AHP/ANP-BASED RISK ASSESSMENT CUSTOMISATION

ABSTRACT

Complex conditions of surrounding environment and a lack of reliable information about it make contemporary decisions prone to risk very much. This is why a reliable risk assessment becomes even more important today. Imperfect nature of available information means that risks can be often expressed in qualitative measures, only. Inherent merits make AHP/ANP technique appropriate for risk assessment in such the case, therefore. Note, however, that adequate risk assessment depends on the ability to address risk attitude. The attitude may deal with both risk neutrality, risk apprehension, and tendency towards risk taking. The common application of the technique for risk assessment is nevertheless based on the use of Saaty's linear judgement scale that is rather adequate of coping with a casual – neutral – attitude to risk, only. The development of the technique provided nevertheless alternative non-linear judgement scales. This is why their applicability for a reliable risk assessment while taking into account alternative – non-neutral – risk attitudes is discussed in the paper. Two scales which provide necessary means with this regard are finally recommended in the paper.

Keywords: AHP/ANP, judgement, application, risk assessment, risk attitude, crisp judgement scale.

1. Introduction

Intangibility-aware nature of AHP/ANP technique makes it appropriate for risk assessment in the case of dependence of imperfect information. However, the application of casual linear Saaty's a judgement scale with this regard seems to cover a single decision maker's attitude towards risk while preparing decision. A neutral attitude is dealt with this regard. It is nevertheless obvious that alternative attitudes also appear in practice of decision making (Fig.1). They cover both inclination towards risk taking as well as risk apprehension. Hopefully, some alternative non-linear judgement scales have been also proposed in course of AHP/ANP technique development. The availability of alternative judgement scales of such the kind makes it reasonable, therefore, to test their applicability to provide necessary means for risk assessment while taking into account covering alternative decision maker's risk attitudes.

2. Literature Review

AHP/ANP used to be often applied to support risk assessment. Although alternative non--crisp judgement scales have been often applied with this regard (McCauley Bell, & Crumption, 1997; Zheng et al., 2022), to name a few, Saaty's (2006) recommendation to use crisp judgements, only, was taken into account in the paper when dealing with the applicability of alternative judgment scales for risk assessment. The above mentioned assumption seems even more justified by the fact that alternative non-crisp representations of judgement scale levels are usually related with to levels of canonical Saaty's linear scale. The alternative crisp judgement scales were presented in different publications and resulted from a specific practical needs of AHP technique application. They have been proposed since the very beginnings of the development of the technique. Some of them were presented much later as in the case of Ishizaka et al. (2006). However, there is still a lack of a publication that would discuss the applicability of alternative judgement scales for AHP/ANP technique-based risk assessment.

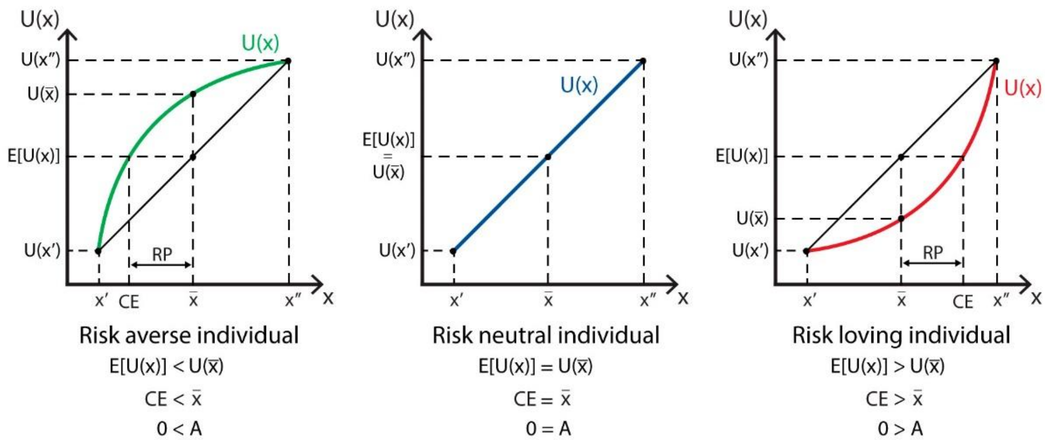

Figure 1. Illustration of different attitudes towards risk – the utility *U* vs. investment amount *x* case (source: https://policonomics.com/wp-content/uploads/2016/02/Risk-aversion.jpg, access date: 2022-08-02)

3. Hypotheses/Objectives

The suitability of alternative judgement scales for dealing with non-standard attitudes towards risk while preparing decisions is discussed in the paper. The considered main hypothesis says that there exist non-standard non-linear judgement scales that are well suited for a reliable risk assessment while taking into account decision maker's risk-inclined or risk- averse attitudes.

