and that it can carry appropriate weight, but, once that is satisfied, the physical aspect may lose importance. The physical goal then becomes a constraint, and so on up the hierarchy. Satisficing, or converting goals into constraints that must be satisfied up to but not necessarily beyond a certain level, is closer to what nomology would suggest. This makes finding solutions more contingent on the situation. If there is only one physically viable place for the bridge, or if there are ten but they all are in danger of being taken away by floods, the physical goal remains paramount. On the other hand, if, having gone through the first six levels, there are still two bridges that qualify equally well, then the discussion could boil down to which is the most satisfying artistically. In a sense, the basis of the inter-changeability of goals and constraints depends, at the higher levels, on decision-makers' subjective evaluation of how realistic are their expectations, and these can be fluid. Furthermore, they are extremely subject to the advice given to the decision-maker by the operational researcher.

Where two or more levels of activities are involved a trade-off has less meaning. This is less of a problem in the case of only two levels than it is with more than two. With two levels, for example economic and social benefits, one could provide syntheses within each alternative and then produce some kind of socio-economic benefit cost score leading to a ranking of alternatives. This is somewhat artificial and does not take account of non-linearities, constraints and thresholds in the two levels. With three levels or more, a good way of excluding poorer (inefficient) alternatives is by using outranking methods. (See Vincke, 1992, for a review of this field which was founded by Bernard Roy.) Using outranking to choose an optimal solution from an efficient set is more difficult because it involves identifying criterion weights. An area of development for outranking methods would be to focus on synthesising criteria in the context of the hierarchy of levels, and then to explore the decision-maker's position on the relative importance of the levels themselves in the context of the situation.

Central to the outranking process is the issue of incomparability. The human mind is highly subtle and can deal with degrees of comparability. Consider the house example above. The *financing* issue is quite distinct from all the others. Within the remainder, location to bus lines (*transportation*), *neighbourhood* and *yard space* form a comparable subgroup; call it *surroundings*. The other subgroup which corresponds to more specifically *house* issues could be similarly divided into those to do with the maintenance, refurbishment and value of the house as distinct from its size, giving a hierarchy such as in Figure 1. The biggest qualitative distinction is at the top of the hierarchy: *financing* determines whether or not one can afford the house or not. The hierarchy could even be extended downwards to the point where some of the criteria are not distinct qualitatively from each other. In the case of the car the differences in criteria were purely quantitative. Differences in payment method could be included under *financing* of the house, the *yard space* areas for the alternative houses could have been included, and so on.

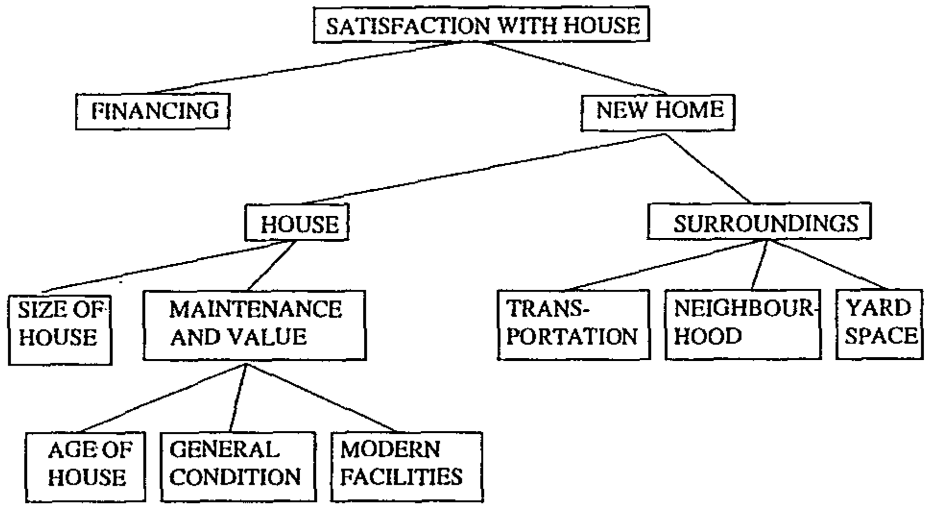

Figure 1. Hierarchy for satisfaction with house

All levels of human activity from the lowest to the highest can be placed in an ordered hierarchy. Likewise, all qualitative distinctions from the broadest to the finest can be used to structure a decision, to the extent that the decision-maker can make such distinctions. Thus, depending on how well the decision-maker realises the differences on any attribute between a particular pair of alternatives, genuine comparisons and evaluations can be made at each level of qualitative distinction.

