INTRODUCING THE ANALYTIC HIERARCHY PROCESS:
A MATHEMATICAL HOMEWORK

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Summary*: This work shows the approach of Analytic Hierarchy Process in a mathematical homework. This homework is an activity that must be developed by Industrial Engineering undergraduation students with specific objectives as to motivate the students and to introduce the use of mathematical tolls. Other great point showed in this work is the contribution of that activity as a part of a greater research involving others professors and students which has needed an alternative, interesting and systematized decision support tool named Analytic Hierarchy Process.*

1. Introduction

It was developed an Industrial Engineering research (Salomon, 2000) involving two professors, three Mechanical Engineering Trainees and three Industrial Engineering undergraduation students. This research aim the decision support for an Industrial Engineering process: the procurement process. The roles for this research participants was the Trainees has developed graduation work approaching the procurement process; the Industrial Engineering students developed a *mathematical homework*; the professors had advised these works.

The *mathematical homework* has got three specific objectives; to motivate the Industrial Engineering study, to show mathematical tools applying, to capacitate the student for the mathematical modeling of problems (Coordination of Industrial Engineering undergraduation course, 1995). In order to achieve these objectives, the coordination of Industrial engineering undergraduation course invites Professors to propose themes for the *mathematical homework*.

The practical example proposed in the *mathematical homework* will be presented followed by some considerations concluding the benefits of their making.

2. Using the Analytic Hierarchy Process

It was proposed, as a real problem regarding to industrial Engineering "to choose the best transportation way for electronic products in Brazil". There was identified five most important criteria:
1. Time of delivery
2. Level of defects
3. Misleading numbers
4. Practicability of this transportation in Brazil
5. Reclamation (refers a bad treatment by drivers or workers)
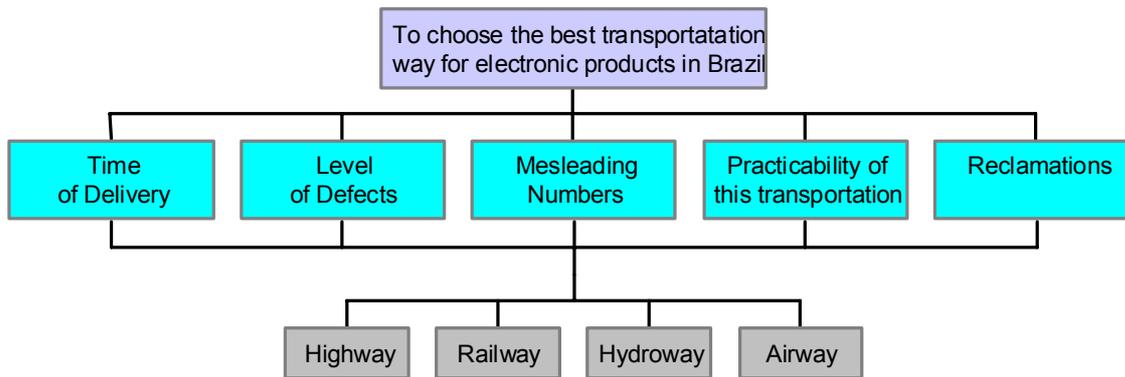We have the four alternatives to selection presented in figure 1.

Figure 1 – Decision hierarchy structure

Table 1 shows the criteria pairwise judgments and their priorities obtained by na eigenvector estimation.. With these data we came to λ_{MAX} and consistency ratio (CR) values equal to 5.442 e 0.099, respectively. These indicate judgments not totally consistent. But, they can be accept, as proposed by Saaty (1980). The table shows *Practicability* as the most important criterion.

	Delivery	*Defects*	*Misleading*	*Practicability*	*Reclamation*	*Eigenvector*
Delivery	1	1/3	3	1/9	1	0.076
Defects	3	1	3	1/9	1/3	0.094
Misleading	1/3	1/3	1	1/9	3	0.061
Practicability	9	9	9	1	9	0.680
Reclamation	1	3	1/3	1/9	1	0.076

Table 1 - Criteria importance

The next step was the alternatives pairwise comparisons for each criterion, in order to obtain the satisfaction ranking which each alternative proportionates to each criterion. This results are showed in tables 2 to 6.