The main objective of the paper is to indicate alternative AHP/ANP judgement scales that would be suitable for a reliable risk assessment while taking into account both risk-averse and risk-inclined attitudes.

4. Research Design/Methodology

There are three kinds of alternative judgement scale types which differ, amongst others, in value expressing the highest scale levels:

- a real number less than 9 in the case of low point judgement scales,
- 9 in the case of regular scales,
- and a real number greater than 9 in the case of high point scales,

as well as in character of a discrete function that define judgments for the consecutive scale levels o = 1, 2…9. Linear, concave and convex functions are registered with this regard. The following AHP/ANP judgement scales are available:

1. Low point scale alternatives L:

   L1: Saaty's root scale: $a_{ij} = c\sqrt{o}$, where: c =1.

   L2: Root scale (Harker & Vargas, 1987): $a_{ij} = \sqrt[c]{o}$, where: c = $\sqrt{2}$.

   L3: Logarithmic scale (Ishizaka et al., 2006): $a_{ij} = \log_c o + 1$, where: $c = e$.

   L4: Multiplicative scale (Dodd et al., 1992): $a_{ij} = \exp\left[\tanh^{-1}\left(\frac{o-1}{H-1}\right)\right]$, where: $H = 1 + 6 \cdot \sqrt{2}$.

   L5: Finan & Hurley (1997) scale: $a_{ij} = (1+s)^{\frac{o-1}{2}}$, where: $s = [0.2, 0.732)$.

2. Regular scales N:

   N1: Standard linear Saaty's scale: $a_{ij} = c \cdot o$, where: c = 1.

   N2: Linear reciprocal scale (Ma & Zheng, 1991): $a_{ij} = \frac{9}{10-o}$.

   N3: Balanced scale (Salo & Hämäläinen, 1997): $a_{ij} = \frac{0{,}5+(o-1)\cdot s}{0{,}5-(o-1)\cdot s}$,
   where: s = 0,05 .

   N4: Equidistant scale (Rašković et al., 2008): $a_{ij} = v_o$, where:
   $v_o = v_{o-1} \cdot s$ for $o > 1, v_1 = 1, s = \sqrt[8]{9}$.

   N5: Finan and Hurley (1997) scale: $a_{ij} = (1+s)^{\frac{o-1}{2}}$, where: $s = 0.732$.

3. Highpoint scales H:

   H1: Saaty's quadratic scale: $a_{ij} = c \cdot o^2$, where: c = 1.

   H2: Exponential scale (Harker, & Vargas, 1987): $a_{ij} = o^c$,
   where: $c = \sqrt{2}$.

   H3: Geometric scale no. 1 (Lootsma, 1996; Lootsma, 1997):
   $a_{ij} = c^{o-1}$, where: $c = e$.

   H4: Geometric scale no. 2 (Lootsma, 1996; Lootsma, 1997):
   $a_{ij} = c^{o-1}$, where: $c = \sqrt{2}$.

   H5: Finan and Hurley (1997) scale: $a_{ij} = (1+s)^{\frac{o-1}{2}}$,
   where: $s = (0.732, 1]$.

   H6: Geometric scale no. 3 (Légrády et al., 1984; Kok & Lootsma, 1985): $a_{ij} = e^{\frac{o-1}{2}} = \exp\left(\frac{o-1}{2}\right)$.

   H7: Multiplicative scale (Dodd et al., 1992):
   $a_{ij} = \exp\left[\tanh^{-1}\left(\frac{o-1}{H-1}\right)\right]$, where: $H = 1 + \frac{14}{\sqrt{3}}$.

