

## THE ISOMORPHISM BETWEEN DOUBLE-ENTRY ACCOUNTING AND THE ANALYTIC HIERARCHY PROCESS

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### 1. Introduction

The Analytic Hierarchy Process (AHP) was introduced into the accounting literature in the early 1980s and it has gradually increased in prominence since that time. The AHP has been used extensively by accounting researchers to solve the complex and ill-structured decision problems of assessing audit risk and choosing between various auditing techniques (Apostolou and Hassell, 1993). Dellmann (1999) showed how evaluation techniques developed for the Analytic Network Process (ANP) can be applied within the general framework of cost accounting. However none of the researchers have ever tried to bridge the gap between decision theory and accounting theory as applied to general ledger accounting. In this paper we will show that there is an isomorphism between the formal structure of double-entry accounting and the construction of the AHP as a decision methodology which has to be further explored for purposes of auditing, corporate risk management and financial statement analysis.

### 2. The Matrix Approach to Double-Entry Accounting

Double-entry provides the basic structure for accounting. It is used in the periodic accumulation of transaction data for the purpose of preparing financial statements. The generic building blocks of the double-entry book-keeping system are the accounts where all transactions are summarized. The account has been used for several hundred years and is traditionally formulated as a so called T-account. On one side of the T, additions to the account are recorded, and on the other side of the T, subtractions are recorded. The closing balance of an account at any point in time is the opening balance, plus the increases, minus the decreases.

Assets/Expenses		Liabilities/Revenues	
	Opening Balance	Opening Balance	
Debit (Increase)	Credit (Decrease)	Debit (Decrease)	Credit (Increase)
Closing Balance			Closing Balance

**Figure 1. The Accounting System**

The left-hand side of a T-account is called the debit side, whereas the right-hand side is called the credit side. For asset accounts increases to assets are recorded as debit entries, and decreases in assets are recorded as credit entries to these accounts. For liabilities and owners' equity accounts, the opposite will be true. Entries to revenue and expense accounts follow a pattern that is consistent with entries to

owners' equity accounts. Revenues are increases in owners' equity, therefore revenue accounts will increase with credit entries and decrease with debit entries. Expenses are decreases in owners' equity, therefore expense accounts will increase with debit entries and decrease with credit entries. Gains and losses are recorded like revenues and expenses respectively.

The duality of double-entry refers to the fact that for every increase in assets or expenses, there is causally related a decrease in assets and/or an increase in liabilities or revenues et vice versa. Therefore with every transaction at least two accounts have to be addressed. A transaction involving the purchase of equipment for \$ 250 cash for example, would be described as a \$ 250 debit to the equipment account and a \$ 250 credit to the cash account. If the company borrowed \$ 250 from a bank in order to finance the equipment purchase, this transaction would result in a \$ 250 debit to the cash account and a \$ 250 credit to the notes payable account.

In terms of graph theory T-accounts can be considered as nodes of a graph whereas transactions are represented by the arcs between a pair of nodes. In general the double-entry bookkeeping system can be considered as an orientied graph or network. Therefore the general ledger of a double-entry bookkeeping system can formally be described as a  $n \times n$ -adjacency matrix, where  $n$  corresponds to the number of accounts used in the general ledger.

	Account 1	Account 2	...	Account n
Account 1	0	$u_{12}$	...	$u_{1n}$
Account 2	$u_{21}$	0	...	$u_{2n}$
...	...	...	...	...
Account n	$u_{n1}$	$u_{n2}$	...	0

**Table 1. The General Ledger Matrix**

Since all entries in the general ledger are made up of debits and credits and the matrix is composed of rows and columns, we can adopt the convention that debits correspond to rows and credits to columns (Mattessich, 1958; Shank, 1972), so that every element of the general ledger matrix has a double designation. Thus if the company buys the equipment mentioned earlier, we need only enter \$ 250 in the cell representing entries which involve a debit to the equipment account and a credit to the cash account. Furthermore, each transaction is processed by adding its dollar amount to the total in the appropriate cell.

In order to get the closing balances one has to compute the row (debit) and column (credit) totals for each account, deduct the column (credit) total of each account from its row (debit) total and add this result to the opening balance. The vector  $\mathbf{v}_t$  containing the closing balances of all accounts in the general ledger equals

$$\mathbf{v}_t = \mathbf{v}_{t-1} + (\mathbf{U} - \mathbf{U}^T) \cdot \mathbf{e} \quad (1)$$

where  $\mathbf{v}_{t-1}$ ,  $\mathbf{U}$ ,  $\mathbf{U}^T$  and  $\mathbf{e}$  are the vector with the opening balances of the accounts, the general ledger matrix, the transpose of the general ledger matrix and a vector containing "1" in every row, respectively.

### 3. The Isomorphism Between Double-Entry Accounting and the AHP

Matrix

$$\mathbf{S} = \mathbf{U} - \mathbf{U}^T \quad (2)$$

contains the net value of transactions for every possible pair of accounts within the general ledger. Matrix

$$\mathbf{S} = \begin{pmatrix} 0 & s_{12} & \dots & s_{1n} \\ -s_{12} & 0 & \dots & s_{2n} \\ \dots & \dots & \dots & \dots \\ -s_{1n} & -s_{2n} & \dots & 0 \end{pmatrix} \quad (3)$$

is anti-symmetric, i.e.

$$s_{ij} = -s_{ji} \text{ for } 1 \leq i, j \leq n. \quad (4)$$

Matrix **A** containing the pairwise comparisons needed within the framework of the AHP equals

$$\mathbf{A} = \begin{pmatrix} 1 & a_{12} & \dots & a_{1n} \\ a_{12}^{-1} & 1 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{1n}^{-1} & a_{2n}^{-1} & \dots & 1 \end{pmatrix} \quad (5)$$

This matrix is reciprocal in the sense of

$$a_{ij} = \frac{1}{a_{ji}} \text{ for } 1 \leq i, j \leq n. \quad (6)$$

The isomorphism between matrix **S** of the double-entry bookkeeping system and matrix **A** of the AHP can be demonstrated by applying the natural logarithm on (6) which equals

$$\ln(a_{ij}) = \ln\left(\frac{1}{a_{ji}}\right) = -\ln(a_{ji}). \quad (7)$$

For the elements of the matrices **S** and **A** follows

$$s_{ij} = \ln(a_{ij}) \quad (7a)$$

or

$$a_{ij} = \exp(s_{ij}), \quad (7b)$$

respectively.

#### 4. Benefits of the Isomorphism in Accounting and Finance

Possible benefits of the isomorphism shown above can be found in the following fields:

- *auditing*: audit planning and aggregation of audit opinions using the risk-oriented audit approach proposed by the American Institute of Certified Public Accountants (AICPA);
- *risk management*: prioritization of transaction risks contained within single accounts or strategic business units of a divisionalized firm stipulated by the German “Gesetz zur Kontrolle und Transparenz im Unternehmensbereich” (KonTraG);
- *business valuation*: prediction of sustainable future cash flows.

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