ORDERS OF MAGNITUDE COST RATIO CONSIDERATIONS IN RESOURCE ALLOCATIONS

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Summary: *Traditional Benefit Cost analysis considers benefits and costs measured in dollars, where benefits are usually expected savings. Since qualitative factors are not considered, it is not really possible to allocate resources in a rational manner. The Analytic Hierarchy Process (AHP) enables the measurement (on a ratio scale) of benefits that include both quantitative and qualitative factors. The AHP hierarchy should include every objective to which the funding alternatives under consideration contribute. The objectives are structured into homogeneous clusters that fit the organization's vision of its strategies and tactics. Elements in each cluster should be within approximately an order of magnitude of importance of one another, so that reasonable pairwise comparisons can be made. A problem arises, however, when alternatives under consideration differ in cost by several orders of magnitude. Unless the ratio of alternative benefits are commensurate with the ratio of their costs, a resource allocation optimization will tend to select the lower cost alternatives. This paper will present approaches for deriving ratio scale measures of alternative benefits that can span the same orders of magnitude as the alternative costs.*

Introduction

Resource allocation is performed in a variety of ways. Perhaps the most common is with a 'BOGSAT' (Bunch of old guys/gals sitting around talking). Proposals are discussed and resources allocated based on the discussion. Of course, there are better ways. Whatever the approach, it is clear that measures of the costs and benefits of the proposed projects/activities be known or estimated. While cost estimating (dollars, space, full time equivalents) is sometimes a difficult undertaking (and to which AHP can be applied), the estimation or evaluation of benefits is a much more formidable task because there are typically many qualitative as well as quantitative benefits that must be synthesized. AHP has been used to derive ratio scale measures of benefits in numerous resource allocations, both in the private and public sectors.

Let us now assume we have estimates of costs and benefits for each of the projects/activities. Given a budget constraint (we will assume a dollar constraint here, although the constraint could also be on space, full time equivalents, or a combination of these), there are three ways to decide on which projects/activities to fund. The first is to sort the projects/activities from high to low benefit and allocate funds until the budget is consumed. This naive approach should never be used. The second approach is to sort the projects/activities from high to low benefit/cost ratio and, again, allocate funds until the budget is consumed. This approach will almost always yield a greater total benefit for the same budget as the first approach. The third, and best approach is to use an optimization approach, which finds that combination of projects/activities that yields the highest total benefit subject to the budget constraint. The second approach is often a good approximation to the optimization, but can, in some cases, yield a much lower total benefit and cannot easily accommodate other constraints for things such as dependencies and exclusivities among the projects/activities. Both the second and third approaches require that both cost and benefit measures be ratio level measure. This is a natural for AHP, and one may even argue that even if a decision maker carefully used a 1-10 or 1-100 scale to estimate project/activity benefits, the resulting estimates might be considered ratio level.

Orders of Magnitude

Suppose now, that the set of projects/activities being considered spans several orders of magnitude – for example from tens of thousands of dollars to hundreds of millions of dollars. Assume that the benefit for each project/activity is somewhat commensurate with the cost. We would then need a way to estimate the benefits such that the highest benefit project/activity would be several orders of magnitude larger than the smallest. This is certainly not possible with a 1-10 or 1-100 scale, and we argue that it would be humanly impossible to accurately estimate benefits even if one were to use a $1\text{-}10^n$ scale. So something like AHP is essential. The question then is how can we use AHP and insure that the range in the measurements of benefit is commensurate with range in the costs of activities/projects being considered?

