Title: Extension of the AHP method to group decision making for determining the best care center of severe cases of covid-19 in Ouagadougou /Burkina Faso:The case of three decision makers.

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Plan

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I. Introduction

- Appeared since March in China, COVID-19 has caused extensive damage, including many deaths all over the world and lower savings. We believe that it is a problem which deserves to be treated by a group and not individually because a decision taken by a group reflect more reality.
- Our contribution is to extend the AHP method to group decision making leading to fewer calculations and less compensation of strong criteria by weak ones than other methods existing in the literature.

II. State of the art

In this part, we are interested in three methods:

- MACASP in [5]
- LON-ZO in [5]
- ELECTRE I in [1]

The willing reader can refer to the references in questions.

III. Research design / Methodology

III.1. Formulation of the problem.

Consider the set belows:

- D = $\{d_1, d_2, \dots, d_N\}$ with $N \geq 2$: set of all N decision makers;
- A = $\{a_1, a_2, \dots, a_M\}$ with M≥ 2 : set of all M alternatives or actions;
- C = $\{c_1, c_2, \dots, c_m\}$ with m≥ 2 : set of all m criteria;
- X = $\left\{g_{ij}^k;\ i = 1, \dots., M;\ j = 1 \dots, m;\ k = 1, \dots, N\right\}$ designating the set of evaluations of the actions affected by the decision makers according to the criteria;
- P = $\left\{w_j^k;\ j = 1 \dots, m;\ k = 1, \dots, N\right\}$ designating the set of weights of the criteria affected by the decision makers.

III.2. Presentation of the extension of the AHP method to the group's decision: MAC-AHP

In this method we have five steps:

Step 1:

Determination of the matrix of the standardized global weights of the criteria.

First , determine the matrix of the overall weights of the criteria and defined as follows:

$$W = (w_1 \;\; w_2 \;\; \dots . w_m)$$

$$w_j = med \left\{w_j^k\right\}_{k=1,...,N} \qquad (1)$$

Then, the matrix of the standardized global weights of criteria is obtain as follows:

$$\widehat{W} = (\widehat{w_1} \;\; \widehat{w_2} \dots . \widehat{w_m})$$

$$\widehat{w_j} = \frac{w_j}{\sum_{j=1}^{m} w_j} \qquad (2)$$

Step 2:

Determination of the matrix of the global evaluation of alternatives.

$$\begin{pmatrix} g_{11} & \cdots & g_{1m} \\ \vdots & \ddots & \vdots \\ g_{M1} & \cdots & g_{Mm} \end{pmatrix}$$

With $g_{ij} = \sqrt[N]{\prod_{k=1}^{N}(g_{ij}^{k})}$ (3)

Step 3:

This step is broken down into several sub-steps described as follows:

Step 3.1: Calculate the ration m_j based on the equation (4):

$$m_j = \frac{max\{g_{ij}\}-min\{g_{ij}\}}{n} \quad (4)$$

n is the number of shares.

Step 3.2: Determine the judgment matrix (J^j) according to each criterion c_j as follows:

$$J = \begin{pmatrix} J_{11}^j & \cdots & J_{1M}^j \\ \vdots & \ddots & \vdots \\ J_{M1}^j & \cdots & J_{MM}^j \end{pmatrix}$$

If the criterion c_j is to be maximized, then:

$$J_{il}^j = \begin{cases} arr\left(\frac{g_{ij}-g_{lj}}{m_j}+1\right) & if\ g_{ij} > g_{lj} \\ \frac{1}{arr\left(\frac{g_{ij}-g_{lj}}{m_j}+1\right)} & else \end{cases} \quad (5)$$

If the criterion c_j is to be minimized, then:

$$J_{il}^j = \begin{cases} arr\left(\frac{g_{ij}-g_{lj}}{m_j}+1\right) & if\ g_{ij} < g_{lj} \\ \frac{1}{arr\left(\frac{g_{ij}-g_{lj}}{m_j}+1\right)} & else \end{cases} \quad (6)$$

'arr' designates a strictly positive integer function with, with a given real, associates the integer which is immediately superior to it.

