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AHP METHOD IN GAME THEORY

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INTRODUCTION

This paper uses the Analytic Hierarchy Process in game theory to evaluate a payoff matrix, and enables a game theory model to be used extensively in an economic competition system. This method overcomes the obstacles produced in payoff matrix, thus, enables people to deal with competition tactics with mathematical method. It can also be applied to the study of business and coordination mechanisms and cooperative behavior in society. Therefore, the combination of AHP and game theory has a bright future.

1. PROBLEMS

Game theory becomes more and more important in a market economy. Especially in the international competitions to improve the wealth in society and whole national strength. Therefore, it is very important to use game theory in economic strategy and scientific decisions.

Nevertheless, the application of game theory is very rare. The reason is that the theory has not developed into its applicable stage, making it difficult to use it.

For example, a social system is usually complicated, there is not sufficient quantitative data, therefore, it's difficult to construct its' payoff matrix. For lack of this most key information, the application of game is very difficult. In addition, there is a misconception about its mathematical application, and the traditional concepts believe that mathematical methods must be very accurate and have strict conditions. Actually, the use of a game model is not merely a problem of quantitative analysis. it is also includes the process to establish the game model to further understand realistic problems and to provide sufficient thinking to solve the problems scientifically. Therefore, it is critical to establish the conception of mathematical model, after that, its technical problems are involved, the model can solve the problems. It is necessary to solve the

problems of the applications of game theory from the point view of conception and techniques. The use of Analytic Hierarchy Process can calculate the payoff matrix, therefore, provides a new way for the use of game theory.

2. BASIC MODEL OF GAME THEORY

Game theory includes three essential factors: the insider, the tactics, and the gain and loss of one game, these factors are given separately in following table.

A \ B	b_1	b_2	···	b_n
a_1	C_{11}	C_{12}	···	C_{1n}
a_2	C_{21}	C_{22}	···	C_{2n}
⋮	⋮	⋮	···	⋮
a_m	C_{m1}	C_{m2}	···	C_{mn}

Table 1

Supposing that insider A synthetically apply his m kinds of tactics by using the probabilities of $X_1, X_2, \ldots, X_m$ separately, and that insider B synthetically use his n kinds of tactics by using the probabilities of $Y_1, Y_2, \ldots, Y_n$ separately, then the income expectation value of A is

$$V_a=E(X,Y)=\sum_i\sum_j X_iC_{ij}Y_j=\sum_i X_i(\sum_j C_{ij}Y_j).$$

A applies synthetic tactics, with the probabilities of $X_1, X_2, \ldots, X_m$, A synthetically uses tactics $a_1, a_2, \ldots, a_m$, and B just uses pure tactics b_j

The income expectation value of A is $V_a=\sum X_iC_{ij}$. Because B is rational, it is necessary to minimize to the income of A, therefore., $V_a=\min_j(\sum_i X_iC_{ij})$, but it is possible to find a best synthetic tactics for A:

$$V_a=\max_{\sum X^*_i=1}[\min_j(\sum_{i=1}^{m} X_iC_{ij})]=\sum_{i=1}^{m} X^*_iC_{ij}$$

For the same reason, B uses the synthetic tactics b_1, b_2, ...b_n of probability Y_j (j=1,2,..., n), A uses the pure tactic a_j, and B pays for

$V_b = \Sigma C_{ij} Y_j$

Surely, A must use a certain tactics to make the payoff expectation value of B become maximum.

$$Vb = \underset{i}{\text{Max}} \left(\sum_{j} C_{ij} Y_j \right).$$

But insider B manages to find a best synthetic tactics Y^*_1, Y^*_2, ... Y^*_n to make his expectation value of loss become minimum, therefore,

$$V_b = \underset{\Sigma Y_j = 1}{\text{Min}} \left[\text{Max} \left(\sum_{j=1}^{n} C_{ij} Y_j \right) \right] = \sum_{j=1}^{n} C_{ij} Y^*_j$$

$V_a = V_b = V$, looking for $(X^*_1, X^*_2, ... X^*_m)$, $(Y^*_1, Y^*_2, ... Y^*_n)$

When A uses $(X_1, X_2, ...X_m)$ to deal with pure tactics b_j, j=1, 2, ...n can be used to learn n inequality, indicating that A manages to find a group of synthetic tactics.

$$C_{11}X_1 + C_{21}X_2 + ... + C_{m1}X_m \geq V$$

. . . . . .

$$C_{1n}X_1 + C_{2n}X_2 + ... + C_{mn}X_m \geq V$$

$X_1 + ... + X_m = 1, X_i \geq 0, \ (i=1, 2, ..., m)$

For the same reason, for the matter of B, B uses $(Y_1, Y_2, ...Y_n)$ to deal with pure tactic a_i of A, so m inequalities can be written out..

$$C_{11}Y_1 + C_{12}Y_2 + ... + C_{1n}Y_n \geq V$$

. . . . . .

$$C_{m1}Y_1 + C_{m2}Y_2 + ... + C_{mn}Y_n \geq V$$

$Y_1 + ... + Y_n = 1, Y_j \geq 0, \ (i=1, 2, ... n)$

A expectation income maximum, $MaxV = V_a = \Sigma \Sigma C_{ij} X_i$

B expectation income minimum, $MinV = V_b = \Sigma \Sigma C_{ij} Y_j$

Game theory for the solution is simplified as linear programming model, therefore, so long as evaluation for payoff matrix, is conducted, the result can be obtained.

3. EVALUATION OF PAYOFF MATRIX AND ITS PRACTICE EXAMPLE

In following table, we can see that from b_i, we can evaluate payoff expenses of a_1, a_2, ..., a_m: C_{1j}, C_{2j}, ...C_{mj}, j=1, 2, ... n.