1. Pairwise Comparisons of Alternatives

The most straightforward approach would be to perform pairwise comparisons of the alternatives with respect to each covering (lowest level) objective to derive priorities. If one pre-sorts the alternatives based on their cost, then the comparisons on the lowest diagonal of the pairwise comparison matrix (the diagonal above what would be the diagonal that would contain all 1's if the entire matrix were displayed) would each tend to be greater than 1 and, if there is no great jump in cost between any two adjacent projects, within one or two orders of magnitude. Making such judgments would be straightforward using either the fundamental AHP 1-9 verbal scale, or extended scales such as the graphical pairwise comparison scale within Expert Choice. Priorities can be calculated based only on the n -1 judgments in this diagonal. However, judgments can also be made on higher diagonals in the matrix in order to improve accuracy. As an example, Figure 1 shows alternatives with costs ranging from over $1000 to $40. Judgments on the two diagonals with respect to one of the covering objectives in the hierarchy are shown in Figure 2. The ratios for judgment in higher diagonals will involve alternatives that are further apart in cost and hence tend to have higher ratios (again assuming that the benefits are approximately commensurate with costs). It might be wise to not make judgments when the ratios exceed an order of magnitude when using the fundamental verbal scale, or two orders of magnitude when using the Expert Choice graphical pairwise scale.

Alternative	Costs
☑ A	43000
☑ B	34000
☑ C	23000
☑ D	18000
☑ E	3000
☑ F	200
☑ G	94
☑ H	83
☑ I	65
☑ J	40
☑ K	28
☑ L	18

Figure 1 – Projects Sorted by Cost

	A	B	C	D	E	F	G	H	I	J	K	L
A		2.0	3.0									
B			3.0	3.0								
C				1.0	8.0							
D					6.0	9.0						
E						9.0	8.0					
F							3.0	4.0				
G								1.0	2.0			
H									2.0	3.0		
I										2.0	4.0	
J											2.0	4.0
K												2.0
L	Incon: 0.04											

Figure 2 – Pairwise Comparisons of Alternatives Under One Covering Objective

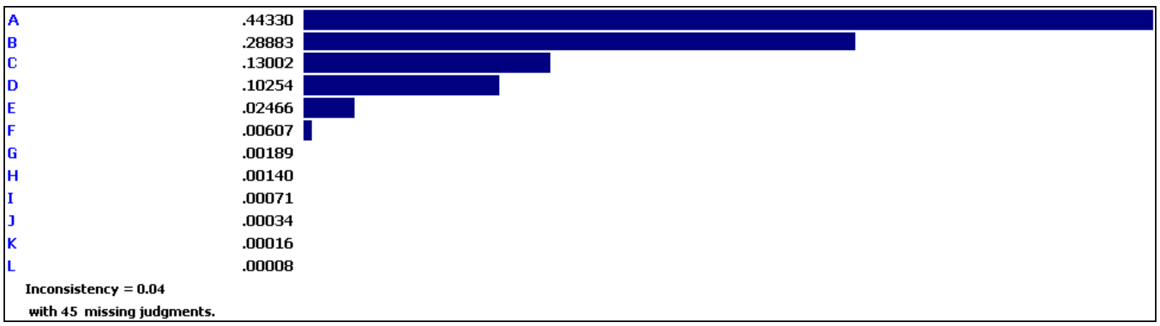


Figure 3 – Derived Priorities of Alternatives for One Covering Objective

Direct Assessment of Alternatives (Ratings)

When considering a large number of alternatives, it is sometimes more expedient to use a ‘ratings approach’ or direct assessment of alternatives instead of making pairwise comparisons as explained above. The ratings approach consists of defining “intensities” of achievement or preference with respect to each of the objectives. These intensities are used in place of alternatives in the first stage of the evaluation. For example, instead of comparing the relative preference for two specific alternatives with respect to VISIBILITY, we would compare the relative preference for a non-specific alternative that possesses GREAT visibility to some other alternative that has LOW visibility. This results in measures of preference for the intensities. A ratings “spreadsheet” is then used to evaluate each alternative as to its intensity on each objective. With the ratings approach, pairwise comparisons are made for the objectives, as well as for the intensities under each objective. The results are ratio scale priorities for the importance of each objective, as well as ratio scale priorities for the intensities below each objective. Then, using the ratings scales in the data grid of Expert Choice, each alternative is evaluated as to its intensity for each objective. The ratio scale priorities are then summed to give an overall ratio scale measure of the preference for the alternatives. We will next look at several ways to do this when the alternatives being considered differ by orders of magnitude in cost and presumably in benefit.