All available scales are considered in course of discussion about their suitability for expressing a non-standard nature of a decision maker's attitude towards risk.
Individual judgement scales are prepared for their direct comparison by means of judgement normalisation (a'_{ij}) according to the number expressing the highest scale level:

$$a'_{ij} = \frac{a_{ij}}{\max_{o}\{a_{ij}\}}. \tag{1}$$

Resulting approximation functions which describe normalised judgements for different judgement scales are presented in Figs.2 – 4. Figs.2 and 3 deal with all low point judgement scales and specific parametric Finan & Hurley's L5 judgement scale, respectively. Fig.4 is devoted to regular judgement scales. Figs.5 and 6 are devoted to all high point scales and a specific Finan & Hurley's H5 judgement scale, respectively.

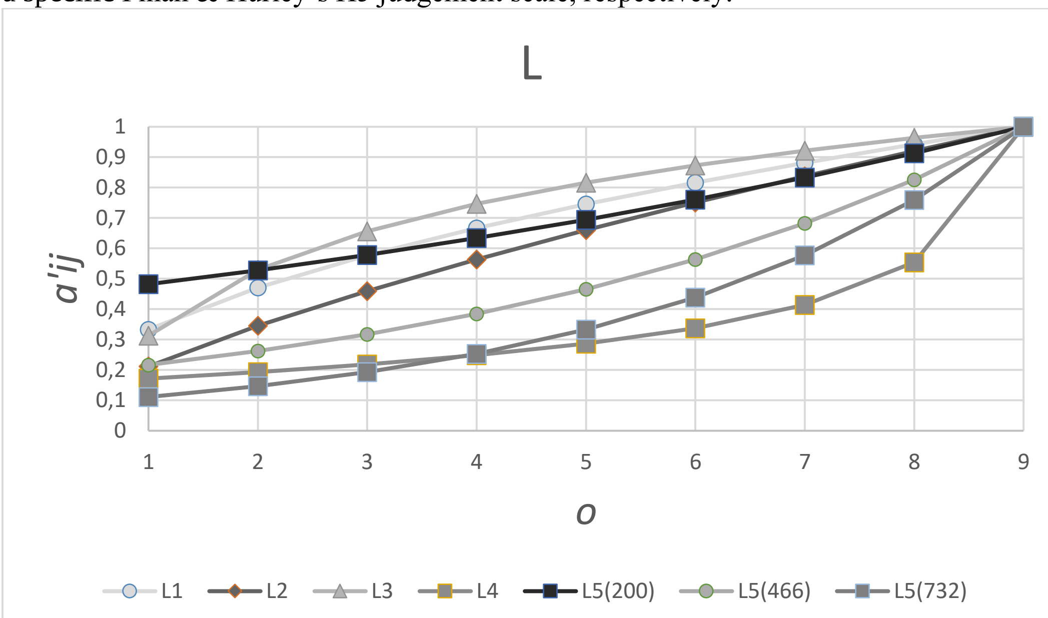

Figure 2. The comparison of normalised low point judgement scales

5. Data/Model Analysis

Shape of functions which express individual judgement scales and their actual performance in the case of a sample risk assessment analysis are applied to test their usability for taking into account diverse decision maker's risk attitudes.
The sample analysis pertains to relative assessment of five risk categories named as R1, R2, R3, R4, and R5. The order of categories corresponds with their names. Thus, R1 is the lowest while R5 is the highest category. The application of fundamental Saaty's scale to pair-wise comparisons of assumed risk categories results in the following judgement matrix:

$$\mathbf{A}=\begin{bmatrix} 1 & 3 & 5 & 7 & 9 \\ 1/3 & 1 & 3 & 5 & 7 \\ 1/5 & 1/3 & 1 & 3 & 5 \\ 1/7 & 1/5 & 1/3 & 1 & 3 \\ 1/9 & 1/7 & 1/5 & 1/3 & 1 \end{bmatrix} \quad (2)$$