An important way to categorise the different MCDA techniques is by how they accommodate qualitative distinctiveness within a modelling process. MCDA methods usually score alternatives on attributes and weight the attributes to provide a synthesis. MAUT, with its easily measured and understood explicit trade-offs, does not take account of differences in qualitative distinctiveness. It converts everything into utilities and presumes that the synthesis will work as well for qualitatively highly distinct attributes as we know it does for similar attributes. Outranking methods and the Analytical Hierarchy Process (AHP) (1980) both address this issue making them more conducive to providing a synthesis of different levels of activity and qualitative distinctiveness. MCDA theoreticians and practitioners need to explore the issue of focus on the qualitatively distinct levels within the mind of the decision-maker because of the inherently different nature of comparisons within and between levels of activity. Jean-Pierre Brans (1994), one of the MCDA leaders in Europe, suggests that the problem facing MCDA is to determine the kind of information that is needed for modelisation within criteria and between criteria.

Intuitively, it makes sense to not mix evaluations which are qualitatively quite distinct from each other, for instance, in the case of the house above, to compare *general condition* with *neighbourhood*, or *yard space* with *financing available*. We would propose that the identification of weights within each category and on each level that is relevant to a choice process should be carried out first, working from the bottom up, if possible. Then the more macro weighting process could be done between the synthesised sets. Occasionally this could lead to only two items being compared at one level. For example, in the house case there would be only one comparison between *house* and *surroundings*, and then between *financing* and *new home*. This has the benefit of requiring the decision-maker to answer fewer questions. It might create anxiety that the result would be over-dependent on single judgements such as on the *financing* versus *new home* question. In our opinion this problem can be dealt with most easily by using sensitivity analysis. As it was, in the actual application (Saaty 1990a, p.17) there was some surprise about the emergence of the least desirable house with respect to *financing* as most preferred. It is possible that our suggested approach might have helped in this case.

The benefit of working from the bottom up is that the decision-maker has the benefit of learning what the criteria mean from having been working with them. We would also suggest that it makes more sense nomologically to have decision-makers work firstly with criteria which are qualitatively close and finish up with the more difficult judgements, that is those which are very different from each other. The other alternative is to compare the importance of the *surroundings* issues within that group and then extend the comparison to the individual house issues. This might be difficult because of the incomparabilities. It also makes more demands on the patience of the decision-maker because of the increased number of questions. At the end of this article we report experience with some alternative approaches.

Another variation would be to make an ordering of the criteria from lower to higher within any level. This should make for easier comparisons between levels. For instance, higher level criteria within an economic category could be compared with lower level criteria within a social category, hopefully producing a seamless join. Within *surroundings*, which is more of a social criterion, *transportation* would be lower than *neighbourhood* or *yard space* and so closer to some of the *house* criteria. Within *maintenance and value*, which is more of an economic type criterion, *modern facilities* is a higher criterion than *age* or *general condition*. Thus one would expect it would be relatively easy for a decision-maker to compare the relative importance of the *transportation* and *modern facilities* criteria. In fact the first is about the convenience of travel to and from the home and the second is about convenience within the home, two qualitatively similar issues. Likewise *age of house* is very close to *size of house*.

Implications for Constructing a Qualitative Decision Function

Having considered the structure of qualitative decision making and its implications for multi-criteria decision aid it is appropriate to develop guidelines that a multi-criteria decision function should follow.

The first issue relates to the question of single criterion versus multiple criteria goals. This is usually taken as read: some problems cannot be forced into a single criterion context. Why does this happen? In fact it is because of qualitative distinctions within the goals. Numerous financial measures can be synthesised into one. Include a non-financial benefit and, at the very least, benefit cost analysis must be done. Include, also, an environmental issue and the problem has independent "dimensions". A good qualitative decision procedure must take account of **qualitative distinctions** between criteria.

