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NOTES ON THE USE OF COMPATIBILITY INDEX IN THE AHP

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1. Introduction

- **compatibility [similarity] index (*S.I.*)**
  - comparing two pairwise comparison matrices
- **correct meaning of *S.I.* and correct use of *S.I.***
  - not to lead to unexpected decision.
- **compatibility for a hierarchy**
- **the sensitivity of the *S.I. to* improve**

Compatibility [Similarity] Metric

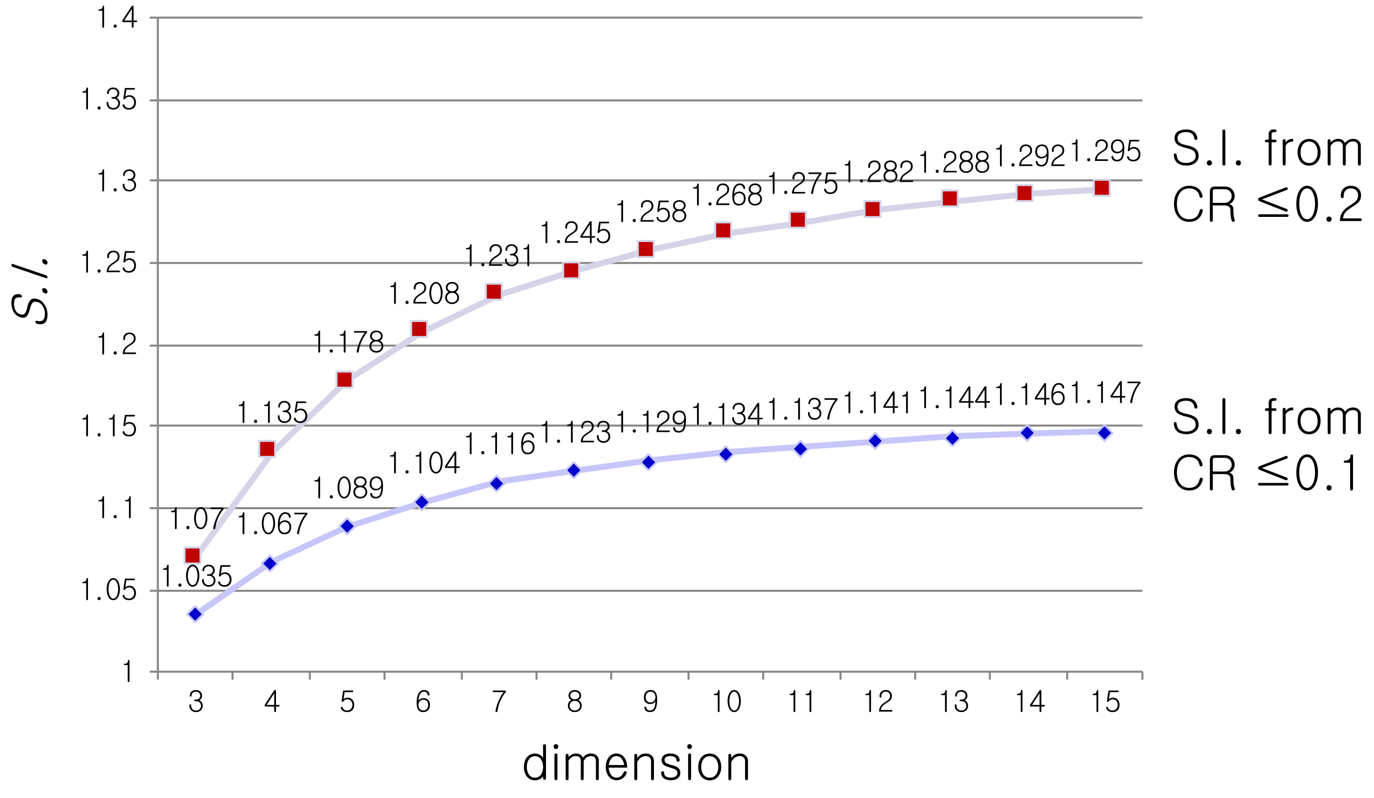
$$S.I. = n^{-2} \cdot e^T A \circ W^T e = \lambda_{max}/n$$

where $\circ$ is the Hadamard product,

W is the matrix of ratios of the principle right eigenvector of a pairwise comparison matrix, A

- *S.I.* becomes 1 if and only if the two matrices are exactly same. Otherwise, *S.I.* goes beyond 1.

Critical Value of S.I.



Source: Saaty 1996, p.63

A. Correct Use of *S.I.* from Inconsistency

	c1	c2	c3	c4	Priori.	CR		c1	c2	c3	c4	Priori.	CR
A	1	2	4	8	0.53	0.0	B	1	1	6	8	0.51	0.1
		1	2	4	0.27				1	1	4	0.28	
			1	2	0.13					1	2	0.14	
				1	0.07						1	0.06	
W						0.0	V						0.0

$S.I.\_AB = 1.073 > 1.067$
$S.I.\_WV = 1.005 \fallingdotseq 1.0$

B. *S.I.* for an Entire Hierarchy

$$DSIH = \frac{SIH_{vw}}{SIcH_{vw}} = \frac{\sum_{h=1}^{H} SI_{vw}^{(h)}}{\sum_{h=1}^{H} SIc_{vw}^{(h)}}$$

where $\sum_{i=1}^{n_h} v_i^{(h)} = \sum_{i=1}^{n_h} w_i^{(h)} = 1$

$SI_{vw}^{(h)}$: S.I. between two priorities vectors v and w in the *h*th layer

$SIc_{vw}^{(h)}$: Critical value of S.I. between two priorities vectors v and w in the hth layer

If *DSIH* is equal or less than 1,
two sets of judgments for an entire hierarchy are not significantly different

C. Sensitivity of *S.I.*

- If priority of element is smaller, the sensitivity even by small change is possibly higher.
- A few ways to measure the difference between the two vectors

$$SI_{vw} = \frac{1}{n^2} \cdot e^T V \circ W^T e = \frac{1}{n^2} \cdot e^T \left[\frac{v_i}{v_j} \cdot \frac{w_j}{w_i} \right] e$$

$$= \frac{1}{n^2} \sum_i \sum_j \left(\frac{v_i}{w_i} \cdot \frac{w_j}{v_j} \right) = \left(\frac{1}{n} \sum_i \frac{v_i}{w_i} \right) \cdot \left(\frac{1}{n} \sum_j \frac{w_j}{v_j} \right)$$

An alternative way: Weighted *S.I.* (*WSI*)

$$WSI = \left(\sum_i \frac{w_i}{v_i} \alpha_i \right) \cdot \left(\sum_j \frac{v_j}{w_j} \alpha_j \right), \quad where \ \sum \alpha_i = 1$$

Concluding Remarks

- The role of S.I.
- From the comparison of two matrices
- Two priorities vectors, similar or not