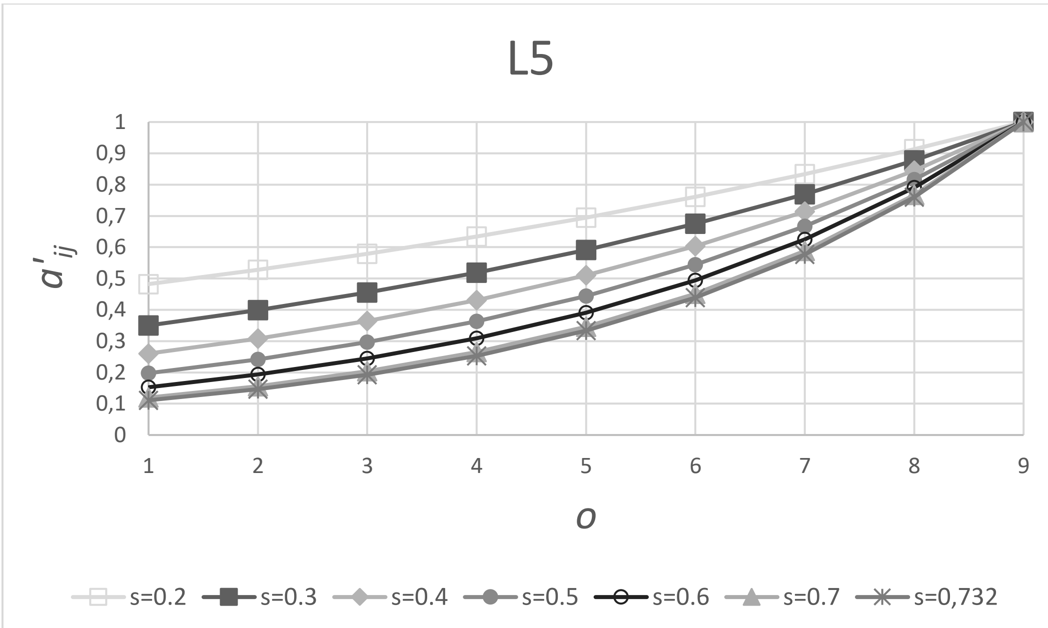

Figure 3. Illustration of parametric Finan & Hurley's low point scale

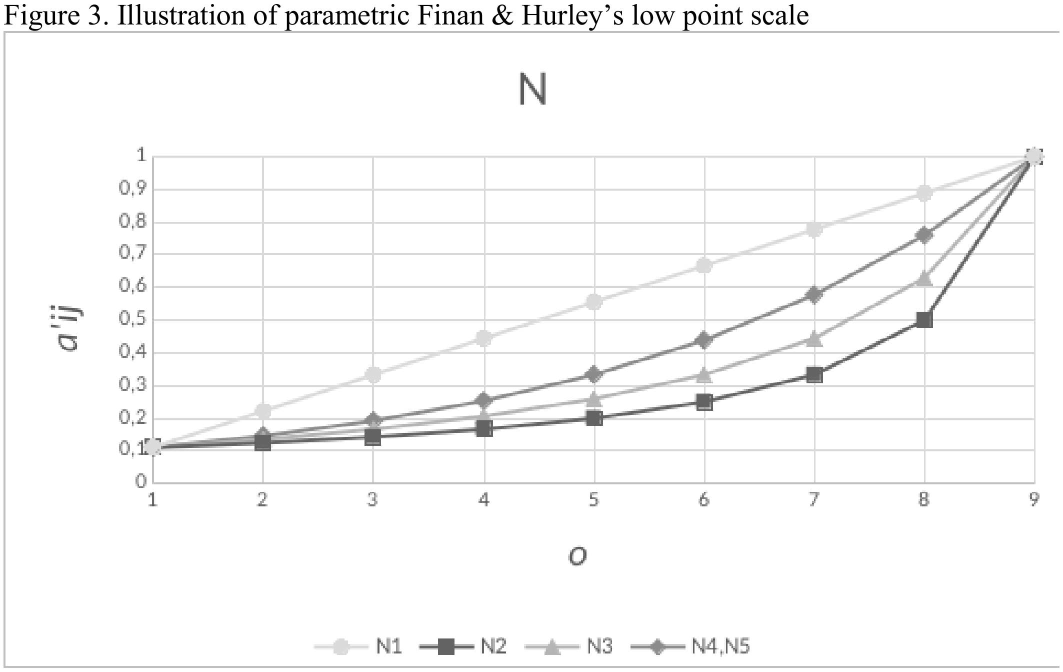

Figure 4. The comparison of regular judgement scales

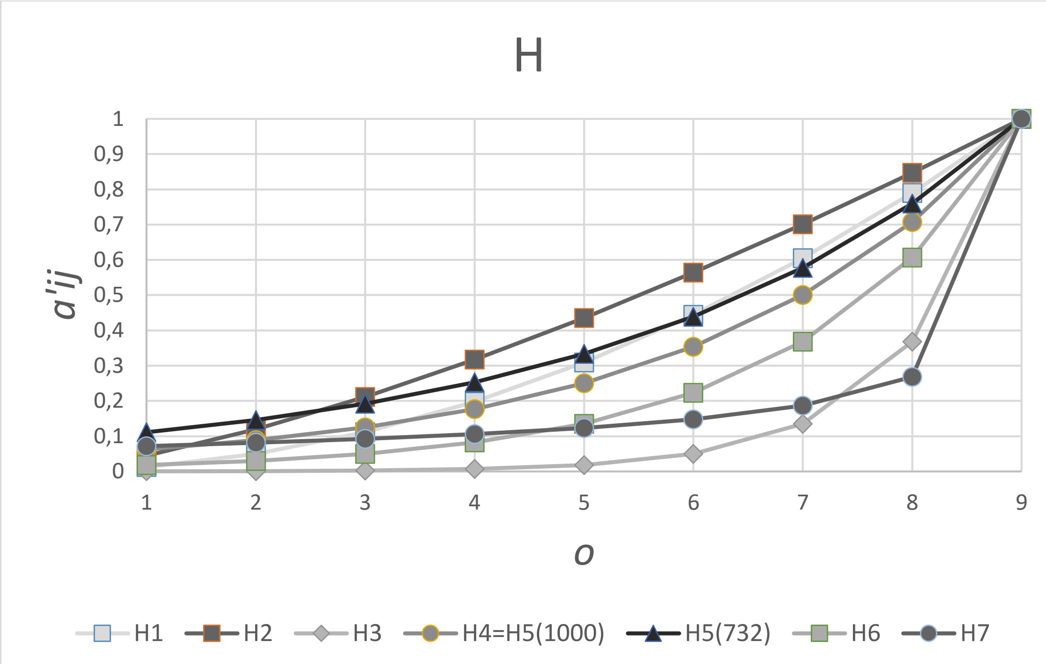

Figure 5. The comparison of high point judgement scales