The more qualitatively distinct the objectives, obviously the more difficult the analysis, and possibly, also, the procedure. But, from the point of view of the decision-maker, there is only one overall goal. Hence, it should be possible, at least, for the decision-maker to be able to think in terms of one goal. But also, it should be possible to visualise the decision process in a single goal context: a **synthesised visualisation.**

The third issue relates to the question of **non-linearity**. This comes in two parts: non-linearity in the scoring of specific attributes, such as diminishing returns and increasing effects, and non-linearity in the treatment of the synthesis of (qualitatively) different objectives. Qualitative differences arise out of the mind of the decision-maker. Consequently we are concerned with utilities which are well-known to be highly non-linear.

Connected to this is a fourth issue, that of **user friendliness**. An excessively complex and unclear system will not be acceptable to decision-makers and their advisors.

Ultimately, we are endeavouring to construct a decision function that reflects the qualitatively different aspects of the decision-makers objectives: a **qualitative decision function.** A fifth issue is that such a function could be usable in the context of main-stream management science, i.e. readily interpreted and easily differentiated so as to facilitate optimisation.

A sixth issue is that the structure of the function be **theoretically well founded** and easy to assess.

Related to this is a final issue for which we use an analogy with physics. Einstein's theory of relativity superseded Newton's theory of gravity, but did not contradict it. The differences become evident only in the context of speeds close to light. The AHP grew out of a need to accommodate qualitative differences between attributes. However, its difficulties and the objections to it have arisen mainly in the simpler area of *quantitative* differences. Saaty (1994), for example, introduced a new form of normalisation, the 'Ideal' form, to avoid rank reversal being caused by the entry of irrelevant alternatives or numerous copies of one alternative. Schoner and Wedley (1989) suggested 'referenced AHP' and 'B-G modified AHP', which was based on Belton and Gear's proposals ((1983, 1985) to deal with quantitative differences in attributes such as in the car case above. Schenkerman (1994) has raised the question of non-additive value functions and shown that the AHP assumption of an additive value function might work when comparing fields on the basis of their perimeters, but would have difficulty if the alternatives were volumes. We suggest that the resolution of the problem will require **compatibility of AHP with multi-attribute utility theory (MAUT)**, an idea proposed by Dyer (1990).

A Model For Relative Comparisons

Consider two qualitatively different attributes A and B which are known to influence Y the value of something which is also different in nature to A and B. A could correspond, for example, to the social consequences of a proposal and B to its cultural implications. How does one measure a change in the value Y due to changes in A or B? Measures of change in A, B and Y can be made only in the context of the sensation of what A, B and Y are currently themselves, i.e. a proportionate change. This is Weber's law (expressed in 1834) and has been introduced into the AHP debate by Lootsma (1993). If the influence of issues A and B can be measured then the corresponding weights of importance **a** and **b** can estimated. This gives the result that the proportionate change in Y is equal to **a** times the proportionate change in A plus b times the proportionate change in B. This result is dependent on a Bayesian assumption of linearity and on the assumption of independence between the two criteria A and B. Both assumptions are reasonable. If ΔY, ΔA and ΔB are small changes in Y, A and B then:

$$\frac{\Delta Y}{Y} = a\frac{\Delta A}{A} + b\frac{\Delta B}{B} \tag{1}$$

This can be integrated to produce the following relationship in which Ln Y is the dependent variable in a simple linear regression model, and Ln A and Ln B are independent variables:

$$\text{Ln } Y = a \text{ Ln } A + b \text{ Ln } B \tag{2}$$

This can be exponentiated to produce the power function:

$$Y = A^a B^b \tag{3}$$

The power function is extremely versatile. If a is negative, then Y decreases as A increases. If a is positive Y increases with A, but with increasing returns if a is greater than 1 and decreasing returns if a is less than 1. In economics this is known as the Cobb-Douglas or log-linear function (Cobb and Douglas, 1928) and has been used to consider the inter-substitutability of labour and capital. In the context of decision making in general it would be used to consider the inter-substitutability of any qualitatively distinct factors which affect some goal.