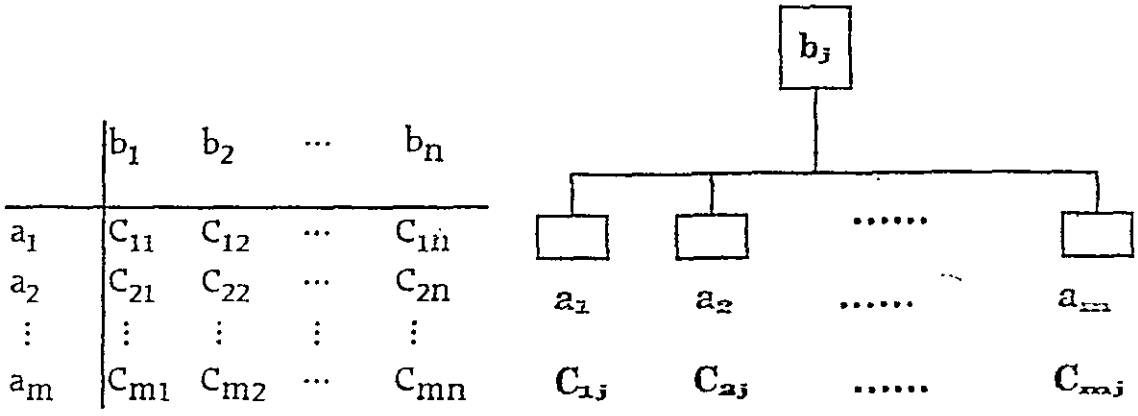

Table 2

In the table, vector $C_J=(C_{1J}, C_{2J}, ..., C_{mJ})^T$ can be evaluated by AHP. Let's use a practical example to indicate above-mentioned application results. Liaoning Province has two major beer corporations, Shenyang and Liaohe corporation, in a certain number of central cities of Liaoning area, they promote the sales for their products respectively. These 8 areas are Shenyang, Dalian, Dandong, Tieling, Chaoyang, Jinzhou, Anshan, Benxi, write respectively is $X_1, X_2, ..., X_8$.

Because of intensive competition, the two corporations have a very high level of expenses for advertisement. Supposing the total business in every city is fixed so as to guarantee supposition of zero sum.

Then, $a_1, ... a_8$ and $b_1, ..., b_8$ indicate respectively the advertisement business of both sides A and B in the 8 cities. Both sides of A and B put funds into advertisement tactics. Because the quantity of beer sold in a city is fixed, if the products of A are sold more, those of B will be sold less, and B will make a loss. So A will make a profit and B a loss. Therefore, it can be approximately looked as a condition to satisfy the zero sum game.

In each city, the sales quantity of corporations A and B can be calculated according to their advertisements, hence, the loss and profit values can be calculated. Assuming that

a_1, b_1 indicate that A, B have bigger advertisement input to Shenyang

a_2, b_2 indicate that A, B have bigger advertisement input to Dalian

. . . . . .

a_8, b_8 indicate that A, B have bigger advertisement input to Benxi

Then, under the condition of b_1, the profit and loss value of $a_1, a_2, ...$ a_8 is respectively evaluated as 0.178, 0.052, 0.089, 0.151, 0.1631, 0.1369, 0.0715, 0.1585;

Under the condition of b_2, the profit and loss value of $a_1, a_2, ..., a_8$ is respectively evaluated as 0.0715, 0.1885, 0.1521, 0.0979, 0.10, 0.0891, 0.1609, 0.150;

Under the condition of b_3, the profit and loss value of $a_1, a_2, ..., a_8$ is respectively evaluated as 0.1410, 0.1490, 0.1190, 0.0710, 0.105, 0.120, 0.155, 0.145;

Under the condition of b_4, the profit and loss value of a_1, a_2, ..., a_8 is respectively evaluated as 0.124, 0.141, 0.1321, 0.1479, 0.0891, 0.0819, 0.1417, 0.1483;

Under the condition of b_5, the profit and loss value of a_1, a_2, ..., a_8 is respectively evaluated as 0.1425, 0.1575, 0.1047, 0.0853, 0.1214, 0.1376, 0.0815, 0.1475;

Under the condition of b_6, the profit and loss value of a_1, a_2, ..., a_8 is respectively evaluated as 0.135, 0.145, 0.121, 0.099, 0.118, 0.0982, 0.1415, 0.1475;

Under the condition of b_7, the profit and loss value of a_1, a_2, ..., a_8 is respectively evaluated as 0.1425, 0.1475, 0.1116, 0.1274, 0.098, 0.082, 0.1435, 0.1475;

Under the condition of b_8, the profit and loss value of a_1, a_2, ..., a_8 is respectively evaluated as 0.1245, 0.1355, 0.1165, 0.1235, 0.0985, 0.092, 0.1453, 0.1647;

The aforementioned data form a C matrix, with which the advertisement tactics of A and B can be solved.

Side A (Shenyang Co.)

0.154, 0.159, 0.098, 0.102, 0.093, 0097, 0.151, 0.146.

Side B (Liaohe Co.)

0.1434, 0.1581, 0.113, 0.113, 0.0787, 0.098, 0.1419, 0.1366.

This result as advertisement tactics is able to provide such a kind of message, which tells us put major into where to advertise. At least it can be audio-visually explained as a distribution for the total value of advertisement investment.

4. CONCLUSION

Based on the use of the knowledge about the tactics of a_i and b_j with AHP method, this paper has calculated payoff matrix C_{ij} after obtaining a judge matrix with specialist advisory method.

This kind of method for determining payoff matrix have certain reliability and provide a new way for tactics research.

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