Vector Compression Method to Convert the Incomplete Matrix of Pairwise Comparisons in the Analytic Hierarchy Process

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Literature Review

- Rubin, D. & Little, R. (1987) and Millet, I. (1997) showed that as the result you can lose important conformities by dropping expert assessments, which often leads to false, but "coordinated assessments”. The next stage in the development of AHP can be considered the methodology of setting an indistinct preference ratio, refusing to assess the proposed pair of alternatives Xu, Z.S. (2004), or even replacing contradictions with omissions.
- The principal applicability and effectiveness of one or another approach depends on the number of data drops and the reasons why they were generated Garcia-Laencina, P.J., Sanco-Gomez, J.-L. & Figueiras-Vidal, A.R. (2009). However, Carmone, F. J., Kara, Jr., A. & Zanakis, S. H. (1997), a specific example shows that "randomly removing up to 50% of the comparisons gives good results without losing accuracy." In the presence of an incomplete matrix of pairwise comparisons, the authors proposed using methods that allow to predetermine the matrix to the full, as confirmed by Ebenbach, D.H. & Moore, C.F. (2000).
- The system that helps to build indistinct preference relationships is proposed Alonso, S., Cabrerizo, F. J., Chiclana, F., Herrera, F. & Herrera-Viedma, E. (2008). Group decision-making, description of procedures correcting the lack of knowledge of a particular expert, using information provided by other experts, together with some aggregation procedures, is described Rehman, A., Hussain, M., Farooq, A. & Akram, M. (2019).

Key Problems

1. There is no method for working with incomplete matrices that allows you not to restore the matrix to full.
2. There are no methods that allow experts to independently determine the ranges of comparison of pairwise estimates.
3. There is no clear "stopping criterion" when you want to achieve maximum consistency of estimates (each of the existing methods has a tendency of getting "false weights" of the analyzed alternatives in the consistency search process)

Indicator That Tracks the Status of Network Connections

Choosing bias

$$\Delta L_i[t] = \frac{E_{max}^{i*}[t] - E_{max}^{*i}[t]}{2} = \frac{E_{max}^{i*}[t] + E_{min}^{i*}[t]}{2},$$

where $E_{max}^{i*}[t]$ is the maximum value of elements of a matrix $E_{i,j}[t]$, $E_{min}^{i*}[t]$ is the minimum value of elements of a matrix $E_{i,j}[t]$ and $E_{max}^{*i}[t]$ is the maximum value of elements of a matrix $E_{j,i}[t]$ for which the indicator function $G_{j,i}[t] = 1$ (a maximum on column i) and influencing a matrix $E[t]$ by convert $S_{\Delta L_i[t]}^{i}(E[t])$ we have:

$E_{i,j}[t+1] = E_{i,j}[t] - \Delta L_i[t]$ where $G_{i,j}[t] = 1$

$E_{max}^{i*}[t+1] = E_{max}^{*i}[t+1] = \frac{E_{max}^{i*}[t] + E_{max}^{*i}[t]}{2}$;