A second attraction is that the power function is constantly elastic. Thus:

$$e_a = \frac{A}{Y}\frac{\partial Y}{\partial A} = \frac{A}{Y} aA^{a-1}B^b = a \tag{4}$$

Thus a relative change in the dependent variable Y due to a relative change in the independent variable A equals the constant elasticity **a**. The idea of constant elasticity is commonly accepted in economics. Hence, if a equals 0.5, then a 6% improvement in Factor A could produce a 3% improvement in the goal function Y. This is particularly relevant when trying to measure the relative impact of changes in several independent variables. If one trades off the improvements in each factor to the point where the marginal benefits are equalised, then:

$$\frac{\partial Y}{\partial A} = \frac{\partial Y}{\partial B} \quad \text{i.e.} \quad \frac{aY}{A} = \frac{bY}{B} \quad \text{thus} \quad \frac{A}{B} = \frac{a}{b} \tag{5}$$

Hence, at optimality, the relative usage of factors A and B should correspond to the ratio of their elasticities. Correspondingly, the degree to which two qualitatively different factors in a multi-criteria decision should be taken into account depends on the ratio of the importance weights of the criteria associated with those factors. This is also the core concept of the AHP, which confirms that the AHP structure is equivalent to a power function as suggested by Lootsma (1993, 1995).

Consider a decision function Y which is modelled on the hierarchy of activities. Hence A could correspond to the relevant physical factors, B to the political, C to the economic, D to the social, E to the cultural and emotional and F to the pneumatic. Let the corresponding elasticities be 0.5, 1.0, 4, 1.25, 0.8 and 0.5. These elasticities have a geometric mean of 1, indicating that only relative values matter.

Now consider the parallels with the AHP. (The use of the geometric normalisation method here is justified below.) The product rule for the weights in AHP is automatically taken into account; for example, if we use for the relative elasticities the term "as important as", then the economic is four times as important as the political, which is twice as important as the physical. Obviously, the economic is eight times as important as the physical.

Next, if we were to break the economic into three subcategories C_1, C_2 and C_3 with elasticities 1, 2 and 0.5 these figures become "local" weights or elasticities. When multiplied by 4, the economic elasticity, C_1, C_2 and C_3 develop "global" elasticities of 4, 8 and 2. This is the same procedure used for hierarchies in the AHP.

The corresponding Qualitative Decision Function, thus, has the following form:

$$Y = cA^{0.5}BC_1^4C_2^8C_3^2D^{1.25}E^{0.8}F^{0.5} \tag{6}$$

By taking logs one can linearise this to produce the format used in the AHP, i.e. "objective" measures of alternatives multiplied by a "subjective" or relative weighting function:

$$LnY = Lnc + 0.5LnA + LnB + 4LnC_1 + 8LnC_2 + 2LnC_3 + 1.25LnD + 0.8LnE + 0.5LnF \qquad (7)$$

This link with the Cobb-Douglas function raises the following points. Firstly, it would deal with the sixth issue, above, of providing a theoretically well founded base in economics for the AHP. Secondly, it makes the use of optimisation techniques such as the Lagrangian more easy to realise. In the context of Dorfman-Steiner (1954) type evaluations there is the possibility of including a cost variable in the goal function. Thirdly, there is the question of negative weights. Consider, for instance, the following typical Dorfman-Steiner function:

$$\text{Sales Q} = kA^{a}P^{-p} \qquad (8)$$

where Q, A and P are sales, advertising and price, respectively, and a and p are constant elasticities, and k is some constant. This is converted into a profit function:

$$Z = (P - c)kA^{a}P^{-p} - A - F \qquad (9)$$

where Z and F represent profit and other fixed costs. This can be optimised for A and P. Here we have two qualitatively distinct criteria within the economic level: price and advertising. Price is a lower level factor than advertising: a good price is necessary for a product to survive; advertising is a higher development factor.

A particular question it poses for the AHP is how would negative weights be measured? Currently, anything negative can be represented verbally in an inverted form, e.g. less destructive to the environment, less expensive, etc., making negative weights redundant. On the other hand, the AHP may be losing out in terms of robustness and flexibility in the context of mixed good and bad criteria. It should be possible to make comparisons between two criteria using language such as "how much more influential is (bad) Criterion A on reducing the chance of achieving the goal than is (good) Criterion B on increasing the chance of achieving it?" In the context of the illustration above, if the advertising elasticity was 0.25 and the price elasticity was -2.0 the answer, then would be their relative (multiplicative) difference, i.e. 2 / 0.25 = 8.0.