	Airway	*Railway*	*Hydroway*	*Highway*	Eigenvector
Airway	1	7	9	5	0.657
Railway	1/7	1	2	1/3	0.087
Hydroway	1/9	½	1	1/5	0.051
Highway	1/5	3	5	1	0.205

Table 2 - Satisfaction to Delivery

The judgments showed in table 2 can be accept because they engender λ_{MAX} and CR values, respectively, equal to 4.16 and 0.058.

	Airway	*Railway*	*Hydroway*	*Highway*	Eigenvector

Airway	1	7	9	4	0.634
Railway	1/7	1	3	1/3	0.098
Hydroway	1/9	1/3	1	1/5	0.047
Highway	1/4	3	5	1	0.221

Table 3 - Satisfaction to Defects

The judgments showed in table 3 can be accept because they engender λ_{MAX} and CR values, respectively, equal to 4.13 and 0.049.

	Airway	*Railway*	*Hydroway*	*Highway*	Eigenvector
Airway	1	3	1/2	8	0.345
Railway	1/3	1	1/2	6	0.185
Hydroway	2	2	1	7	0.426
Highway	1/8	1/6	1/7	1	0.043

Table 4 - Satisfaction to Misleading

The judgments showed in table 4 can be accept because they engender λ_{MAX} and CR values, respectively, equal to 4.20 and 0.076.

	Airway	*Railway*	*Hydroway*	*Highway*	Eigenvector
Airway	1	4	4	1/4	0.223
Railway	1/4	1	3	1/8	0.087
Hydroway	1/4	1/3	1	1/8	0.058
Highway	4	8	8	1	0.632

Table 5 - Satisfaction to Practicability

The judgments showed in table 5 can be accept because they engender λ_{MAX} and CR values, respectively equal to 4.32 e 0.12.

	Airway	*Railway*	*Hydroway*	*Highway*	Eigenvector
Airway	1	2	2	4	0.433
Railway	1/2	1	1	3	0.239
Hydroway	1/2	1	1	3	0.239
Highway	1/4	1/3	1/3	1	0.088

Table 6 - Satisfaction to Reclamation

The judgments showed in table 5 can be accept because they engender λ_{MAX} and CR values, respectively equal to 4.02 and 0.008.

The table 7 summarizes the priorities for the alternatives from all criteria.

Criteria		Alternatives			
		Airway	*Railway*	*Hydroway*	*Highway*
Delivery	0.076	0.657	0.087	0.051	0.205
Defects	0.094	0.634	0.098	0.047	0.221
Misleading	0.061	0.345	0.185	0.426	0.043
Practicability	0.680	0.223	0.087	0.058	0.632
Reclamation	0.076	0.433	0.239	0.239	0.088

Table 7 - Decision matrix

Weighting the elements of the Decision matrix with the criteria priorities showed in table 1 we have the decision vector presented by table 8. Based on this results, we may conclude that *Highway* was the best transportation way for electronic products in Brazil, with priority around 47%.

	Global Priorities
Airway	0.315
Railway	0.104
Hydroway	0.092
Highway	0.474

Table 8 - Decision Vector

3. Conclusions

The judgments provide a high global consistency ratio, but still acceptable (0.098). However, the alternatives pairwise comparisons for practicability should be reviewed: they provide a CR equal to 0.12. This high unacceptable value can clearly justified to the low level of knowledge of the students (that performed the comparisons) about this topic. This fail can be reduced with this topic research (economics and technical features of the transportation way), but, this not inside the original scope of objectives of the homework. Judgments review highlights as a new topic to be studied by students in later works.

It is very important to relate that the works performed by the Trainees has been considered successful. The Analytic Hierarchy Process application were a key factor for all of them. It was possible to better select suppliers (Caçador, 1999), equipment (Ivo, 1999) and so on. The applications by the Trainees was facilitated by the *mathematical homework*: in the middle of the year the students who developed that *homework* has presented a theoretical introduction about this method to the other participants of the research.

References[1]

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Salomon, V. A. P. (2000), *Decision Support For The New Procurement Policies Adoption*, Guaratinguetá (Brazil): Universidade Estadual Paulista

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[1] For better understanding we have translated the original tittles of Brazilian references to English