2. Broad Hierarchy of Objectives

When the ratings approach is used to evaluate alternatives that differ by several orders of magnitude in cost, we must somehow allow for the derived benefits to differ by several orders of magnitude as well. There are several approaches to achieve this. First, it is likely that larger more costly projects/activities will contribute to more of the covering objectives in a broad hierarchy of objectives than smaller lower cost projects/activities. Even if the priorities for the rating intensities under most or all of the covering objectives differ by only one or two orders of magnitude, it is possible for the synthesized alternative priorities to differ by a greater amount because the will be more instances where no contribution is made to a covering objective for the lower cost alternatives than for the more costly alternatives. This will depend greatly on the breadth and detail of the AHP hierarchy itself.

3. Use More Rating Intensities to Allow For Greater Differentiation in Priorities

It is advisable to define a sufficiently large number of rating intensities so that when larger cost projects are (typically) rated with the higher priority intensities and lower costing projects are (typically) rated with lower priority intensities, there is a significant ratio between the higher priority intensities and the lower priority intensities. The derivation of the priorities for the rating intensities can, even with the verbal fundamental scale of 1-9, span several orders of magnitude. For example, the results of the pairwise judgments between the elements of the top two diagonals for the rating intensities shown in Figure 4 and Figure 5 span a range with a ratio of 5000:1!

	Outstanding	Excellent	VG to Exce	Very Good	Good to Ve	Good	Fair to Goo	Fair	Just a Tad
Outstanding		3.0	7.0						
Excellent			4.0	6.0					
VG to Excellent				3.0	8.0				
Very Good					3.0	7.0			
Good to Very Good						4.0	7.0		
Good							7.0	8.0	
Fair to Good								5.0	9.0
Fair									5.0
Just a Tad	Incon: 0.05								

Figure 4 – Pairwise Comparisons of Rating Intensities

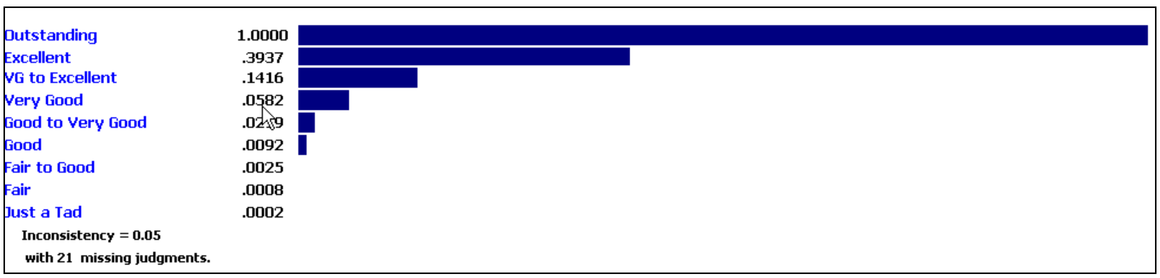


Figure 5 – Derived Priorities of Rating Intensities

Outstanding	Excellent	VG to Excell	Very Good	Good to Very	Good	Fair to Good	Fair	Just a Tad
1 (1.00000)	2 (.39365)	3 (.14164)	4 (.05816)	5 (.02191)	6 (.00916)	7 (.00252)	8 (.00085)	9 (.00021)

Ideal mode			RATINGS
Alternative	Total	Costs	obj1 (L: 1.000)
A	1.00000	43000	Outstanding
B	.39365	34000	Excellent
C	.14164	23000	VG to Excellent
D	.14164	18000	VG to Excellent
E	.05816	3000	Very Good
F	.00916	200	Good
G	.00916	94	Good
H	.00252	83	Fair to Good
I	.00085	65	Fair
J	.00252	40	Fair to Good
K	.00085	28	Fair
L	.00021	18	Just a Tad

Figure 6 – Alternatives Rated Under One Covering Objective

4. Use Linked Sets of Ratings for High and Low Cost Projects

Another approach is to define one set of rating intensities to be used for high cost or 'large' alternatives and another for low cost or 'small' alternatives. The pairwise comparisons for the ratings scale is shown in Figure 7 and includes one set of comparisons among the large alternatives (upper left), another set among the small alternatives (lower right) and one comparison that links the large and small. The resulting priorities are shown in Figure 8.