If it were possible to get to this point, then it opens another area of application. Usually, in constructing the above profit goal the most difficult part is evaluating the price and advertising elasticities. Difficulties with measuring elasticity data have impeded the use of the Dorfman-Steiner theorem; it is not included in the later edition of Kotler's text on Marketing Models. (See Lilien and Kotler, 1983 and Lilien, Kotler and Moorthy, 1992)' Such calibration has usually been done using historical data; however, acceptable assessments of coefficients in such functions are often made subjectively by experts (Lilien et al, 1992). In a marketing context of changing markets and products there often is no alternative but to use expert opinion. The AHP method of subjective measurement of relative scores could be used to facilitate this, thereby assisting in the application of management science to marketing model building.

Implications for the Analytical Hierarchy Process

We indicated above that, for measurements of change in a dependent variable due to qualitatively different independent variables, the mind finds it most convenient to think in terms of changes relative to the size of the variables. It follows that comparisons of two amounts of such a variable should be most easily considered in relation to one another, e.g. A_1 divided by A_2. Thus when making a comparison between the scores of alternatives with respect to different attributes it is appropriate to use relative comparisons as shown below.

$$\frac{Y_1}{Y_2} = \left(\frac{A_1}{A_2}\right)^{a}\left(\frac{B_1}{B_2}\right)^{b} \qquad (10)$$

Likewise, the same follows for the measurement of elasticities, which correspond to the measures of relative importance of the different criteria. Equation (2) above can be divided by B's elasticity **b** to give (11), showing that the important information with regard to elasticities is their values relative to the other elasticities.

$$1/b \text{ Ln } Y = a/b \text{ Ln } A + \text{Ln } B \tag{11}$$

Furthermore, the synthesis of effects due to variables A and B in equation (10) is also multiplied. Consequently all the key measurements are relative, i.e. they use either multiplication or division. *This should be the central feature of the AHP.* Hence there is no justification to use any method of synthesis of multiple scores of attributes or of elasticities other than the geometric mean as suggested by Crawford and Williams (1985), Barzilai et al (1987, 1992, 1994), Holder (1990) and Brugha and Alphonce (1996).

The original normalisation method used for AHP was the distributive mode (Saaty, 1977). The weights produced using this mode were percentages of overall relative priorities, or probabilities of different outcomes (Saaty, 1990b). Because of difficulties with the distributive mode providing acceptable probabilities the supermatrix formulation was developed (Harker and Vargas, 1987; Saaty, 1994). This formulation has also been criticised (Dyer, 1990) and has been shown to fail to eliminate rank reversals unless particular restrictions are placed on its inputs (Salo and Hamalainen, 1993). We would suggest that it has been a mistake to use probabilities as part of a synthesis of the model. Probabilities can be derived from the constant elasticities in (2) by using arithmetic normalisation. Using them to synthesise the full model would require working with the log of the attribute as in (11); these figures would have many places of decimals and would be difficult to interpret. We would suggest that it would be better to work with (10) and normalise both attribute scores and elasticities geometrically:

$$Y_{1\bullet} = \frac{Y_1}{Y_{GM}} = \left(\frac{A_1}{A_{GM}}\right)^{\frac{a}{a_{GM}}} \left(\frac{B_1}{B_{GM}}\right)^{\frac{b}{b_{GM}}} = A_{1\bullet}^{a^*} B_{1\bullet}^{b^*} \tag{12}$$

This has the same effect as working with relative values as in (10) but with the added information that the relative influence of attributes and elasticities (which are the importance weights of the criteria) can be readily seen. The major difference between this and Saaty's normalisation methods is that syntheses of attribute scores and criteria weights are done separately; the entire score for an alternative is then synthesised according to the structure of the qualitative decision function as in (12) (otherwise known as the Cobb-Douglas or power function).

The essence of this process is that it provides a means to synthesise scores on attributes which are measured in terms of qualitatively different criteria. The resolution to Schenkerman's (1994) problem is to distinguish between measures and attributes. Lengths and breadths of fields are arbitrary measures, not criteria; so are the ways whereby one produces the volume of a box. The AHP could be used to synthesise some value measure of a field which incorporates an area aspect with some other aspects such as nearness to the farmhouse or quality of the land. The same could be done with boxes to synthesise their volumes with their colours etc.