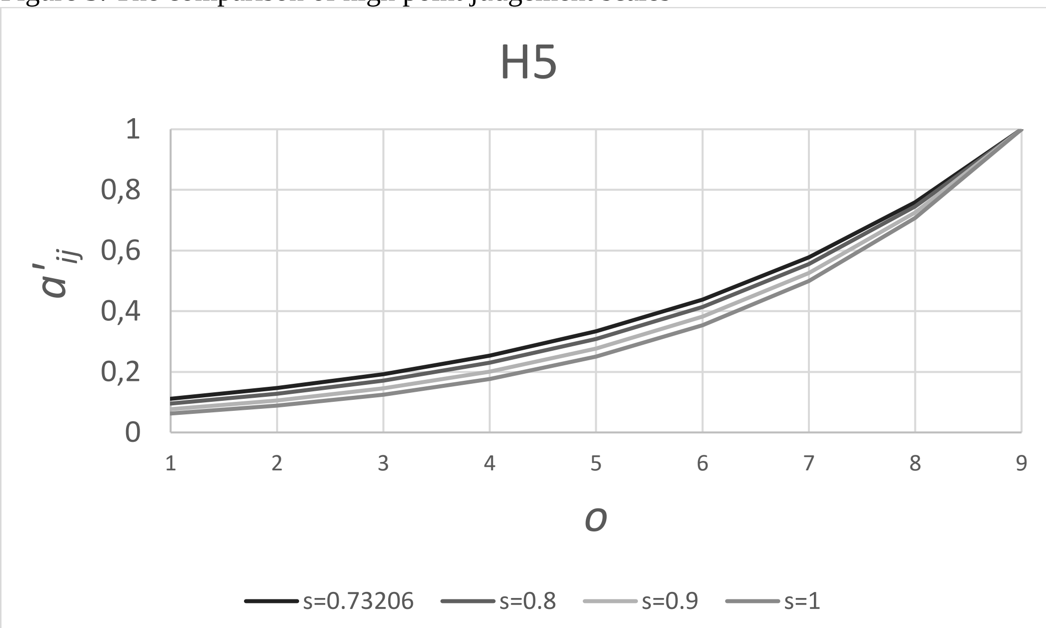

Figure 6. Illustration of parametric Finan & Hurley's high point scale

Standard REV is then applied to obtain relative priorities for risk categories (maximum right matrix **A** eigenvalue λ_{max} is equal to 5.237, and consistency ratio *c.r.* is equal to 0.053). Ideal priorities are as follows:

$$\mathbf{p} = \begin{bmatrix} 0.065 \\ 0.124 \\ 0.252 \\ 0.510 \\ 1 \end{bmatrix}. \tag{3}$$

In order to compare the effects of using of different kinds of AHP/ANP judgement scale on priorities for risk categories R1–R5, the following general matrix **A** scheme was used in the calculations for all non-standard AHP/ANP judgement scales:

$$\mathbf{A} = \begin{bmatrix} 1 & a_{ij}(o=3) & a_{ij}(o=5) & a_{ij}(o=7) & a_{ij}(o=9) \\ 1/a_{ij}(o=3) & 1 & a_{ij}(o=3) & a_{ij}(o=5) & a_{ij}(o=7) \\ 1/a_{ij}(o=5) & 1/a_{ij}(o=3) & 1 & a_{ij}(o=3) & a_{ij}(o=5) \\ 1/a_{ij}(o=7) & 1/a_{ij}(o=5) & 1/a_{ij}(o=3) & 1 & a_{ij}(o=3) \\ 1/a_{ij}(o=9) & 1/a_{ij}(o=7) & 1/a_{ij}(o=5) & 1/a_{ij}(o=3) & 1 \end{bmatrix} \tag{4}$$

Results of the application of different judgement scales are presented in Tab.1 where the highlighted rows relate to the most interesting judgement scales.

Table 1. Detailed results of calculations –risk category priorities (ideals)

Judgement scale	R1	R2	R3	R4	R5
L1	0.2543	0.3521	0.5024	0.7164	1
L2	0.1444	0.2282	0.3773	0.6227	1
L3	0.2070	0.3009	0.4523	0.6791	1
L4	0.2554	0.3753	0.4974	0.6558	1
L5 (s = 0.200)	0.4823	0.5787	0.6944	0.8333	1
L5 (s = 0.466)	0.2165	0.3174	0.4653	0.6821	1
L5 (s = 0.73204)	0.1111	0.1925	0.3333	0.5774	1
N1	0.0650	0.1236	0.2515	0.5099	1
N2	0.1885	0.2995	0.4226	0.5897	1
N3	0.1442	0.2410	0.3774	0.5891	1
N4	0.1111	0.1925	0.3333	0.5774	1
N5 (s = 0.73205)	0.1111	0.1925	0.3333	0.5774	1
H1	0.0043	0.0154	0.0633	0.2561	1
H2	0.0211	0.0520	0.1418	0.3836	1
H3	0.0003	0.0024	0.0183	0.1353	1
H4	0.0625	0.1250	0.2500	0.5000	1
H5 (s = 0.73205)	0.1111	0.1925	0.3333	0.5774	1
H5 (s = 0.73206)	0.1111	0.1925	0.3333	0.5774	1
H5 (s = 0.866)	0.0825	0.1539	0.2872	0.5350	1
H5 (s =1)	0.0625	0.1250	0.2500	0.5000	1
H6	0.0183	0.0498	0.1353	0.3679	1
H7	0.1630	0.2754	0.3755	0.5040	1

Based on Figs.2–6, it can be concluded that only one judgement scale alternative is capable of expressing a neutral attitude towards risk. It is represented by the most popular – standard – judgement scale N1. This is because it is the only one judgement scale described by means of a linear function.

Figs.2–6 also show that most AHP/ANP judgement scales express convex functions. This character of the judgement function favours a much higher assessment for a more severe risk level than low or even average risk level. Since, convex scales correspond to the needs of decision-makers who are inclined to risk. The comparison of the convex scales illustrated in Figures 2–6 reveals two of them that specifically differentiate the relative risk assessment across the full range of AHP/ANP assessments, namely H3 scale and H6 scale. It also seems that H6 scale provides a better balance between the assessments obtained for different risk levels (see also Tab.1 with this regard). This is mainly due to the fact that it gives more distinct judgements in the case of intermediate scale levels. This is why its use in risk assessment is recommended to decision makers who are particularly inclined to risk.