L: Outstanding		3.0	4.0	5.0				
L: Excellent			2.0	3.0				
L: Good				4.0				
L: Fair					9.0			
S: Outstanding						2.2	6.1	9.0
S: Excellent							2.6	4.6
S: Good								10.1
S: Fair	Incon: 0.11							

Figure 7 – Rating Intensities for Large and Small Alternatives

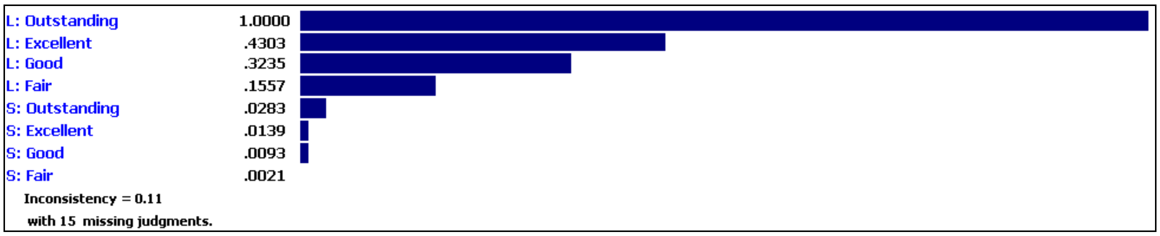


Figure 8 – Priorities for Rating Intensities for Large and Small Alternatives

Figure 9 shows the resulting rating scale applied to large alternatives (A – E) and small alternatives (F-L).

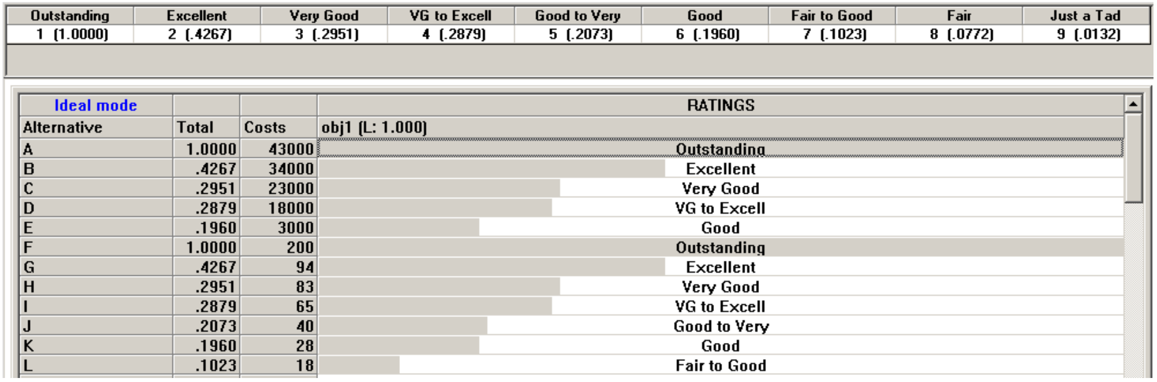
L: Outstandi	L: Excellent	L: Good	L: Fair	S: Outstandi	S: Excellent	S: Good	S: Fair
1 (1.0000)	2 (.4303)	3 (.3235)	4 (.1557)	5 (.0283)	6 (.0139)	7 (.0093)	8 (.0021)

Ideal mode			RATINGS
Alternative	Total	Costs	obj1 (L: 1.000)
A	1.0000	43000	L: Outstandi
B	.4303	34000	L: Excellent
C	.3235	23000	L: Good
D	.3235	18000	L: Good
E	.1557	3000	L: Fair
F	.0283	200	S: Outstanding
G	.0139	94	S: Excellent
H	.0139	83	S: Excellent
I	.0093	65	S: Good
J	.0093	40	S: Good
K	.0021	28	S: Fair
L	.0021	18	S: Fair