With the house example one should synthesise the known estimates of the costs linearly and feed the result into an AHP process if some other non-cost attributes need to be incorporated. This process can be extended to using utilities and a MAUT procedure such as SMART to synthesise attributes that are qualitatively similar. This includes using non-linear utility functions. Because the AHP should not be used to synthesise qualitatively similar attributes there is, therefore, no need to use Schoner and Wedley's (1989) *referenced AHP* or *B-G modified AHP* which they adapted from Belton and Gear (1983, 1985).

Some further points remain to show that this approach satisfies the guidelines above. Firstly, replacing the distributive form of normalisation by a geometric form will eliminate one of the main causes of rank reversal (Barzilai and Golany, 1994; Lootsma, 1993; Brugha, 1996b). Next, there is the issue of nonlinearities in the synthesising of the function. This is inherent in the power function where elasticities greater than one indicate increasing return and leass than one decreasing returns. Finally, providing the decision-maker a visualisation of his or her mental model is described below.

Synthesised Portrayal of Multi-Criteria Goal

Another implication of nomology for MCDA arises from the usual MCDA assumption that the multiple criteria are distinct and unconnected. In nomology there is a hierarchy of activities starting with the physical, then the political and so on upwards. Consequently, the decision-maker, who ultimately thinks in terms of a single "goal" anyway, can be given a visual portrayal of his or her goal as in the following cumulative effect diagram in Figure 2 which corresponds broadly to the function in equation (6). Here the greatest increase in satisfaction comes from the economic with its high elasticities.

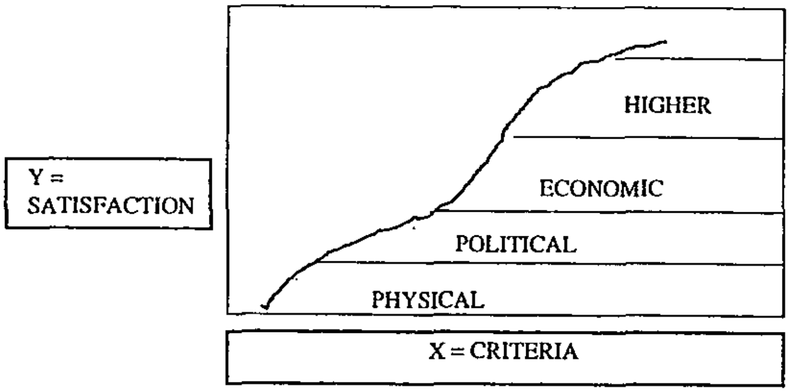

Figure 2: Satisfaction - Criteria Relationship

This diagram is not directly useful for comparing alternatives because they would contain different mixes of the various criteria. It does provide useful feedback about the main thrust of the goal. It also leads to some nomological guidelines for the application of the AHP.

1. When making comparisons between criteria for rating purposes it would be better to put the criteria in an ordered hierarchy so as not to cause the decision-maker confusion by having to move backwards and forwards between different types of criterion.

2. It also would be better, when comparing criteria in a group, to work from the most important (highly weighted) criterion outwards down to lesser important. It is easier to start with the key criterion in a group and compare the others with it.

3. When comparing groups of criteria it would also be better to work from lower level criteria up the hierarchy of activities. As one moves up one synthesises the information and opinions about groups of criteria at lower levels. Sometimes the decision-maker may fill in gaps which are missing at a lower level. The totality of a criterion such as *new home*, above, may be more than *house* and *surroundings*; there may be an intangible "cosy factor".

These three suggestions combine to provide the following: start first with the most important criterion (in this illustration the economic) and work down to the political and then the physical. Then work from the economic upwards. Afterwards, for confirmation, one could do some cross-comparisons such as political to social etc.

4. If, as was done for the economic level in this example, one of the levels has been broken up into a new layer, it should be easier for an end-user to make comparisons across different criteria on the same level. Thus, the three economic subcriteria could be synthesised into one economic criterion, and have it named, and then compared with the political, physical, social and other criteria, as was done with the house example in Figure 1.