Step 3.3: Build the matix of the sum of the judgments by binary comparison of each column and noted (S^j) as follows:

$$S^j = (s_1 \ \ s_2 \ \ \ldots . \ s_M)$$

$$s_i = \sum_{l=1}^{M} J_{il} \qquad (7)$$

Step 3.4: Determine the normalized judgment matrix (B) expressed as follows:

$$B^{j} = \begin{pmatrix} b_{11} & \cdots & b_{1M} \\ \vdots & \ddots & \vdots \\ b_{M1} & \cdots & b_{MM} \end{pmatrix}$$

$$b_{li} = \frac{J_{li}}{s_i} \qquad (8)$$

Step 3.5: The priority matrix has build as follows:

$$I^{j} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_M \end{pmatrix}$$

$$\mu_l = \frac{\sum_{i=1}^{M}(\mu_{li})}{M} \qquad (9)$$

Step 3.6 : Determie the eigenvalue λ_{max}

First, determine the eigenvalue matrice (λ) expressed as follows:

$$\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_M \end{pmatrix}$$

$$J^j . I^j = \lambda . I^j \qquad (10)$$

Then, calculate $the\ eigenvalue\ \lambda_{max}$ based on the equation (11):

$$\lambda_{max} = \frac{\sum_{i=1}^{M}(\lambda_i)}{M} \qquad (11)$$

Step 3.7: Calculate the consistency index based on the equation (12):

$$CI = \frac{\lambda_{max} - n}{n - 1} \qquad (12)$$

n is the size of the judgments matrix.

Step 3.8 : Determine the randomized index (RI) based on the size of the judgments matrix and given by the table below:

Table1: Table of random indices

size of the matrix	3	4	5	6	7	8	9	10
RI	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Step 3.9: Calculate the consistency ratio (CR) based on the equation (13):

$$CR = \frac{CI}{RI} \tag{13}$$

Step 4:

Build the priority matrix according to all the criteria (M) as follows:

$$M = (I^1 \;\; I^2 \ldots I^m)$$

Step 5:

Determine the score matrix (M) of the actions:

$$\alpha = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_M \end{pmatrix}$$

$$M \,.\, t_{\widehat{W}} = \alpha \qquad (14)$$

$t_{\widehat{W}}$ is the transpose of the normalized global weights matrix.

After determing the score matrix, the best alternative is their have the more score.

IV. Data /Model Analysis

Here we relied on data from an article to apply.

IV.1. Position of problem

The problem is to determine the best center for the management of severe cases of COVID-19 among the following health centers: Yalgado hospital, Tengandogo hospital, Bogodogo district hospital, peace clinc according to the following criteria: equipment in respirators, equipment in beds, quality of staff, quality of receptionn, accessibility and with the following decision makers: the order of doctors, the COVID-19 management unit, and the national assembly.

Here are the table which give evaluation matrices:

Table 1: Assessment matrix of the order of doctors

Criterion	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Acces
weight	7	5	3	3	4
Yalg hosp	7	6	2	3	3
dist.Bog hosp	6	5	2	5	3
Ting hosp	5	7	3	6	4
clin.pe	5	4	4	4	3

Table 2: Assessment matrix of the COVID-19 management unit

Criterion	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Acces
weight	6	3	2	4	3
yalg hos	6	5	2	4	5
dist.Bog hosp	5	6	3	3	4
Ting hosp	7	5	4	6	3
clin.pe	6	4	5	3	6

Table 3: Assessment matrix of the national assembly

Criterion	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Acces
weight	6	4	2	3	3
Yalg hosp	6	5	2	4	4
dist.Bog hosp	7	6	3	5	3
Ting hosp	6	5	4	3	5
clin.pe	5	4	3	6	4

By applying the MAC-AHP method with its data, this is what we get:

Step 1 :

Table 4: Global weight matrix

Criterion	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Acces
weight	6	4	2	3	3

Table 5: Global normalized weight matrix

Criterion	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Acces
standardized weight	0.33	0.22	0.11	0.16	0.16

Step 2:

Table 6: Assessment synthesis matrix

Criterion	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Acces
Yalg hosp.	6.31	5.31	2	3.63	3.91
dist.Bog hosp	5.94	5.64	2.62	4.21	3.30
Ting hosp.	5.94	5.59	3.63	4.76	3.91
clin.pe	5.31	4.00	3.91	4.16	4.16

Step 3:

- Comparison based on the respirator equipement criterion.

Step 3.1 : Clculation of m_1: m_1= 0,25

Step 3.2: matrix of Judgments according to the respirator equipment criterion.

Equi.Resp	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	3	3	5
dist.Bog hosp	$\frac{1}{3}$	1	1	4
Ting hosp	$\frac{1}{3}$	1	1	4
clin.pe	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	1

Step 3.3: matrix of Judgments and sum by column according to the respirator equipment criterion.

Equi.Resp	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	3	3	5
dist.Bog hosp	$\frac{1}{3}$	1	1	4
Ting hosp	$\frac{1}{3}$	1	1	4
clin.pe	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	1
Sum	1.86	5.25	5.25	14

Step 3.4: Normalize judgments according to the respirator equipment criterion.