Figure 9 – Ratings using Intensities for Large and Small Alternatives

5. Cluster Alternatives by Cost and Link Alternatives Near Borders

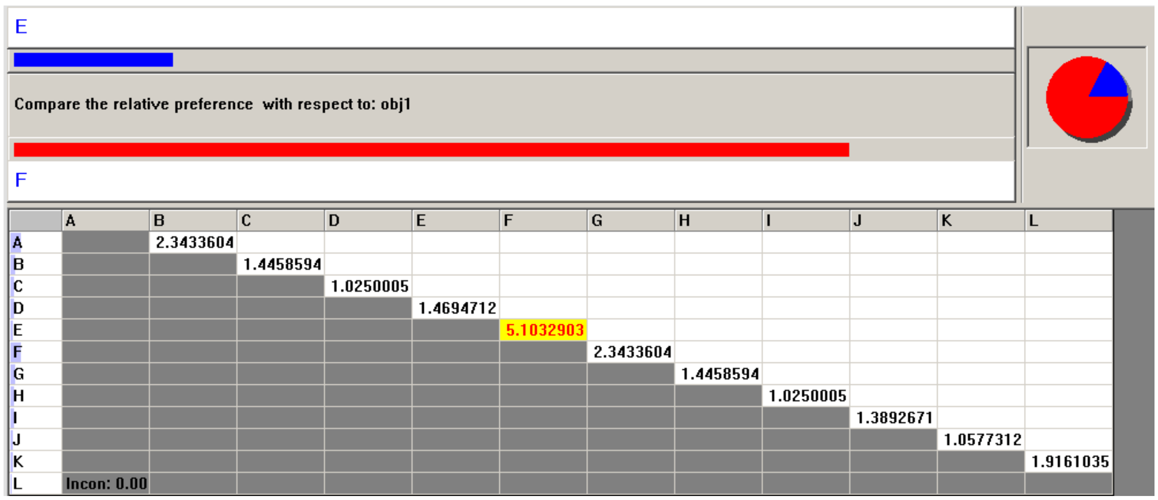
Finally, large and small alternatives can initially be rated using the same rating scales and subsequently adjusting the priorities under each covering objective by converting the priorities to pairwise comparisons and replacing the pairwise comparisons in cells at the borders of the large and small alternative sets. Figure 10 shows the initial ratings of both large and small alternatives using the same ratings scale for each. However, what might be considered Outstanding for a 'large' alternative might be much more than an outstanding contribution for a 'small' alternative. The ratios of the priorities for each of the alternatives, is shown in Figure 11, where the highlighted cell contains a ratio based on rating intensities applied to a large alternative and a small alternative, and therefore needs to be replaced, as shown in Figure 12. The resulting priorities are shown in Figure 13. These priorities are shown in the data grid, normalized on percent of maximum, in Figure 14.



Outstanding	Excellent	Very Good	VG to Excell	Good to Very	Good	Fair to Good	Fair	Just a Tad
1 (1.0000)	2 (.4267)	3 (.2951)	4 (.2879)	5 (.2073)	6 (.1960)	7 (.1023)	8 (.0772)	9 (.0132)