Developing The Thinking Process Of The Decision-Maker

MCDA is on the border of operational research, and very close to systems development. Roy (1994) describes the traditional O.R. model as reductionist and unproductive for Decision Aid. We support his proposal that the new label OR-DA should be adopted. The natural consequence of having multiple criteria is to include the decision-maker in the modelling process as Brans (1994) and others have suggested. Brans goes on to talk about a consequential "space of freedom" and meta-models which help decision-makers determine the freedom they have within this space. Nomology is presented as one such meta model.

Incorporating the decision-maker into the decision has far reaching consequences to which it will take time for operations researchers to adjust. Mathematical elegance in MCDA modelling cannot be substituted for the actual processes of the decision-maker, who thinks in terms of qualitative distinctions, thresholds, goals which are conditional on the situation, and hierarchies of needs, preferences and values. Furthermore, simplistic assumptions about clear-cut distinctions between stages in a model's development must be dropped. In production planning one has the expectation that all relevant data can be assembled before modelling. With MCDA an inherent part of the model is in the mind of the decision-maker who has to identify relevant criteria, their weights, and feasible alternatives as part of a development process. It has technical aspects: the decision-maker has to become proficient in the modelling process, with the guidance of the MCDA specialist. It involves others: the MCDA specialist, the people involved in the (possibly group) decision process, clients, affected parties, customers, etc. It also involves relating to the particular situation: proposals for a bridge across a river, a new town, or whatever. All this takes time. The first cut of the model may be simple and the goals at the start may be quite different to those at the finish. The mind is a highly sophisticated computer that can continually revise its assessments, or return to earlier stages of the solution development cycle. The role of the MCDA advisor is to bring the decision-maker through this process. This requirement may have unpalatable consequences for the more technically orientated operations researchers. However, it is the route that MCDA has found itself on.

It also has implications for the development of MCDA software because such packages commonly incorporate much of the minds of the developers and so can restrict the advisor's freedom to explore and help develop the mental model of the decision-maker. Software which would facilitate the decision-maker entering information in alternative ways, sometimes via values and needs, othertimes via liked and disliked alternatives, would be very helpful. It may be that the goal function was constructed with a limited number of alternatives in mind. A change in the mix of feasible alternatives may require a revision of the goal function. The ability to enquire interactively to determine what the current situation was with regard to criteria or alternatives, and then refine them, would also help the process. Increasing the decision-maker's opportunities to explore different aspects, to control the process themselves, and to build from what they know may be the best route for MCDA to take.

As one moves through the stages of developing a solution there is a deepening in the thinking that the decision-maker does, and also on the part of the decision-aid expert/advisor, starting with an intuition and, hopefully, reaching a climax with a full realisation of what the situation was about. The process of ordering and clustering criteria, and then of synthesising them at different levels, as was suggested above for the house case, will contribute to an enhanced understanding of the decision-maker's goal. Each such insight has potential implications for evaluating alternatives and how they interact with the various levels of activity.

This process should be seen as a form of fine tuning or focusing on the core of the decision. As such, this may mean, in some situations, not the continual creation of bigger and more complex models, but possibly the stripping away of less viable alternatives and less significant criteria. This can mean the implied or explicit incorporation of constraints, i.e. the satisficing concept. Using the construction example again, initially one may be concerned that the alternative bridge candidates will not fall down; subsequently one may be considering only physically viable candidates. At that point the "physical" objective could be excluded from the set of objectives that will be "traded of" against each other. To be fair to the decision-maker, some objectives, such as physical viability or political acceptability, simply may not be tradable. Similarly, some low level activities may not be comparable with high level activities, for example economic issues versus artistic. At the very least this may be context dependent: if there is a lot of money available, then a beautiful bridge is more of an option.

As the decision-maker moves through the process he or she should become better at making assessments. MCDA software such as Expert Choice should take this into account. The decision team (advisor plus maker) should take account of the degree of certainty with which a judgement is made. If a decision-maker indicates a weight of 5 is relevant for some comparison this should be returned to the software as a score of 5 if there is total certainty about it, or as "somewhere between 4.5 and 5.5" if there is a moderate degree of uncertainty, and even a "between 4 and 6" if there is a high degree of uncertainty. The software could then be used to synthesise a score for this and the other uncertain assessments which minimised inconsistency.

