IMPROVED STRUCTURAL WEIGHT ADJUSTMENT IN TOP DOWN ORIENTED CONVENTIONAL AHP HIERARCHIES

Diederik J.D. Wijnmalen
Division for Operational Research and Business Management
TNO Physics and Electronics Laboratory
P.O. Box 96864, NL-2509 JG The Hague, The Netherlands
wijnmalen@fel.tno.nl

Keywords: conventional AHP, weight normalisation, weight adjustment, hierarchy orientation

1. Introduction

In a fairly recent paper Jonathan Barzilai (1998) showed that equivalent hierarchies can produce non-equivalent results. He used an example where two marketing strategies are compared on total annual revenue. The four executives of the company are each using their own model; the models differ in the way the five stores of the company are grouped together according to three territories. These four hierarchies are equivalent in the sense that they all are equivalent models or descriptions of the problem of choosing the best marketing strategy. Under the assumption that it does not matter from which store one extra dollar revenue originates and no other circumstantial reasons exist for making any difference in importance, all weights are made equal within each family of hierarchy elements. According to conventional AHP a weighted sum function is set up to compute the composite alternative priorities after the usual unity sum normalisations per node. Barzilai then shows that the weighted sum functions differ in the global weights of the stores thus producing different results, even rank reversals, despite the consistency of the judgements. He concludes that AHP's incorrect decomposition rule involving multiple normalisations must be the cause of the non-equivalent results and rank reversals that go with them, thereby invalidating AHP.

2. The improved structural weight adjustment procedure

Three of the four marketing hierarchies are incomplete hierarchies where the nodes on a level are not connected to all nodes on its adjacent higher or lower level. In such hierarchies a structural imbalance exists if this incompleteness results in node families of unequal size on the same level, or if paths of unequal length exist from the hierarchy top to its bottom level. The former type of incompleteness occurs in Barzilai's example. This is particularly important in view of the unity sum normalisation. There, the average weight $1/n$ of the child of a parent-node depends on the size n of that node's family, and so does each individual child's weight. The global weights of the bottom level criteria therefore depend on the sizes of their (ancestor-)families. Conceptually equivalent hierarchies, but having different clustering of nodes can thus produce different global weights resulting in non-equivalent final priorities.

A structural adjustment procedure adjusting the weights according to the hierarchical structure was proposed by Saaty in 1980 and implemented in the Expert Choice package, but presumably largely ignored thus far. It is good enough for our purpose. It is, however, not good enough for general situations with incomplete hierarchies of more than three levels or unequal path lengths from top to bottom. The adjustment is a very local procedure as it only considers two adjacent hierarchy levels at a time, and makes no distinction between alternatives and criteria. An improved procedure will be presented; it adjusts all weights at all levels above the bottom criterion level (i.e. the criterion leaf level), thus removing, as it were, the influence of the structure on that bottom criteria's global weights. The improved adjustment boils down to giving more weight to larger sub-hierarchies, and less weight to smaller ones according to the relative number of criterion leaves they are encompassing. This will avoid weight dilution by large node families. Barzilai's equivalent hierarchies will produce equivalent results when using structural weight adjustment.

What the adjustment in fact does, is making the average of the global weights within each combination of criterion leaf clusters reflect the weight of that combination's ancestor criterion. It is a size-based adjustment based on a weighted averaging process where the weights reflect sub-hierarchy sizes. Should an extra dollar from one territory for strategic reasons of company growth be more important than an extra dollar from another territory, then the adjustment would make the global weights of the stores reflect that difference; not necessarily the individual global store weights but their average per territory. Without the adjustment, the hierarchical structure would interfere as it did in the Barzilai example. This will be shown; the importance of the issue surpasses this specific example, though.

3. Top down and bottom up oriented hierarchies

In a top down oriented hierarchy, importance originates from the hierarchy top, the goal, or rather the context of the decision problem; it is then distributed downward. For this type of hierarchy, conventional AHP's independence axiom holds: importance of lower level elements is related to higher level elements, not vice versa. In a bottom up hierarchy, importance originates from the alternatives (or the criterion leaves). This is the situation for which either MAVT is valid, where weighting values reflect the relative value of swings on score scales, or linking pin AHP (ref. Schoner et al., 1993), where relative attractiveness of referent alternatives is assessed and AHP's independence axiom is or, rather, should be violated.

In *top down* hierarchies the global weights are breakdowns or *decomposites* of the global weights of ancestors using local weights. The global weights of the criterion leaves therefore are a result of the decomposition process. While the multiple unity sum normalisations serve as a means to decide upon how to distribute the parent weights among their children, they are distorting this process in the sense that they make the global weights dependent on family sizes. This can be off-set by the size-based adjustment described above. In *bottom up* hierarchies global weights are *composites* of lower level global weights. Now, comparison of criterion leaves is a starting-point, with the global weights of higher-levelled elements being the result. MAVT assumes that a non-hierarchical weighting method is used at the criterion leaf level with no need for further weighting at higher levels. The node-wise comparisons of the AHP however imply that a hierarchical weighting method is used. This should relate, or rather *apportion*, the different node families in such a way that a common ratio scale is derived when computing the composite alternative priorities in multi-level hierarchies. We conjecture without elaborating that in bottom up hierarchies: a) either linking pin AHP (preferred because easier to do) or total-referenced AHP or average-referenced AHP with the size-based adjustment should be applied to ensure this commensurability; and b) a weighting mass-based (rather than size-based) adjustment procedure may be used to correctly incorporate pre-established weights at higher levels when the orientation is ambiguous.

We conclude that Barzilai has confused the two approaches, and either should not have ignored AHP's (original) structural adjustment or should have used the proper AHP variant for his example.

References

Barzilai, J. (1998), "On the decomposition of value functions", *Operations Research Letters*, 22, 159-170.
Saaty, T.L. (1980), *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation*, Mc-Graw-Hill, 42-43.
Schoner, B., Wedley, W.C., Choo, E.U. (1993), "A unified approach to AHP with linking pins", *European Journal of Operational Research*, 64, 384-392.