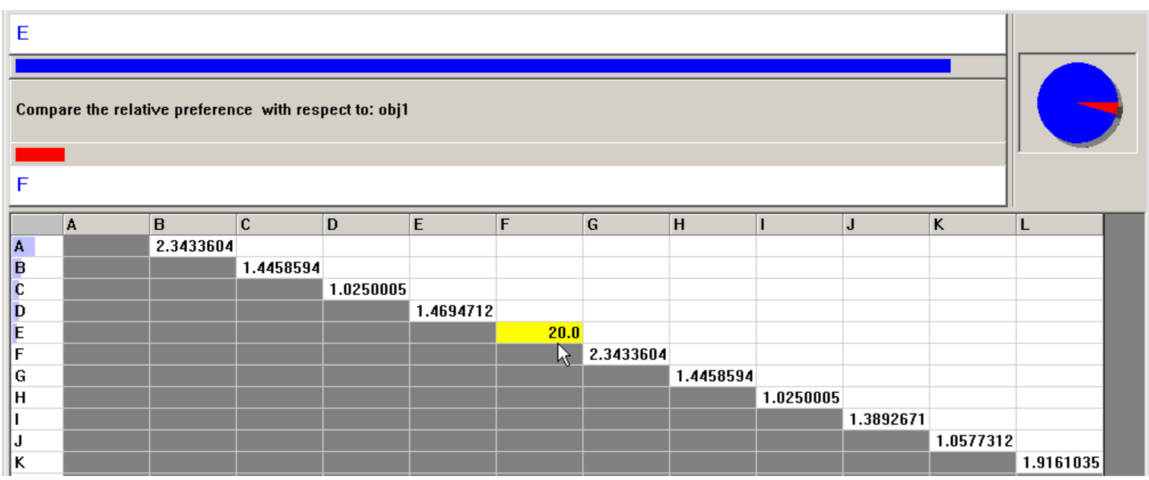
Ideal mode			RATINGS
Alternative	Total	Costs	obj1 (L: 1.000)
A	1.0000	43000	Outstanding
B	.4267	34000	Excellent
C	.2951	23000	Very Good
D	.2879	18000	VG to Excell
E	.1960	3000	Good
F	1.0000	200	Outstanding
G	.4267	94	Excellent
H	.2951	83	Very Good
I	.2879	65	VG to Excell
J	.2073	40	Good to Very
K	.1960	28	Good
L	.1023	18	Fair to Good

Figure 10 – Rating High and Low Cost Projects Using Same Rating Intensities



	A	B	C	D	E	F	G	H	I	J	K	L
A		2.3433604										
B			1.4458594									
C				1.0250005								
D					1.4694712							
E						5.1032903						
F							2.3433604					
G								1.4458594				
H									1.0250005			
I										1.3892671		
J											1.0577312	
K												1.9161035
L	Incon: 0.00											

Figure 11 – Converting to Pairwise; Must Replace Judgment Between E and F



	A	B	C	D	E	F	G	H	I	J	K	L
A		2.3433604										
B			1.4458594									
C				1.0250005								
D					1.4694712							
E						20.0						
F							2.3433604					
G								1.4458594				
H									1.0250005			
I										1.3892671		
J											1.0577312	
K												1.9161035

Figure 12 – Judgment between a High Cost and Low Cost Alternative

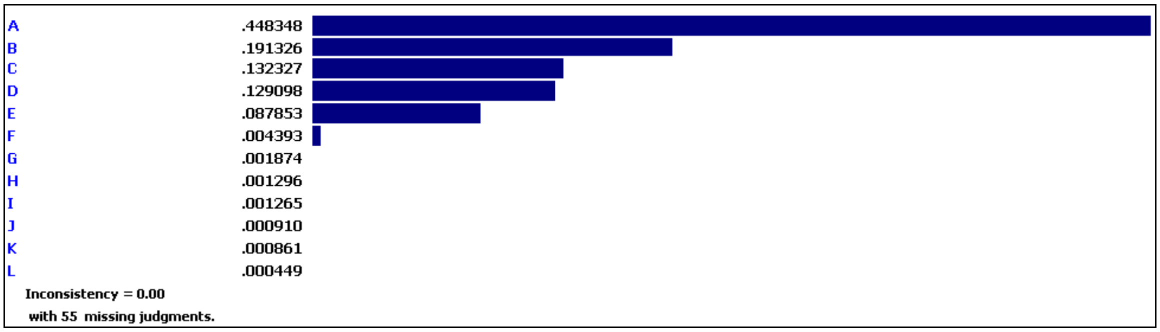


Figure 13 – Priorities Adjusted Based on Judgment Between a High Cost and Low Cost Alternative