The software could also be usable as a review mechanism as part of the focusing process. If, for instance, one particular alternative was scoring much lower than the others the software could prompt the question "do you want to consider excluding this alternative?". Further, if after excluding some alternatives those remaining appear to score similarly on one particular criterion, it could prompt the question "how necessary is this criterion as a differentiator between the remaining alternatives?". The process should be able to move from the macro to the micro, ultimately focusing on the key viable alternatives and the key differentiating issues. It may happen that a unique best alternative appears. If not, it might be appropriate to re-cast the differentiating criteria in terms which are meaningful in the context of the remaining alternatives. The key idea is that of focus and facilitating the mind of the decision-maker to make judgements.

To the extent that the decision-aid advisor fails or succeeds in helping the decision-maker reach this climax, which might be measured by the degree of satisfaction felt by the decision-maker, he or she could be deemed to be contributing to the changing of a situation of a OR-DA problem (see Roy, 1994, p.24). Also, there must remain a subjective aspect in the determination that the best solution has been realised. What seemed perfect this evening might seem faulty tomorrow if one of the decision participants discovers overnight that some important element has been missed. This does not prove that procedures and methods in OR-DA cannot have a scientific foundation, just that there can never be certainty that the best solution has been found.

Improvements in solutions are found in the relationship between the DA advisor and the decision-maker. The skilled advisor should try to assess how the decision-maker relates to the problem, i.e. where he or she is in the solution process. This involves diagnosing what stage of thinking the decision-maker has reached (Brugha, 1995c) and doing this at a variety of levels such as: of handling MCDA procedures, of working with the particular model or software, of dealing with the industry or problem type, and of handling the particular situation. A decision-maker might understand the industry, be learning how to work with the software, and yet only have an intuition about how to deal with the problem at hand. The decision advisor diagnoses which aspects needs development and gently moves the decision-maker along. This is totally different to traditional O.R. where, once the problem is recognised, the process is much less dependent on input from the decision-maker.

Experiments With Alternative Approaches

During the Spring of 1996 some of the above ideas were tested by groups of final year students of the Bachelor of Commerce Degree in University College Dublin as part of their course assignments. The decisions modelled mainly had to do with choice of mode in the final year of the degree or choice of route on graduation: Master of Management Science, MIS option in the Master in Business Studies, etc. Part of their task was to compare *Naive AHP* with *Structured AHP*, and then to compare both with SMART. *Naive AHP* corresponds to the usual approach as in Saaty's (1990a) house example, which means pairwise comparisons of each criterion with every other criteria. *Structured AHP* meant following the approach of Figure 1 above and using qualitative similarities to group the criteria.

The most telling comment the students reported was the positive feedback they got from those who participated. The process helped to focus their minds about their educational and career choices. The importance of the process as distinct from the traditional "black box" solution came as a surprise to some. It actually helped some of their colleagues to develop and focus their thoughts. This led to suggestions about the process, that it should start with a broad discussion of alternatives and criteria, then move into simple rankings of both, and finally do more structured modelling.

SMART was seen as user friendly, a good way to start, possibly to help reduce the problem and help to identify unimportant criteria roughly and quickly. The visible scale was an advantage over the AHP. SMART was not good for modelling decisions with more than three criteria.

The *Naive AHP* caused a problem when there were many criteria. This led to an excessive number of questions, a drop off in interest towards the end of the questioning process and higher inconsistencies. The *Structured AHP* led to higher consistency scores. The grouping of criteria helped respondents become aware of criteria or attributes that they might have forgotten. The questions were more specific and easily understood. However, if the structuring was poorly done there was a danger of confusion. It required a greater understanding of the issues in order to do the structuring. Also, the added levels might led to distortions in the global weightings. Generally with the AHP there were some initial difficulties with the scoring of 1 to 9 causing respondents to wish to return and review some of their first answers. However, it was felt that the AHP gives an accurate result which was also understandable. The Expert Choice software was helpful especially if the respondent answered using the software. It allowed for easy and quick interpretation and showed up inconsistent answers.

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