The analysis of illustrations of individual alternatives of AHP/ANP assessment scale also reveals the existence of only three concave grading scales: L1, L2, and L3 (see: Fig.2). These scales correspond to the needs of decision-makers which are risk averse. Fig.7 shows that the widest range of differentiation in the relative risk assessment corresponds to the use of L2 scale. However, it is L3 scale that allows for more evident differentiation in the relative assessment for intermediate scale levels. Therefore, it seems to be more appropriate for an informed decision-maker who is risk averse.

Table 1 proves that the same ordering of the risk categories is obtained – from the least significant category R1 to the most important category – R5 for all considered scales. Moreover, the use of almost all judgement scales gives higher non-unitary priorities than the use of the standard N1 scale. A significant exception in this context concerns high-point scales H1, H2, H3 and H6, This is because they give lower values of non-unitary priorities. The application of H4 scale gives practically the same priorities as those obtained using the N1 scale. On the other hand, the use of H7 scale results in inconsistent priorities which only partially exceed the results of using N1 scale.

Note that the presence of H6 scale among the scales that behave properly in the context of risk inclination practically confirms its – already signalled – recommendation for practical application by decision-makers who are inclined to take risks. All concave judgement scales L1, L2, L3 behave appropriately in the context of risk averse attitude. This fact also confirms the correctness of the initial recommendation to use the L3 scale to assess the risk.

The usability of recommended L3 and H6 judgement scales is also confirmed by the graphical illustration of the results of the sample analysis which are presented in Fig.7. These judgement scales are finally suggested, therefore for a reliable expression of non-standard decision maker's attitude towards risk: logarithmic scale L3 by Ishizaka et al. (2006) for risk adverse attitude and geometric scale H6 by Légrády et al. (1996) in the case of risk inclination.

6. Limitations

Although shapes of functions describing suggested judgement scales and a simple analysis confirm their usability for non-standard attitudes towards risk it nevertheless seems that they should be also examined by means of the application in the case of more comprehensive and practical analysis.

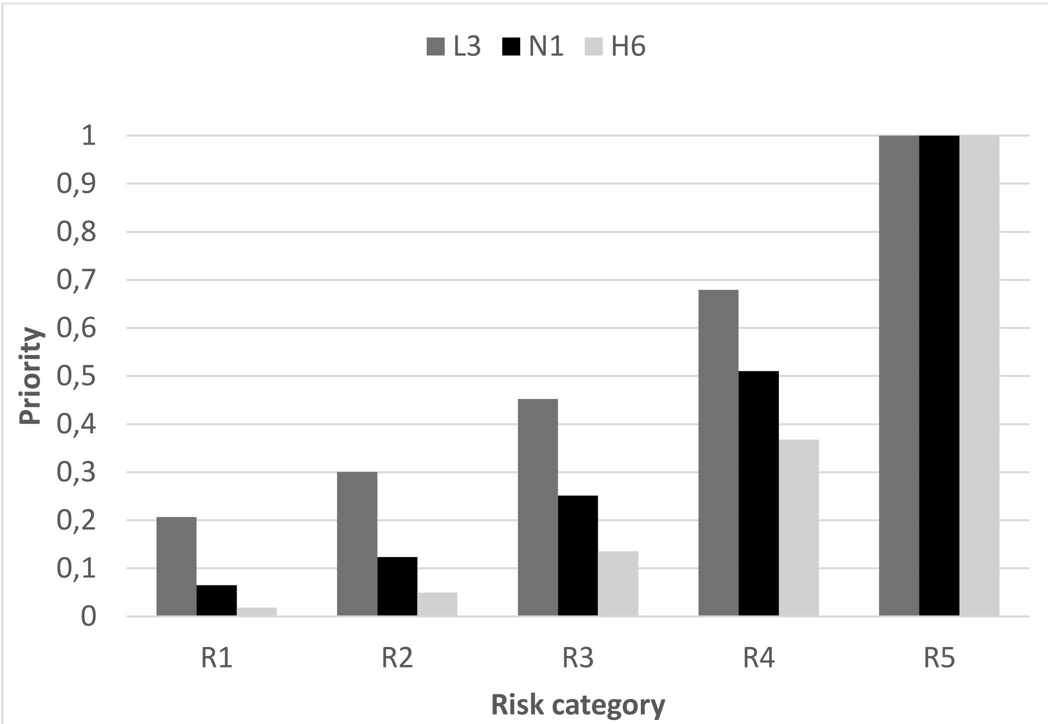

Figure 7. The comparison of the results of using the recommended scales (L3, H6, N1)