Ideal mode	%Max		Pairwise
Alternative	Total	Costs	obj1 (L: 1.000)
A	1.0000	43000	1.00000
B	.4267	34000	.42673
C	.2950	23000	.29500
D	.2877	18000	.28767
E	.1957	3000	.19571
F	.0098	200	.00978
G	.0042	94	.00417
H	.0029	83	.00288
I	.0028	65	.00281
J	.0020	40	.00202
K	.0019	28	.00191
L	.0010	18	.00100

Figure 14 – Priorities Normalized on %Maximum in Data Grid

Additional accuracy can be obtained by making judgments for more than just the two alternatives at the border between large and small alternatives (or between any two adjacent clusters in a more elaborate example.) For example, judgments between two large alternatives (D and E) and two small alternatives (F and G) are show in Figure 15. The resultant priorities are shown in Figure 16. When these judgments are combined with those for all the alternatives (using a linking feature of Expert Choice) the priorities obtained are shown in Figure 17. These priorities are similar to, but more accurate than those shown in Figure 14. In either case, it is easy to see that the priorities can span several orders of magnitude if necessary.

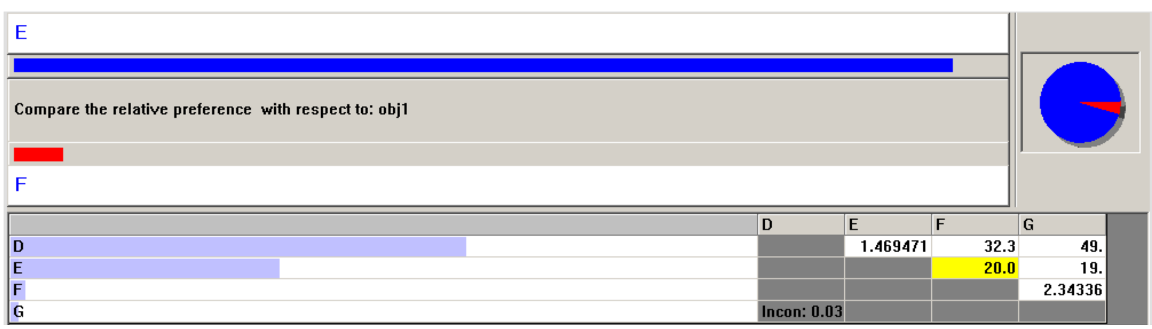


Figure 15 – Pairwise Comparisons of Two Large and Two Small Alternatives

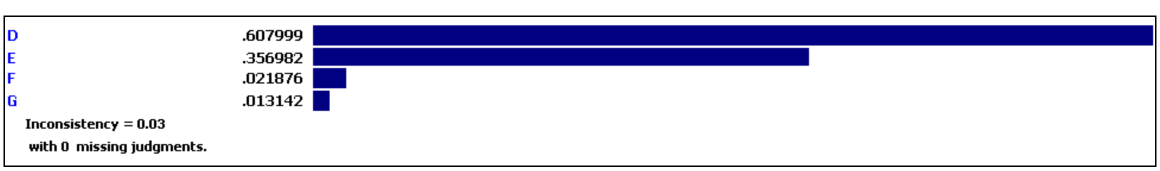


Figure 16 – Relative Priorities of Two Large and Two Small Alternatives

Ideal mode	%Max		Pairwise
Alternative	Total	Costs	obj1 (L: 1.000)
A	1.00000	43000	1.00000
B	.44080	34000	.44080
C	.32452	23000	.32452
D	.34621	18000	.34621
E	.20798	3000	.20798
F	.01270	200	.01270
G	.00743	94	.00743
H	.00451	83	.00451
I	.00393	65	.00393
J	.00258	40	.00258
K	.00229	28	.00229
L	.00116	18	.00116

Figure 17 – Priorities After Linking with Judgments for Two Large and Two Small Alternatives

Summary and Conclusion

We have addressed the situation that can arise when alternatives under consideration for resource allocation differ in cost by several orders of magnitude. The solution must allow for the ratio of alternative benefits to be measured on a scale that is commensurate with the ratio of their costs, otherwise there will be a tendency to fund the low cost alternatives. We have presented several approaches that individually, or in combination, can be effectively employed.