$E_{j,i}[t+1] = E_{j,i}[t] + \Delta L_i[t]$ where $G_{j,i}[t] = 1$

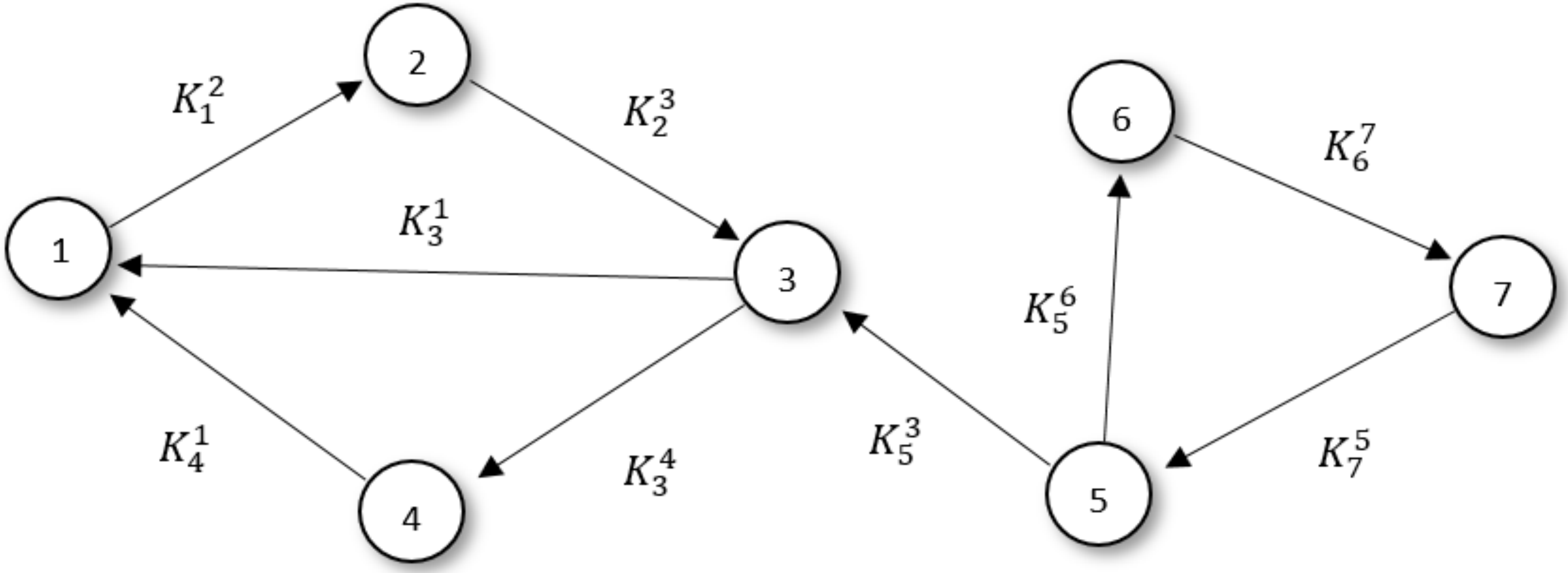
$E_{min}^{i*}[t+1] = -E_{max}^{i*}[t+1]$

The values $E_{max}^{i*}[t+1]$ and $E_{max}^{*i}[t+1]$ are fitted.

Table 1. Initial data

Number of the line	Maximum value in the line	Minimum value in the line	Value of coordinate wise bias
1	$E_{max}^{1*}[t]$	$E_{min}^{1*}[t]$	$(E_{max}^{1*}[t] + E_{min}^{1*}[t])/2$
2	$E_{max}^{2*}[t]$	$E_{min}^{2*}[t]$	$(E_{max}^{2*}[t] + E_{min}^{2*}[t])/2$
…			
N	$E_{max}^{N*}[t]$	$E_{min}^{N*}[t]$	$E_{max}^{N*}[t] + E_{min}^{N*}[t])/2$

Example of a graph of links between compared alternatives



The Vector Compression Method

Expert data we will create a connecting network in the form of a block matrix $\bar{A}$ of dimension $[M\times N, M\times N]$ and a zero vector $v_i^m[0]$ of dimension $[M\times N]$.

1. Local maximum E_{max}^{i*} and minimum E_{min}^{i*} are calculated in rows.
2. Recalculate $v_i^m[t+1] = v_i^m[t] + \frac{E_{max}^{i*} + E_{min}^{i*}}{3}$.
3. If $\frac{1}{2}\sum_{i=1}^{N} \left| E_{max}^{i*} + E_{min}^{i*} \right| > \varepsilon$, go to point Otherwise:
4. If $E_{max}^{**} > \varepsilon_1$ — correction $\bar{A} = \bar{A} - \theta E$, the previous v_i^m can be left. Go to point 1.
5. Otherwise, the specified accuracy is reached.
6. Calculation of v_i^m for each expert advisor.
7. Forming the upper T_i and lower B_i estimates of the sets of weights $B_i \le v_i^m \le T_i$.

	Expert Data 1							Expert Data M						
	1		i		j		N	1		i		j		N
1	0							$\text{Э}_{1M;11}$		NA		NA		NA
		0												
i			0		$\bar{a}_{11;ij}^{1}$			NA		$\text{Э}_{1M;ii}$		NA		NA
				0										
j			$-\bar{a}_{11;ij}^{1}$		0			NA		NA		$\text{Э}_{1M;jj}$		NA
						0								
N							0	NA		NA		NA		$\text{Э}_{1M;NN}$
1	$\text{Э}_{M1;11}$		NA		NA		NA	0						
									0					
i	NA		$\text{Э}_{M1;ii}$		NA		NA			0		NA		
											0			
j	NA		NA		$\text{Э}_{M1;jj}$		NA			NA		0		
													0	
N	NA		NA		NA		$\text{Э}_{M1;NN}$							0

Conclusions

The described approach can be applied to methods of processing data of incomplete examinations, when new connections and new nodes (objects) may appear.

This is quite real in practice — the appearance of new objects, which must be quickly tied to the current general picture.

The proposed method can also be used in a wide range of decision-making tasks, including analysis and quantitative assessment of risks, security management of complex systems and objects, etc.

Thank you for your attention!