7. Conclusions

AHP/ANP lacked ability to express non-standard decision maker's risk attitudes. The outcomes of analysis presented in the paper show that there are alternative judgement scales available which allow for addressing other attitudes reliably. This fact is extremely important for universal AHP/ANP technique usability for risk assessment.

Key References

Mccauley Bell, P., Crumpton, L. (1997) A fuzzy linguistic model for the prediction of carpal tunnel syndrome risks in an occupational environment. *Ergonomics, 40(8),* 790-799.

Zheng, Q., Shen, S.-L., Zhou, A., Lyu, H.-M. (2022). Inundation risk assessment based on G-DEMATEL-AHP and its application to Zhengzhou flooding disaster. *Sustainable Cities and Society*, *86,* art. no. 104138.

Saaty, T.L. (2006). There is no mathematical validity for using fuzzy number crunching in the analytic hierarchy process. *Journal of Systems Science and Systems Engineering*, *15(2),* 457-464.

Harker, P.T., Vargas, L.G. (1987): The theory of ratio scale estimation, *Management Science*, 33, 1383-1403.

Ishizaka A., Balkenborg D., & Kaplan T. (2006). Influence of agregation and preference scale on ranking a compromise alternative in AHP. In: *ECAI-06, Multidisciplinary Workshop on Advances in Preference Handling*, Garda, Italy, 51-57.

Dodd, F., Donegan, H., & McMaster, T.M.B. (1992). Reassessment of consistency criteria in judgment matrices. *The Statistician*, 44(1), 31-41.

Finan, J.S., & Hurley, W.J. (1997). The analytic hierarchy process: Does adjusting a pairwise comparison matrix to improve the consistency ratio help? *Computers & Operations Research*, 24:749-755.

Ma, D., & Zheng, X. (1991): 9/9-9/1 scale method of AHP. In: *Proceedings of the 2nd International Symposium on AHP* (pp.197-202), 1, Pittsburgh. Retrived from http://isahp.org/uploads/197-scale.pdf (date: 2022-08-25).

Salo A.A., Hämäläinen R.P. (1997): On the measurement of preference in the Analytic Hierarchy Process, *J Multi-Crit Decis Anal* 6:309-319.

Rašković, S., Decker R., & Meißner M. (2008): An Investigation of Saaty's Consistency Ratio with Respect to Alternative Scales in AHP, in: *Conference Handbook, OR50*, The OR Society, The University of York, 2008, p.48.

Lootsma, F.A. (1996). A model for the relative importance of criteria in the multiplicative AHP and SMART. *European Journal of Operational Research*, 94:467-476.

Lootsma, F.A. (1997). Multicriteria decision analysis in a decision tree. *European Journal of Operational Research*, 101, 442-451.

Légrády, K., Lootsma, F.A., Meisner, J., & Schellemans F. (1984). Multicriteria Decision Analysis to Aid Budget Allocation. In: M. Grauer, A.P. Wierzbicki (Eds.) *Interactive Decision Analysis, Lecture Notes in Economics and Mathematical Systems, 229*, Springer Verlag Berlin, 164-174.

Kok, M, Lootsma, F.A. (1985). Pairwise-comparison methods in multiple objective programming, with applications in a long-term energy-planning model. *European Journal of Operational Research*, 22(1), 41-47.

Acknowledgments

The paper has been financed by a subsidy for scientific research provided by AGH University of Technology, Cracow, Poland – no. 16.16.200.396.

Authors' affiliations

Vigneshkumar, C. – Indian Institute of Technology Guwahati, Guwahati, India.
Ginda, G. – AGH University of Science and Technology, Cracow, Poland.