Table 7: matrix of normalized judgments

Equi.Resp	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	0.54	0.57	0.57	0.36
dist.Bog hosp	0.18	0.19	0.19	0.29
Ting hosp	0.18	0.19	0.19	0.29
clin.pe	0.11	0.05	0.05	0.07

Step 3.5: Priority according to the respirator equipment criterion.

Table 8: Priority vector

Equi.Resp	priorité
Yalg hosp	0.52
dist.Bog hosp	0.21
Ting hosp	0.21
clin.pe	0.07

With Step 3.6 , Step 3.7, Step 3.8, Step 3.9, we obtain:

$$\lambda_1 = 4.09\ ,\ \lambda_2 = 4.14,\ \lambda_3 = 4.14,\ \lambda_4 = 3.98$$

$$\lambda_{max} = 4.08$$

$$CI = 0.02$$

$$RI = 0.90$$

$$CR = 0.02 < 0.1$$

- Matrix of judgments according to the beds equipment criterion.

Step 3.1 : Clculation of m_2: m_2 = 0,41

Step 3.2: matrix of Judgments according to the beds equipment criterion.

Equi.Lit	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	$\frac{1}{2}$	$\frac{1}{2}$	5
dist.Bog hosp	2	1	2	5
Ting hosp	2	$\frac{1}{2}$	1	5
clin.pe	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	1

Step 3.3: matrix of Judgments and sum by column according to the beds equipment criterion .

Equi.Lit	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	$\frac{1}{2}$	$\frac{1}{2}$	5
dist.Bog hosp	2	1	2	5
Ting hosp	2	$\frac{1}{2}$	1	5
clin.pe	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	1
Sum	5.20	2.20	3.70	16

Step 3.4: Normalize judgments according to the beds equipment criterion.

Table 9: matrix of normalized judgments

Equi.Lit	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	0.19	0.23	0.13	0.31
dist.Bog hosp	0.38	0.45	0.54	0.31
Ting hosp	0.38	0.23	0.27	0.31
clin.pe	0.04	0.09	0.05	0.06

Step 3.5: Priority according to the beds equipment criterion.

Table 10: Priority vector

Equi.Lit	priorité
Yalg hosp	0.21
dist.Bog hosp	0.42
Ting hosp	0.29
clin.pe	0.06

With Step 3.6 , Step 3.7, Step 3.8, Step 3.9, we obtain:

$$\lambda_1 = 4.06 \ , \lambda_2 = 4.15, \ \lambda_3 = 4.16, \ \lambda_4 = 4.10$$
$$\lambda_{max} = 4.12$$
$$CI = 0.04$$
$$RI = 0.90$$
$$CR = 0.04 < 0.1$$

- Matrix of judgments according to the personnel quality criterion.

Step 3.1 : Clculation of m_3: m_3 = 0,47

Step 3.2: matrix of Judgments according to the personnel quality criterion.

Qual.Pers	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{6}$
dist.Bog hosp	3	1	$\frac{1}{4}$	$\frac{1}{4}$
Ting hosp	5	4	1	$\frac{1}{2}$
clin.pe	6	4	2	1

Step 3.3: matrix of Judgments and sum by column according to the personnel quality criterion .

Qual.Pers	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{6}$
dist.Bog hosp	3	1	$\frac{1}{4}$	$\frac{1}{4}$
Ting hosp	5	4	1	$\frac{1}{2}$
clin.pe	6	4	2	1
Sum	15	9.33	3.45	1.61

Step 3.4: Normalize judgments according to the personnel quality criterion.

Table 11: matrix of normalized judgments

Qual.Pers	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	0.06	0.03	0.05	0.09
dist.Bog hosp	0.20	0.10	0.07	0.15
Ting hosp	0.33	0.42	0.28	0.31
clin.pe	0.40	0.42	0.57	0.62

Step 3.5: Priority according to the personnel quality criterion.

Table 12: Priority vector

Qual.Pers	priorité
Yalg hosp	0.06
dist.Bog hosp	0.13
Ting hosp	0.33
clin.pe	0.50

With Step 3.6 , Step 3.7, Step 3.8, Step 3.9, we obtain:

$$\lambda_1 = 4.06\ ,\ \lambda_2 = 3.99,\ \lambda_3 = 4.25,\ \lambda_4 = 4.13$$
$$\lambda_{max} = 4.10$$
$$CI = 0.03$$
$$RI = 0.90$$
$$CR = 0.04 < 0.1$$

- Matrix of judgments according to the reception quality criterion.

Step 3.1 : Clculation of m_4: $m_4 = 0{,}28$

Step 3.2: matrix of Judgments according to the reception quality criterion.

Qual.Accu	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$
dist.Bog hosp	2	1	$\frac{1}{2}$	2
Ting hosp	3	2	1	2
clin.pe	2	$\frac{1}{2}$	$\frac{1}{2}$	1

Step 3.3: matrix of Judgments and sum by column according to the reception quality criterion .

Qual.Accu	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{2}$
dist.Bog hosp	2	1	$\frac{1}{2}$	2
Ting hosp	3	2	1	2
clin.pe	2	$\frac{1}{2}$	$\frac{1}{2}$	1
Sum	8	4	2.33	5.5

Step 3.4: Normalize judgments according to the reception quality criterion

Table 13: matrix of normalized judgments

Qual.Accu	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	0.12	0.12	0.14	0.09
dist.Bog hosp	0.25	0.25	0.21	0.36
Ting hosp	0.37	0.50	0.42	0.36
clin.pe	0.25	0.12	0.21	0.18

Step 3.5: Priority according to the reception quality criterion.

Table 14: Prioritv vector

Qual.Accu	priorité
Yalg hosp	0.12
dist.Bog hosp	0.26
Ting hosp	0.41
clin.pe	0.19

With Step 3.6 , Step 3.7, Step 3.8, Step 3.9, we obtain:

$$\lambda_1 = 4.05 \;,\; \lambda_2 = 4.08,\; \lambda_3 = 4.06,\; \lambda_4 = 4.06$$
$$\lambda_{max} = 4.06$$
$$CI = 0.02$$
$$RI = 0.90$$
$$CR = 0.02 < 0.1$$

- Matrix of judgments according to the accessibility criterion.

Step 3.1 : Clculation of m_5: m_5 = 0,21

Step 3.2: matrix of Judgments according to the accessibility criterion.

Acces	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	2	$\frac{1}{2}$	$\frac{1}{2}$
dist.Bog hosp	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{3}$
Ting hosp	2	2	1	$\frac{1}{2}$
clin.pe	2	3	2	1

Step 3.3: matrix of Judgments and sum by column according to the accessibility criterion .

Acces	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	1	2	$\frac{1}{2}$	$\frac{1}{2}$
dist.Bog hosp	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{3}$
Ting hosp	2	2	1	$\frac{1}{2}$
clin.pe	2	3	2	1
Sum	5.5	8	4	2.33

Step 3.4: Normalize judgments according to the accessibility criterion.

Table 15: matrix of normalized judgments

Acces	Yalg hosp	dist.Bog hosp	Ting hosp	clin.pe
Yalg hosp	0.18	0.25	0.12	0.21
dist.Bog hosp	0.09	0.12	0.12	0.14
Ting hosp	0.36	0.25	0.25	0.21
clin.pe	0.36	0.37	0.50	0.42

Step 3.5: Priority according to the accessibility criterion.

Table 16: Priority vector

Acces	priorité
Yalg hosp	0.19
dist.Bog hosp	0.12
Ting hosp	0.26
clin.pe	0.41

With Step 3.6 , Step 3.7, Step 3.8, Step 3.9, we obtain:

$$\lambda_1 = 4.04 \ , \ \lambda_2 = 4.06, \ \lambda_3 = 4.10, \ \lambda_4 = 4.06$$
$$\lambda_{max} = 4.06$$
$$CI = 0.02$$
$$RI = 0.90$$
$$CR = 0.02 < 0.1$$

Step 4:

Table 17: Priority matrix according to all the driteria.

Criterion	Equi.Resp	Equi.Lit	Qual.Pers	Qual.Accu	Acces
Yalg hosp	0.52	0.21	0.06	0.12	0.19
dist.Bog hosp	0.21	0.42	0.13	0.26	0.12
Ting hosp	0.21	0.29	0.33	0.41	0.26
clin.pe	0.07	0.06	0.50	0.19	0.41

Step 5:

Table 18: Score matrix

Hospital	global Score
Yalg hosp	0.27
dist.Bog hosp	0.24
Ting hosp	0.28
clin.pe	0.19

After all these steps we find that Tengandogo hospital is the best treatment center for severe cases of COVID-19.

V. Conclusion

We note that we have obtained interesting results with the MAC-AHP method. In addition we got the same conclusions as those generated by MACASP, LONZO and ELECTRE I. But given that any method has its advantages and disadvantages. We ask the question of knowing, in case we have a large number criteria, won’t the calculations be enormous with MAC-AHP ?

Thank you dear participants and organizers, thank you

Dr Zoïnabo SAVADOGO !

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